# REEXAMINATION OF THE NUCLEAR STABILITY OF $Z = 114$ TO $Z = 126$ SUPERHEAVY NUCLEI WITH THE USE OF THE DEFORMED WOODS-SAXON POTENTIAL

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Various sets of the Woods-Saxon potential parameters are extrapolated to the $Z = 114$ to 126 superheavy element region. It is shown that the behaviour of the single particle level density may indicate magic numbers different from those indicated simply by the appearance of energy gaps in the single particle spectrum. The possible readjustment of the nuclear skin thickness parameter is discussed in the case of protons. Deformation of the Woods-Saxon potential is introduced in terms of the $\beta_2$ and $\beta_4$ parameters and the resulting fission barriers are calculated together with the corresponding life-times; the relative stability of the elements with $Z$ lower than $Z = 126$ is pointed out. The computed fission life-times for the most stable nuclei vary from $3.0 \times 10^{+15}$ sec to $2.0 \times 10^{+26}$ sec, while the corresponding life-times with respect to $\alpha$-decay vary from $2.0 \times 10^{+11}$ sec to $9.0 \times 10^{+5}$ sec.

## 1. Introduction

The success of the shell correction method in calculation of nuclear masses and potential energy surfaces has shown that we are able to predict theoretically some areas of the increased nuclear stability not only as a function of both the proton number, $Z$, and the neutron number, $N$, but also as a function of the nuclear deformation. This fact supports the idea of extrapolating this powerful method to the region of hypothetic superheavy nuclei.

On the other hand several teams of researchers have undertaken experiments to find superheavy nuclei in nature or to produce them in heavy ion collisions. None of these studies did succeed until in report [1] the authors claimed that they have found superheavy nuclei corresponding to $Z = 116$ and $Z = 124$, 126, 127 in measurement of the proton induced X-rays from microscopic crystalline monazite inclusions in biotite mica. However,

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up to our knowledge, no other team succeeded in repeating these measurements; contrary, alternative interpretation of the data of Ref. [1] has been given. According to this newer interpretation the radiation observed was due to contaminants.

Despite this fact, several theoretical studies have been undertaken in order to reexamine possible physical effects which could stabilize nuclei with $Z$ around $Z = 126$; for instance, nuclear shapes other than spherical (e. g. toroidal, Ref. [2]) or possibility of the nuclear skin thickness readjustment (Ref. [3]) have been considered. The half-lives of the nuclei in the new region ($Z \sim 126$) were calculated in Ref. [3] using the modified oscillator single particle potential and also in Ref. [4] where the folded Yukawa potential has been used. However, the neglect of the important macroscopic restoring force responsible for stabilizing the neutron and proton skin thicknesses around certain average value has led the authors of Ref. [3] to the half life estimates which should be regarded as extreme upper limits [4].

The aim of the present paper is to recalculate half-lives of the superheavy nuclei with $Z$ around $Z = 126$ basing on the deformed Woods-Saxon potential with the aid of some new ideas concerning the extrapolation of the potential parameters. In particular, the slope of the effective potential well for protons deserves more careful treatment [3, 5], and the possible extrapolation starting from various sets of the Woods-Saxon potential parameters known in literature should, in our opinion, be discussed in more detail. It is also important to relate the results concerning the $Z = 126$ superheavy nuclei with the results of the previous calculations which dealt with $Z = 114$ nuclei rather than those with $Z = 126$.

The possible existence of $Z = 126$ shell closure for protons attracts our attention also because of the important symmetry $Z_m = N_m = 2, 8, 20, 28, 50, 82$ observed in real nuclei ($Z_m$ and $N_m$ denote the so-called "magic numbers" for protons and neutrons, respectively) and thus it is interesting to look for a similar symmetry for, possibly, $Z_m = N_m = 126$.

## 2. *Choice of the potential*

In most calculations concerning superheavy nuclei the realistic shell model potentials, i. e. modified oscillator, Woods-Saxon and folded Yukawa potentials have been employed. We chose the deformed Woods-Saxon potential whose parameters can be extrapolated in a rather straightforward way to a region of large proton number, $Z$, and large neutron number, $N$, nuclei.

In the case of spherical shape the Woods-Saxon potential contains usually six to eight adjusted parameters [6–8]. These are: the potential depth, $V_0$, the diffuseness parameter $a$, and the radius $R_0 = r_0 A^{1/3}$ for the central part of the potential and the three corresponding parameters of the spin-orbit term: $V_{s.o.}$, $a_{s.o.}$ and $R_{s.o.}$; sometimes, as e. g. in Ref. [8], two additional parameters are introduced, related to the Coulomb potential for protons: $R_c = r_c A^{1/3}$ and the nuclear charge diffuseness parameter $a_c$.

There is a possibility to include even more adjustable parameters by introducing a concept of the mass field and allowing to vary the charge density distribution (see e. g.

Ref. [5]); we do not adopt these concepts in the present paper following more traditional approach of Refs [6–8] and Ref. [9] (see below).

All the parameters listed above were adjusted by fitting the single particle level spectra of doubly magic nuclei to the experimental data [6–8]. Several sets of parameters obtained in this way are listed in Table I for the sake of completness. They are treated as the starting point for the present investigation.

It is worth emphasizing that the parameters $r_0$ and $r_{s.o.}$ as well as the diffuseness $a$ are, in a very good approximation, independent of the mass number $A$, Refs [8, 10], in real nuclei. The potential depth, on the other hand, depends only very weakly on $Z$ and $N$ and this dependence can be expressed in the form [11]

$$V_0 = V\left(1 \mp \kappa \frac{N-Z}{N+Z}\right), \tag{1}$$

where $\kappa = 0.67$ and $V = 51.0$ MeV for the set of Blomqvist and Wahlborn parameters [6], or $\kappa = 0.86$ and $V = 49.6$ MeV for the set of Rost parameters [7], or else $\kappa = 0.63$ and $V = 53.3$ MeV for the set of Chepurnov parameters [8]. The strength of the spin-orbit potential, $\lambda$, is almost independent of the neutron excess (see Ref. [11] and references quoted therein).

The above facts can be used as arguments for employing the Woods-Saxon potential when extrapolating its parameters to large proton and neutron number nuclei. Special attention should, however, be paid to the choice of the proton diffuseness parameter in the case of superheavy nuclei. In the case of experimentally known nuclei, the proton diffuseness parameter was usually put equal to that of neutrons, and both were independent of $A$. Consequences of such an assumption for the extrapolated spectra together with some alternative concepts will be discussed below.

### 3. *Extrapolation of the potential parameters*

We assume that some of the parameters are the same for superheavy and for the experimentally known nuclei. Among these are all the parameters: $r_0$, $r_{s.o.}$ and $r_c$ appearing in radii: $R_0 = r_0 A^{1/3}$, $R_{s.o.} = r_{s.o.} A^{1/3}$ and $R_c = r_c A^{1/3}$. The parameters are set to be equal to one of the values quoted in Table I. The same concerns also the strength $\lambda$ of the spin-orbit potential. The neutron diffuseness parameter is kept for superheavy nuclei the same as for the experimentally known nuclei (see Table I).

It is rather difficult to give a rigorous argumentation for extrapolating the potential parameters to any specific value and in particular for extrapolation of the proton diffuseness parameter, $a_p$. We doubt, however, that keeping proton and neutron diffusenesses equal and independent of $Z$ and $N$ in calculation of lifetimes of superheavy nuclei is a justified procedure. Let us list here some consequences of the assumption $a_p = a_n$ for the behaviour of the effective proton $V_{eff}^{(p)} = V_{Coul} + V_{nucl}^{(p)}$ and neutron $V_{nucl}^{(n)}$ potentials and the corresponding spectra ($V_{Coul}$ denotes the Coulomb potential).

*i*) The presence of the Coulomb part in the effective potential makes $V_{eff}^{(p)}$ steeper than the corresponding pure nuclear part. Thus, as a consequence of increase of $Z$, $V_{eff}^{(p)}$

TABLE I

Various sets of the Woods-Saxon potential parameters summarized after Refs [6–8]; the lowest two lines correspond to extrapolation applied in this paper for the calculations of half-lives of superheavy nuclei with $Z$ around 126 and $N$ around 228. The extrapolated radii and the strength of the spin-orbit term coincide with those of the Rost parametrization

| Ref. | Particles | $-V_0$ (MeV) | $\lambda$ | $r_0$ (fm) | $(r_0)_{s.o.}$ (fm) | $(r_0)_c$ (fm) | $a$ (fm) | $a_{s.o.}$ (fm) | $a_c$ (fm) |
|---|---|---|---|---|---|---|---|---|---|
| [6] | neutrons | 44.0 | 32.0 | 1.270 | 1.270 | — | 0.67 | 0.67 | — |
| | protons | 58.0 | 32.0 | 1.270 | 1.270 | 1.270 | 0.67 | 0.67 | — |
| [7] | neutrons | 40.6 | 31.5 | 1.347 | 1.280 | — | 0.70 | 0.70 | — |
| | protons | 58.7 | 17.5 | 1.275 | 0.932 | 1.275 | 0.70 | 0.70 | — |
| [8] | neutrons | 46.2 | 33.6 | 1.240 | 1.240 | — | 0.63 | 0.63 | — |
| | protons | 60.4 | 33.6 | 1.240 | 1.240 | 1.070 | 0.63 | 0.63 | 0.56 |
| (present paper) | neutrons | 37.3 | 31.5 | 1.347 | 1.280 | — | 0.70 | 0.70 | — |
| | protons | 63.4 | 17.5 | 1.275 | 0.932 | 1.275 | 0.85 | 0.85 | — |

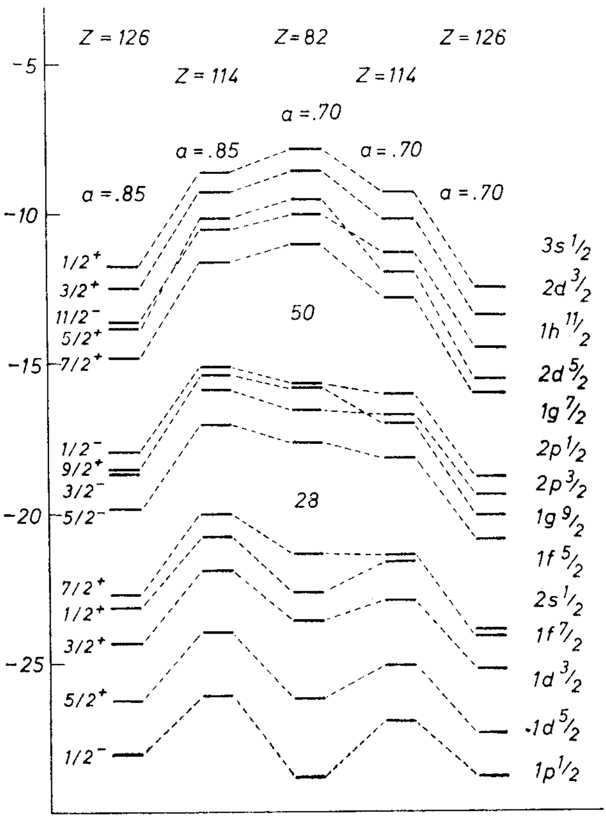


Fig. 1. Comparison of the proton single particle spectra (energy in MeV) corresponding to the Rost parametrization of the Woods-Saxon potential with and without increase of the diffuseness parameter, $a$. The depth of the potential well was calculated from formula (1). $Z = 82$ corresponds to $N = 126$, $Z = 114$ to $N = 164$ and $Z = 126$ to $N = 228$

becomes more and more square-well-like and the difference in bunching between the proton and neutron spectra gets more and more pronounced. If we assume, keeping in mind that the proton and neutron magic numbers are the same for the light and medium heavy nuclei (up to $Z_m = N_m = 82$), that $N_m = 126$ should have an analog, $Z_m = 126$, then the most natural way to get it in the calculations is to increase the proton potential diffuseness as in such a case the corresponding $V_{eff}^{(p)}$ becomes less steep (see also illustration of the single particle spectra in Figs 3–6). This does not necessarily mean that the presence of other, even well pronounced gaps, is excluded.

*ii*) The assumption $a_p = a_n$ has an interesting consequence connected immediately with the behaviour of the single particle levels as it is illustrated in Fig. 1 where the low-lying parts of the spectra are shown. Note that the low-lying levels calculated with $a_p = a_n = 0.70$ fm vary stronger with an increase of $A$ (crossing of levels, stronger changes in the relative positions of the individual levels) than the levels corresponding to increased $a_p$. Deciding to increase $a_p$ we also less influence the spatial behaviour of the effective potential and in particular, the potential slope, when increasing $Z$ and $A$. We can interpret this fact by saying that the nucleons residing in the deeper nuclear interior are less affected by adding new and new particles into the nucleus.

*iii*) Increase of the proton diffuseness tends to increase the level bunching which in $^{208}$Pb caused improvement of the agreement between calculated and experimental *gross structure* properties of the spectra, Ref. [5]; according to these results, the best effect is

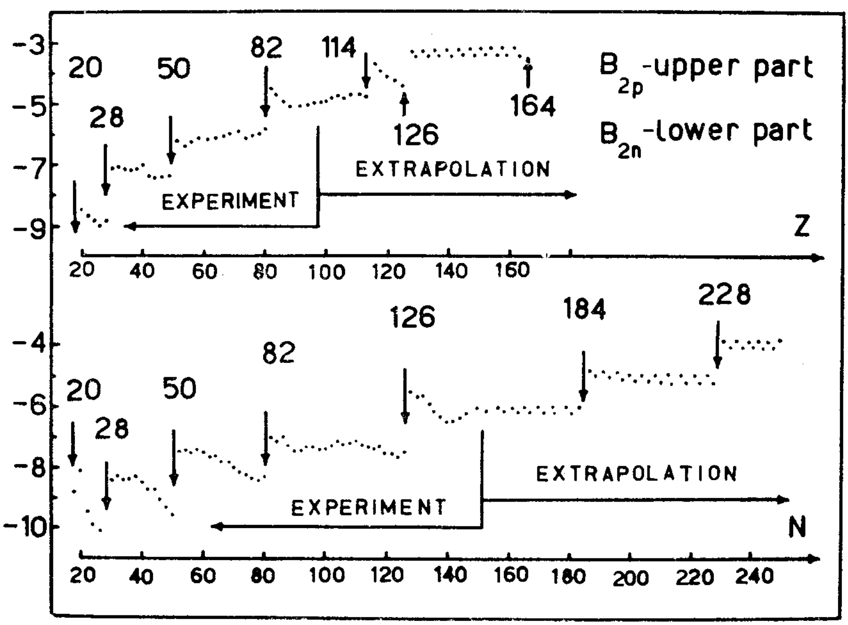


Fig. 2. The mean separation energies, in MeV, for protons (upper part) and for neutrons (lower part).The step-like behaviour of both curves should be noted; the height of the step being of the order of 1.2 MeV in both cases. The right-hand parts of the figures illustrate exrtapolated values of the mean separation energies, and are used for extrapolating the potential depth

achieved for $a_p$ increased by about 15% to 25% with respect to its standard value 0.66 fm. Assuming that the gross structure properties are the most important for calculating shell correction effects one can take 20% increase of $a_p$ standard value as reasonable.

Finally, we assumed a change in parameter $a_p$ which is approximately 20% of the value given in Table I (e. g. in the case of the Rost parameters $a_p$ is increased to 0.85 fm).

The procedure employed for extrapolating the potential depth is illustrated in Fig. 2. Here the experimental mean separation energies ($B(Z, N)$ denotes nuclear binding energy)

$$B_{2p}(Z, N) = \frac{1}{2}[B(Z+1, N)-B(Z-1, N)], \tag{2a}$$

$$B_{2n}(Z, N) = \frac{1}{2}[B(Z, N+1)-B(Z, N-1)], \tag{2b}$$

are plotted for protons and neutrons, respectively, and the characteristic jumps corresponding to the known magic numbers and the average step-like behaviour of these curves deserve noting. We assume that the jumps resembling very much those marked in the figure with arrows and corresponding to normal nuclei would take place also for the hypothetic superheavy magic nuclei. This assumption allowes us to fix the approximate position of the Fermi level for superheavy nuclei and choose the potential depth in such a way that the resulting theoretical Fermi level coincides with the extrapolated experimental one.

## 4. *Results*

First we should like to compare the single particle level spectra corresponding to the differently chosen potential parameters.

Figure 3 shows the proton single particle levels computed using: (a) the Rost parameters, (b) the Blomqvist and Wahlborn parameters, (c) the Chepurnov parameters, (d) the results concerning folded Yukawa potential, taken from Ref. [12]; the last column shows the experimental results. Figure 4 shows the analogous results for neutrons. Both figures contain a part of positive energy spectrum (resonances) in order to reveal possible candidates for the new magic numbers. It can immediately be seen from the figure that $N_m = 184$ and $Z_m = 114$ together with $Z_m = 126$ are candidates for magic numbers.

In the next step we calculate the single particle spectrum for $A$ corresponding to the expected superheavy nucleus; anticipating the results of calculations of $\beta$ and $\alpha$-decay probabilities, we start with ${}_{126}X_{228}$ rather than ${}_{126}X_{184}$. The corresponding results are shown in Figs 5 and 6 for protons and neutrons, respectively. The levels in Fig. 5 are grouped in the following way: columns (a) and (b) correspond to Rost parameters, (c) and (d) to Blomqvist and Wahlborn parameters, and (e) and (f) to Chepurnov parameters; the right column in each group contains the results corresponding to the increased proton diffuseness parameter in terms of the arguments given in the previous section. It is easy to notice that for both $Z_m = 114$ and $Z_m = 164$ the magic numbers seem to be very well established while the $Z_m = 126$ energy gap is sensitive to the choice of the proton diffuseness parameter.

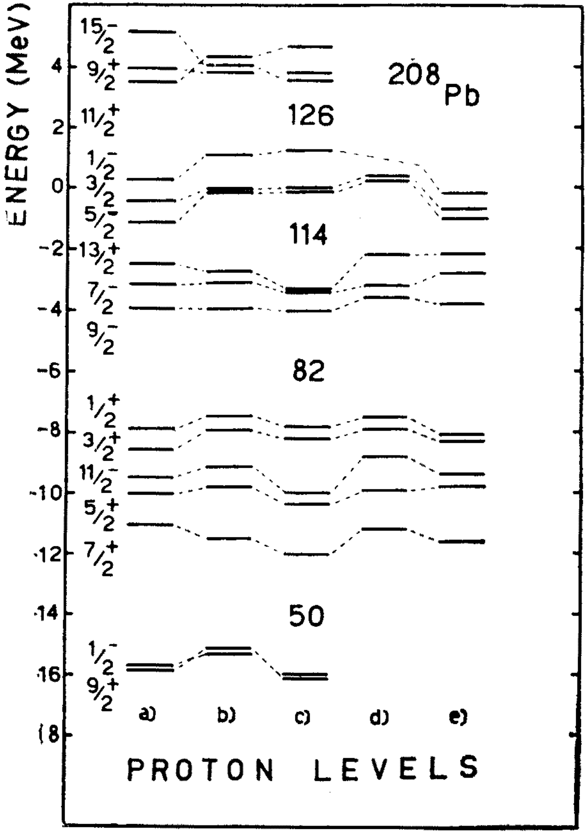


Fig. 3

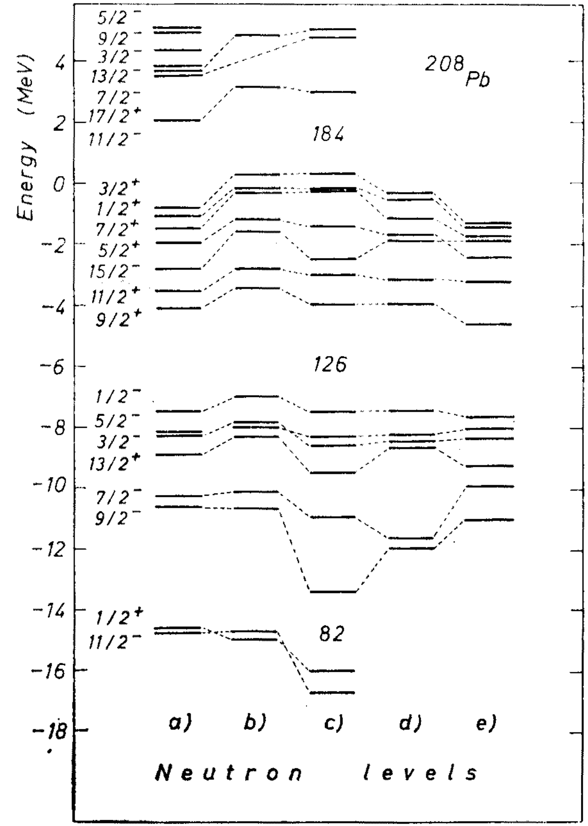


Fig. 4

Fig. 3. The single particle proton spectrum for $^{208}$Pb nucleus: (a) Rost parameters, (b) Blomqvist and Wahlborn parameters, (c) Chepurnov parameters, (d) folded Yukawa potential of Ref. [17], (e) experimental results. The appearance of magic structure is marked by the numbers $Z = 50$, 82, 114 and clearly 126. It should be noted that the last number corresponds to the positive energy part of the spectrum

Fig. 4. The same as in Fig. 3 but for neutrons

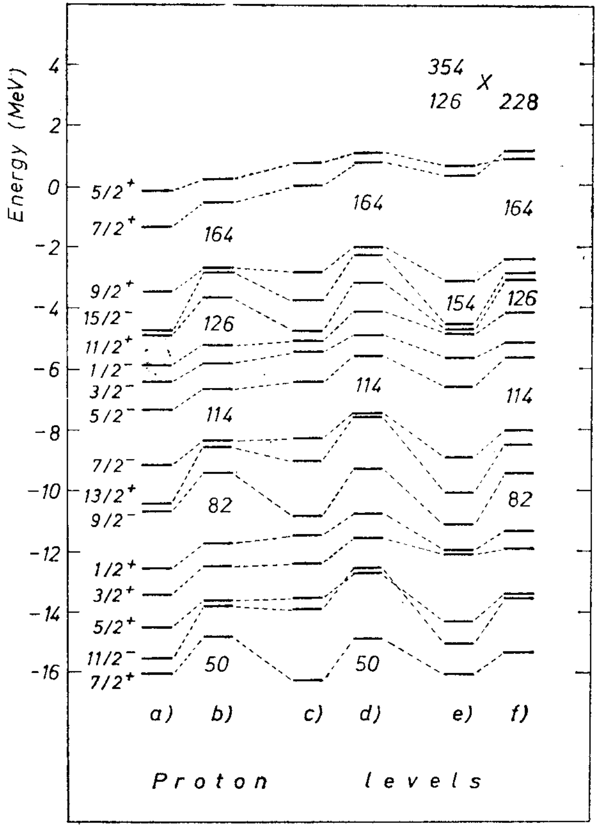


Fig. 5

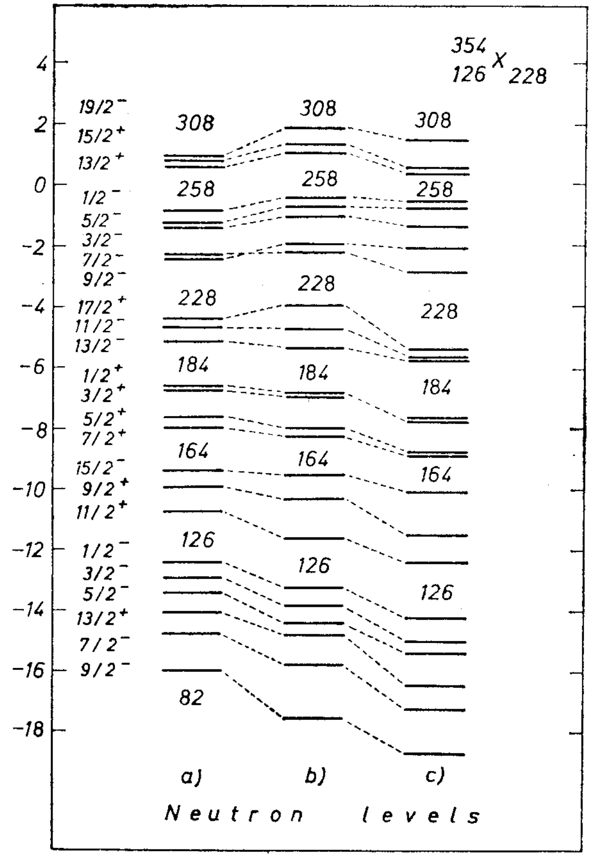


Fig. 6

Fig. 5. Proton single particle levels with the extrapolated Woods-Saxon potential parameters for the $^{354}_{126}X_{228}$ nucleus. Columns (a), (c) and (e) correspond to the proton diffusenesses quoted in Table I after Refs [6–8], while columns (b), (d) and (f) those with the diffuseness parameter increased about 20%, as discussed in the text. The results in (a) and (b) correspond to the extrapolated Rost parameters, (c) and (d) to the extrapolated Blomqvist and Wahlborn parameters, and (e) and (f) to the extrapolated Chepurnov parameters

Fig. 6. Neutron single particle levels with the extrapolated Woods-Saxon potential parameters according to the references quoted in caption to Fig. 4

We should like to emphasize, however, that the analysis of the single particle level *density* in addition to analysis of the spectra of Figs 3–6 shows a characteristic decrease of the average number of levels per 1 MeV just for $Z_m = 126$, or, as in the case of Chepurnov parameters a wide minimum corresponding to proton numbers from $Z = 114$ to $Z = 126$ (see Fig. 7). This characteristic behaviour follows from the fact that $Z = 114$

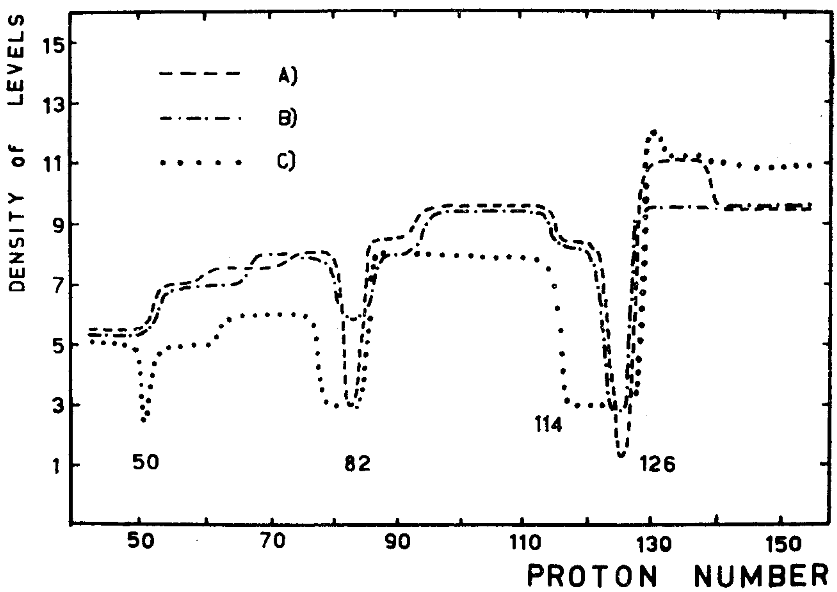


Fig. 7. The single particle level density $\varrho = \Delta n/\Delta E$ for protons with extrapolated Woods-Saxon potential parameters of Refs [6–8] for the cases (A), (B) and (C), respectively with proton diffuseness increased. The energy window $\Delta E$ is put equal to the average shell spacing, roughly about $41/A^{1/3}$ MeV; $\Delta n$ is the number of levels appearing in the energy window. The characteristic decrease of the density for $Z = 114$ to $Z = 126$ proton numbers which indicates the strongest stability of the corresponding nuclei due to the shell correction effect deserves noting

and $Z = 126$ gaps are separated by low degeneracy levels $2f_{5/2}$, $3p_{3/2}$ and $3p_{1/2}$ while in the neighbourhood highly degenerated levels occur e. g. $1i_{13/2}$, $1i_{11/2}$, $1j_{15/2}$ or $2g_{9/2}$. Thus not only $Z = 114$ but also, owing to the characteristic behaviour of the single particle level density, $Z = 126$ and some neighbouring nuclei are candidates for more stable elements in this region.

In order to illustrate the consequences of this fact we plot the proton shell correction obtained in terms of the Strutinsky method as a function of the particle number $Z$, Fig. 8, ($a_p$ increased). The results presented here are fully consistent with those in Fig. 7 and argue once again for the existence of a relatively big island of nuclei with increased stability.

In order to calculate fission barriers we introduce the potential deformation expressing the nuclear surface in terms of the spherical harmonics $Y_{20}$ and $Y_{40}$

$$R(\Theta) = c(\beta_2, \beta_4)R_0[1+\beta_2 Y_{20}+\beta_4 Y_{40}], \tag{3}$$

where $R(\Theta)$ is the distance of a point on the nuclear surface from the origin of the coordinate system. Constant $c(\beta_2, \beta_4)$ is calculated taking account of the requirement that the volume of the nucleus is independent of deformation.

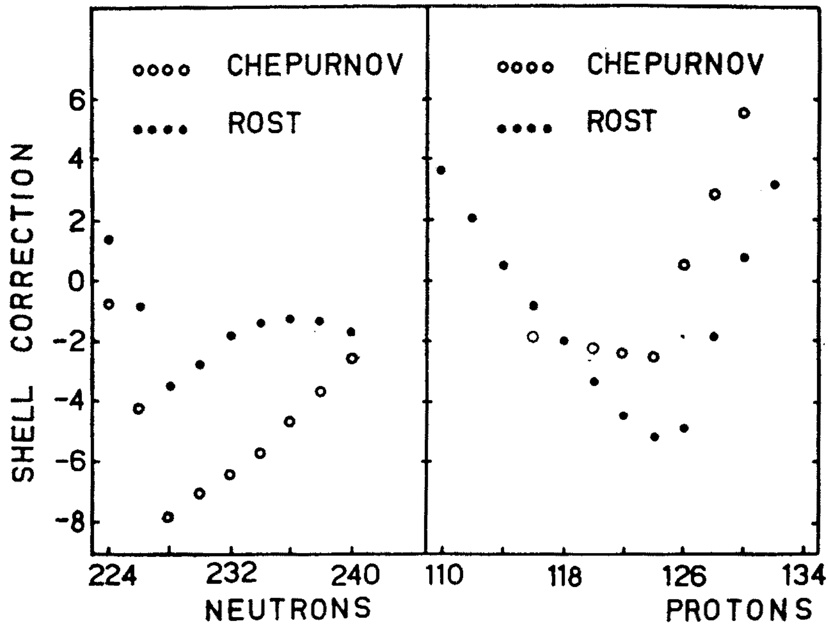


Fig. 8. The total single particle (shell+pairing) correction to the liquid drop model energy for neutrons and protons in the vicinity of $N = 228$ and $Z = 126$ magic numbers with the two alternative sets of parameters marked in the figure (compare with Fig. 7). Proton diffuseness increased

The potential well was formed in a similar way as in Ref. [9]. First Eq. (3) is transformed into cylindrical coordinate system (axial symmetry is assumed) and the equation of the nuclear surface is obtained in the form

$$\pi(u, v) \stackrel{\text{df}}{=} u^2+v^2-[1+\beta_2 Y_{20}(\cos \Theta_{u,v})+\beta_4 Y_{40}(\cos \Theta_{u,v})] = 0 \tag{4}$$

with

$$u \stackrel{\text{df}}{=} \frac{z}{c(\beta_2, \beta_4)}, \quad v \stackrel{\text{df}}{=} \frac{\sqrt{x^2+y^2}}{c(\beta_2, \beta_4)}$$

and

$$\cos \Theta_{u,v} = \frac{u}{\sqrt{u^2+v^2}}.$$

Then we introduce the auxiliary function [9]

$$\mathscr{M}(u, v; \beta_2, \beta_4) = \sqrt{\pi(u, v)-\pi^*} -\sqrt{-\pi^*} \tag{5}$$

where $\pi^* = \pi^*(\beta_2, \beta_4)$ is by definition a function equal to the minimum of $\pi$ over the variables $u$ and $v$; function $\mathcal{M}(u, v; \beta_2, \beta_4)$ of Eq. (5) can be used for constructing the potential well

$$V(\varrho, z; \beta_2, \beta_4) \Leftrightarrow V(u, v; \beta_2, \beta_4) = \frac{V_0}{1+\exp\left[l(u, v; \beta_2, \beta_4)/a\right]}, \tag{6}$$

$$\varrho \overset{\mathrm{df}}{=} \sqrt{x^2+y^2}$$

with

$$l(u, v; \beta_2, \beta_4) = \frac{\mathcal{M}(u, v; \beta_2, \beta_4)}{|\nabla \mathcal{M}(u, v; \beta_2, \beta_4)|}. \tag{7}$$

This parametrization of the deformed Woods-Saxon potential having been extensively examined in a series of papers [9], we cease discussing here in further details, wishing only

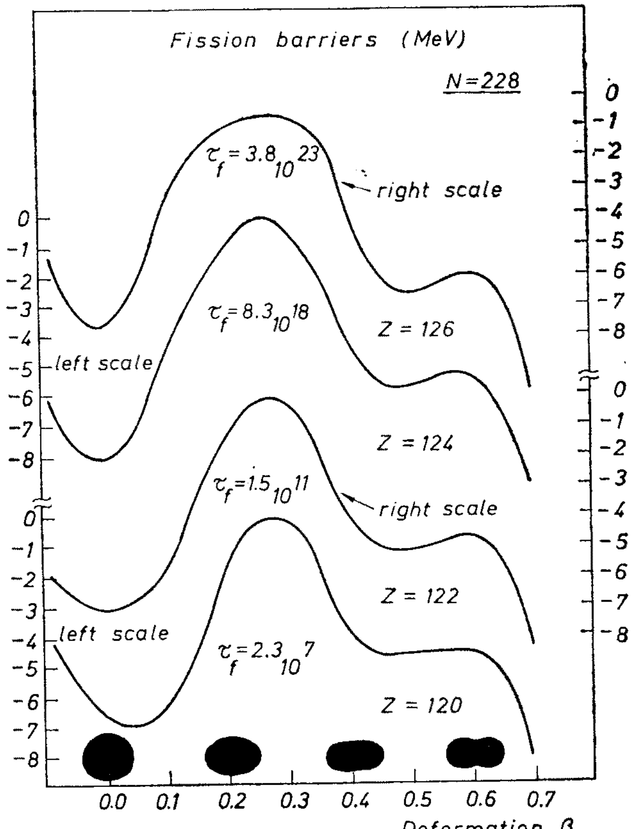


Fig. 9. The fission barriers calculated using the deformed Woods-Saxon potential with the parameters extrapolated according to the discussion in the text (proton diffuseness increased). The corresponding fission half-lives $T_f$ are marked in the figure. The barriers are minimized with respect to $\beta_4$ deformation, but the path to fission obtained in this way is very close to the one with $\beta_4 = 0$. Lifetimes, given in seconds, were calculated using simple one-dimensional WKB formula for the probability of the barrier penetration

to remind that due to its construction, function $l$ can be interpreted as the distance from the nuclear surface $\pi(u, v; \beta_2, \beta_4) = 0$ measured along the direction normal to this surface, provided function $\pi$ is smooth enough and $(u, v)$ are coordinates of a point not very far from the surface.

Strictly speaking, our approach differs slightly from that of Ref. [9] since we take

$$L(u, v; \beta_2, \beta_4) = l(u, v; \beta_2, \beta_4) + h(u, v; \beta_2, \beta_4) \tag{8}$$

instead of function $l$; the new function $h(u, v; \beta_2, \beta_4)$ differs from zero only in a small region of the nuclear interior ensuring that the bottom of the nuclear potential is flat like in the usual Woods-Saxon case.

Finally, the Schrödinger equation with the deformed potential is solved numerically by the diagonalization method using the anisotropic oscillator basis.

The collective potential energy surface were calculated by making use of the Strutinsky shell correction method [13] with the macroscopic part of the total energy formula taken in a form of the usual *liquid drop model* with parameters of Ref. [14]. The pairing effects were accounted for by employing the pairing correction in a way described in details by Bolsterli et al. [12]. The final values of the extrapolated parameters of the Woods-Saxon potential are shown in Table I.

The resulting fission barriers for the most stable nuclei studied in this paper are given in Fig. 9. The fission and $\alpha$-decay lifetimes are shown in Table II. For the $\alpha$-decay probabil-

TABLE II

Superheavy nuclei with $N = 228$ and the corresponding theoretical fission, $T_f$, and $\alpha$-decay, $T_\alpha(1)$ and $T_\alpha(2)$, half-lives in seconds. The quantities $T_\alpha(1)$ and $T_\alpha(2)$ correspond to formulas of Refs [15] and [16], respectively. The frequency $\omega_{vib}$ defines the number of assaults (in unit time) of the nucleus into the fission barrier; $\hbar\omega_{vib} = \sqrt{C/B}$, where the stiffness parameter $C = \frac{1}{2}\frac{\partial^2 V}{\partial \beta_2^2}$ was calculated using the Strutinsky method. The mass parameter, $B$, was estimated like in Ref. [18] and was proved to be close, on average, to the microscopic calculation results of cranking model. Actually the constant mass parameter $B = 0.054\, A^{5/3}\, \hbar^2\mathrm{MeV}^{-1}$ was used in our estimates of $\omega_{vib}$ and fission half lives (see Ref. [18] and references quoted therein). Although change of the Nilsson parameters $(\varepsilon_2, \varepsilon_4)$ into $(\beta_2, \beta_4)$ modifies $B$ of Ref. [18] within about 15% in the deformation range considered, nevertheless the basic uncertainty of $B$, being perhaps much larger, does not influence the $\alpha$-decay half-lives which seem to be most decisive for the decay probabilities of $\beta$-stable nuclei around $Z = 126$. The values $\hbar\omega_{vib}$ and $Q_\alpha$ are in MeV, $B$ in $\hbar^2$ MeV$^{-1}$

| $Z$ | $A$ | $\hbar\omega_{vib}$ | $B$ | $T_f$ | $Q_\alpha$ | $T_\alpha(1)$ | $T_\alpha(2)$ |
|---|---|---|---|---|---|---|---|
| 120 | 348 | ~0.1 | 928 | $3.0\times10^{14}$ | 5.21 | $1.7\times10^{11}$ | $5.9\times10^{6}$ |
| 122 | 350 | 0.175 | 939 | $5.0\times10^{18}$ | 5.90 | $2.0\times10^{9}$ | $9.5\times10^{4}$ |
| 124 | 352 | 0.297 | 948 | $2.0\times10^{25}$ | 6.69 | $2.5\times10^{7}$ | $1.7\times10^{3}$ |
| 126 | 354 | 0.337 | 957 | $5.8\times10^{23}$ | 7.40 | $9.6\times10^{5}$ | $8.6\times10^{1}$ |

ities we use two alternative formulae; the one derived by Viola and Seaborg [15]

$$\log T_\alpha = (2.11329Z-48.9879)Q_\alpha^{-1/2}+(-0.39004Z-16.9543) \tag{9}$$

and that of Taagapera and Nurmia [16]

$$\log T_\alpha = 1.61\left[\frac{Z-2}{\sqrt{Q_\alpha}}-(Z-2)^{2/3}\right]-28.9, \tag{10}$$

where $Q_\alpha$ denotes the $\alpha$-decay energy and $Z$ is the proton number of the parent nucleus.

From Table II we can immediately see that our results are much less optimistic than those of Ref. [3], in spite of the fact that the simple analysis of the single particle spectrum indicates a certain increase of stability around $Z = 122$ to $Z = 126$ nuclei. The estimated half-lifes do not exceed $10^4$ years in the most favourable case, even without taking into account the possible competition of $\beta$ decay (for the estimates of the $\beta$-decay half-lifes see Ref. [4] noting that results concerning ground state masses are similar for deformed Woods-Saxon and folded Yukawa potentials).

## 5. *Concluding remarks*

We have applied the most favourable, but in our opinion still reasonable extrapolation of the parameters of the Woods-Saxon potential to the region of superheavy nuclei around $Z = 126$. Although the increase of the proton diffuseness parameter (relative to the standard or to the corresponding neutron values) seems to be physically reasonable in a sense of the discussion in Section 3, and in spite of the fact that it favours the $Z = 126$ magic number, no effect strong enough in fission and $\alpha$-decay lifetimes is produced, and consequently the total lifetimes are predicted much shorter than the age of the Earth. Also our assumption about the pairing force (we put the strength constant proportional to the nuclear surface area which overestimates the pairing energy gaps and thus most probably decreases the fission barriers) does not change our pessimistic conclusions since the $\alpha$-decay probabilities are not influenced in this way (cf. Table II).

The second aspect of the problem also deserves emphasis; the single particle spectrum of the Woods-Saxon potential is rather a sensitive function of the nuclear size for large $A$ (cf. Figs 3–6). As a consequence, the single particle level *density* may be substantially changed when $Z$ and $N$ are decreased from $Z = 126$ and $N = 228$ to the lower proton and neutron magic numbers, and again the nuclei with $Z$ around $Z = 114$ might appear relatively stable although the effect seems to be sensitive to extrapolation of the diffuseness parameters.

The uncertainties in half-lives quoted in Table II are mainly due to uncertainties of $Q_\alpha$ and $B$ values, however the estimates of $\alpha$-decay probabilities (formulae (9) and (10)) and one-dimensional treatment of the barrier penetration also introduce certain errors. For instance, 1 MeV change in $Q_\alpha$ may cause change of the corresponding $\alpha$-decay half-lives even as much as about five orders of magnitude whereas 20% variation of $B$ leads to a variation of the fission lifetimes within a factor of $10^{\pm 4}$. Consequently, our calculations based on the Woods-Saxon scheme do not support hypothesis of a possible existence of superheavy nuclei with $Z = 126$ in natural sources on the Earth (cf. Ref. [4] where the slightly different microscopic approach was employed).

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