ODD-EVEN EFFECT IN THE NUCLEAR SHELL-MODEL FOR NUCLEI WITH $N = 28$ AND $N = 50$

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(Received April 7, 1978)

The effective interaction energies were deduced from the experimental data for single neighbouring nuclei with $N = 28$ and $N = 50$. Odd-even effect, which was found, manifests itself by the dependence of the effective two-particle interaction energy on the number of particles which are filling the same orbit. The results were compared with the predictions obtained for δ-function interaction with a varied value of spin exchange term. Comparison with the experiment is given.

1. Introduction

It is well known [1, 2] that nuclei with $N = 28$ and $N = 50$ are well described by the nuclear shell-model in which the ^{48}Ca and ^{88}Sr can be treated as cores. Using the phenomenological approach, in which the number of matrix-elements can be reduced by considering only the lowest dominant configurations, one can investigate the two-body interactions defined by $V_J = \langle j_1 j_2 J|V|j_1 j_2 J\rangle$. These interactions are treated then as parameters to fit the experimental data. Any many-body matrix-element can be expressed as the linear combination of V_J [3]. For any state described by a pure configuration and energy E_I one gets an equation of the type

$$\sum a_J V_J = f(E_I, \text{DNBE}, E^C), \tag{1}$$

where a_J are coefficients, DNBE denotes difference of the nuclear binding energy [4] and E^C denotes the Coulomb energy of the valence particles. If energy states are described by the mixed configurations, the equation (1) should be modified a little. For a large number of states belonging to several nuclei, one can obtain effective interactions which should reproduce positions of all the model-levels within the region in question. Results obtained from the single neighbouring nuclei [4] show the odd-even effect in the $f_{7/2}$ region. A similar effect was observed in the previous work [5], where V_0 and V_2 were different for ^{90}Zr and ^{91}Nb. The aim of this work is to show the odd-even effect in the

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$p_{1/2}\, g_{9/2}$ region (^{90}Zr [6], ^{91}Nb [7], ^{92}Mo [8]). Results of the calculations presented here are more exact and more complete than in [5]. The odd-even effect is discussed. The δ-function interaction with a small spin-exchange term can be taken as a zero-order approximation of the two-particle interaction. Values predicted by this interaction are compared with our results in Section 3. Comparison with the experiment is given in Section 4.

2. *Derivation of the effective two-body interactions*

Notation, sign convention and method of obtaining the effective two-body interactions from experimental data for the simple case of $f_{7/2}$ orbit only, were presented in work [4]. In that case the set of equations of type (1) allows one to calculate directly the two-body interactions. When two or more orbits are included into considerations, the method is the following. Experimental energies corrected for DNBE's and single-particle energies (or E^C also) should be equal to the eigen values of many-body interaction matrix. The method of calculation requires that the energy of any model-state should be expressed as a combination of the parameters V_l (V_l denotes the necessary two-body matrix-element; the number of the parameters V_l depends on the configuration space). If the configuration space is restricted to the $2p_{1/2}$ and $1g_{9/2}$ orbits, one needs 9 two-body parameters and 2 single particle energies. These parameters for proton-proton interaction are

$$\langle g_{9/2}^2 J|V|g_{9/2}^2 J\rangle = V_J \quad \text{with} \quad J = 0, 2, 4, 6, 8,$$

$$\langle p_{1/2}^2 J = 0|V|p_{1/2}^2 J = 0\rangle = V_{pp}, \quad \langle p_{1/2}^2 J = 0|V|g_{9/2}^2 J = 0\rangle = V_{pg},$$

$$\langle p_{1/2} g_{9/2} J = 4|V|p_{1/2} g_{9/2} J = 4\rangle = V_{4M},$$

$$\langle p_{1/2} g_{9/2} J = 5|V|p_{1/2} g_{9/2} J = 5\rangle = V_{5M}. \tag{2}$$

The single proton energies $\mathscr{E}_p$ and $\mathscr{E}_g$, treated as constants, were calculated from the energies of $1/2^-$ and $9/2^+$ states of ^{89}Y [9] yielding the values

$$\mathscr{E}_g = -(6149 \pm 14)\ \text{keV}, \quad \mathscr{E}_p = -(7063 \pm 13)\ \text{keV}. \tag{3}$$

The method requires these two values to be constant because fits were made separately for different $N = 50$ isotones. Keeping in mind the last remark, we express any many-body matrix-element for mixed configurations as a sum of two matrices

$$\langle \psi_k|V|\psi_j\rangle \equiv H_{kj}^M + D_{kj} = H_{kj}, \tag{4}$$

where

$$H_{kj}^M = \sum_{l=1}^{9} \alpha_{lkj} V_l, \tag{5}$$

and D_{kj} depends on single particle energies. Starting from the equation for the eigen value problem we can write down

$$HX = EX, \tag{6}$$

$$\sum_{j=1}^{N} H_{kj} A_{jk} = E_k A_{kk}, \tag{7}$$

where the matrix A is built up from the eigen vectors as columns and N denotes the order of the interaction matrix (for specific spin value). After substituting Eq. (4) to Eq. (7) one gets

$$E_k = \sum_{l=1}^{9} C_{kl} V_l + C_k^D, \tag{8}$$

where

$$C_{kl} = \sum_{j=1}^{N} \alpha_{lkj} A_{jk} / A_{kk}, \tag{9}$$

and

$$C_k^D = \sum_{j=1}^{N} D_{kj} A_{jk} / A_{kk}. \tag{10}$$

In our case the matrix D_{kj} contains only zeros with the exception of the element $D_{NN} = -2\delta = 2(\mathscr{E}_g - \mathscr{E}_p)$ which corresponds to the configuration $\pi p_{1/2}^2 g_{9/2}^{n-2}$. In Eq. (8) E_k should be corrected for DNBE and single particle energies (or E^C also).

As an example for $9/2^+$ ground state in ^{91}Nb we have

$$E_k = (E_{gs} = 0) - C_{3g}, \tag{11}$$

where $C_{3g} = \text{DNBE(Nb, Sr)} + 3\mathscr{E}_g = (2128 \pm 28)$ keV. The energy of any odd-parity state in ^{91}Nb arising from the $p_{1/2}\, g_{9/2}^{n-1}$ configuration can be expressed as [10]

$$E_I = \text{DNBE} + \mathscr{E}_p + (n-1)\mathscr{E}_g + \langle p_{1/2} g_{9/2}^{n-1} I | V | p_{1/2} g_{9/2}^{n-1} I \rangle. \tag{12}$$

Finally, we have a set of equations of the type

$$E_I = f(V_l, \text{DNBE}, \mathscr{E}_p, \mathscr{E}_g, A) \tag{13}$$

which can be solved by the method proposed by Glaudemans et al. [11]. This method requires the knowledge of the initial values of the parameters V_l, which can be taken e.g. from Ref. [12]. The initial values of V_l serve to construct the interaction matrix. After diagonalizing procedure one gets sets of eigen vectors (necessary to construct an equation of the type (13)) and eigen values, which can be compared with the experimental energies. If the required agreement is not achieved, these values are treated as starting parameters for the next step of iteration. Two computer programs NIOB and MOLB were written to solve this problem for ^{91}Nb and ^{92}Mo.

The fitting procedure described above gives a set of parameters, the final values of which depend on the set of levels taken into account. It should be mentioned at this point that if we take into consideration different levels of some nucleus, the final parameters do not change their values significantly [4]. The fluctuations are much smaller than those observed in the case when the parameters were obtained from levels belonging to different nuclei. For more complicated level scheme, it was not obvious which level should be included into analysis. In the first step all model-levels were taken into account. For such a set we obtained a RMS deviation per level, which was often large. RMS we define as

$$\text{RMS} = \sqrt{\frac{(E_{\text{cal}} - E_{\text{exp}})^2}{n-k}}, \tag{14}$$

where n denotes the number of levels and k the number of parameters. In the next step we skipped some levels. The smaller RMS value obtained in this step attests that the missed level has more complicated structure or its interpretation was wrong.

3. *Results of calculation and discussion*

The final values of the obtained parameters are presented in Tables I and II. The mean interaction energies defined by

$$\bar{V}(2) = \sum_{J\ \mathrm{even}} (2J+1)V_J / \sum_{J\ \mathrm{even}} (2J+1) \tag{15}$$

are also given in Tables I and II. A graphical representation of the parameters is shown in Fig. 1.

TABLE I

Effective two-proton interaction energies for $f_{7/2}$ orbit (in keV). Only statistical errors are indicated. $\bar{V}(2)$ is defined as a mean interaction energy for states with $T = 1$. $\bar{E}(\delta, \alpha)$ are predictions for δ-forces with spin-exchange term. For comparison between the $\bar{E}(\delta, \alpha)$ and others see text

	^{50}Ti	^{51}V	^{52}Cr	^{53}Mn	^{54}Fe	$E(\delta, \alpha)$
V_0	-2934 ± 26	-2493 ± 49	-2618 ± 34	-2288 ± 72	-2484 ± 65	-2335
V_2	-1379 ± 27	-1085 ± 20	-1209 ± 31	-965 ± 29	-1075 ± 57	-556
V_4	-257 ± 28	-53 ± 20	-34 ± 32	-53 ± 28	55 ± 49	-273
V_6	268 ± 28	313 ± 16	482 ± 30	428 ± 22	467 ± 38	-136
$\bar{V}(2)$	-309	-155	-97	-72	-46	-336

TABLE II

Effective two-proton interaction energies for $p_{1/2}\, g_{9/2}$ configuration space (in keV). See comment to Table I

	^{90}Zr	^{91}Nb	^{92}Mo	$E(\delta, \alpha)$	SIG [13]
V_0	-2100 ± 150	-1772 ± 30	-1865 ± 37	-1772	-1705
V_2	-939 ± 24	-661 ± 13	-688 ± 24	-430	-616
V_4	-48 ± 24	53 ± 15	48 ± 23	-223	155
V_6	323 ± 24	300 ± 15	400 ± 26	-132	437
V_8	464 ± 24	479 ± 9	538 ± 30	-71	570
$\bar{V}(2)$–327	-219	-162	-116	-197	-61
V_{pp}	-561 ± 150	-495 ± 46	-802 ± 128		-544
V_{pg}	868 ± 50	827 ± 69	832 ± 100		853
V_{4M}	528 ± 23	525 ± 10	887 ± 96		716
V_{5M}	108 ± 23	173 ± 10	68 ± 36		194

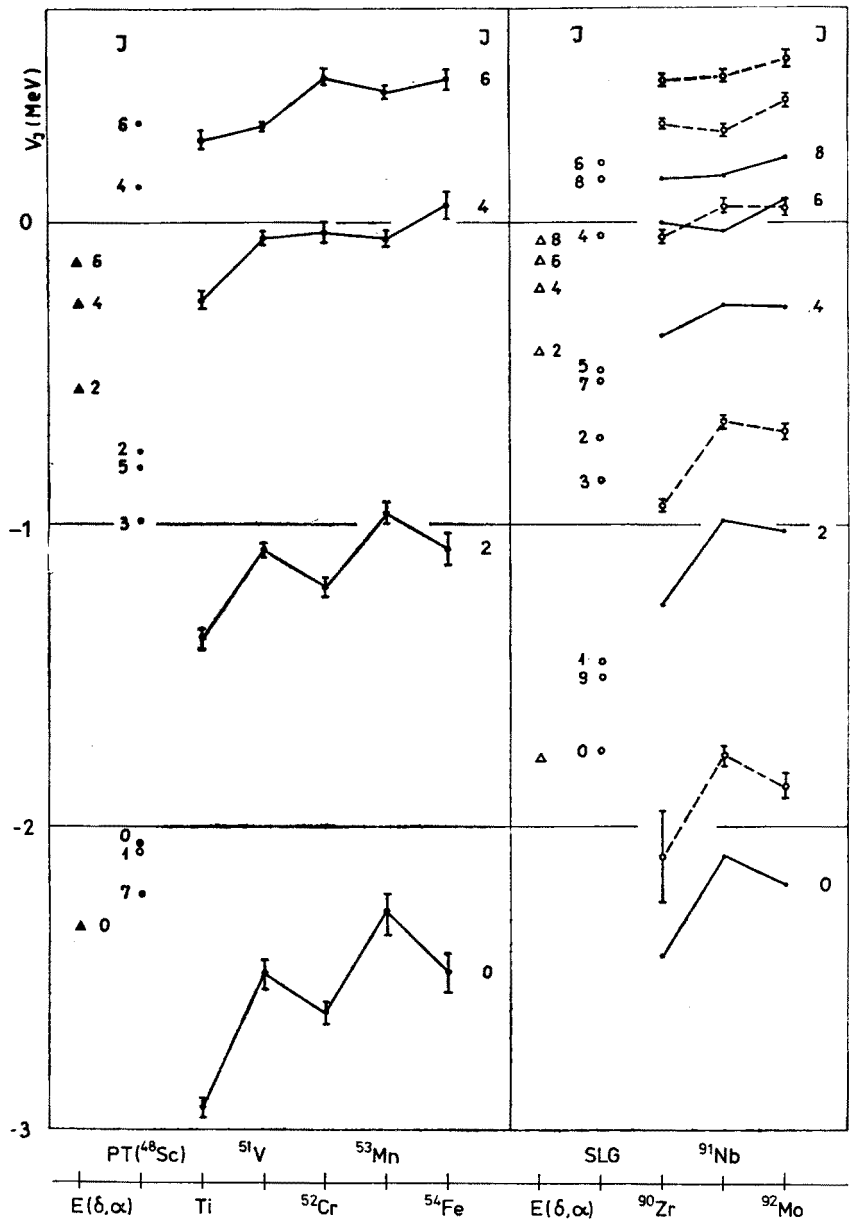

Fig. 1. Effective two-body interaction energies for $f_{7/2}$ and $g_{9/2}$ orbits. Only statistical errors are denoted. The shift of parameters for $g_{9/2}$ is explained in text. Data for neutron-proton interactions obtained for the $f_{7/2}$ orbit (PT(^{48}Sc)) from ^{48}Sc using Pandya transformation, and for $g_{9/2}$ orbit (SLG) taken from literature (Ref. [13]) are included

Results show the odd-even effect which is prominent, especially for V_0 and V_2. When the Coulomb energy is not taken into account the parameters are shifted towards positive values (less attractive interactions). To obtain nuclear effective parameters which can be compared with the δ-force interactions we should subtract a constant C_g from the values presented in Table II. The constant C_g is defined as follows

$$C_g = \langle g^2(\pi)|V|g^2(\pi)\rangle_J - \langle g(\pi)g(\nu)|V|g(\pi)g(\nu)\rangle_J, \tag{16}$$

with $J = 0, 2, 4, 6$ and 8, where π and ν denote proton and neutron, respectively. It has been shown by Gross and Frenkel that C_g equals 327 keV [12]. Figure 1 also contains values of neutron-proton interactions. These values were obtained for the $f_{7/2}$ orbit from ^{48}Sc using the Pandya transformation and those for $g_{9/2}$ orbit were taken from Ref. [13].

For V_6 and V_8 i.e. for the case when the interaction is repulsive, it seems that the effect changes the sign. One can say that the odd-even effect manifests itself in such a way that the interaction energy of the two particles depends on the number of particles which additionally are filling up the same orbit. It does not mean that real interactions should have the same behaviour. The effect accounts for the assumptions made in the calculations. We do not consider the origin of this effect here, but only comment on it in relation to the works published earlier. It was noticed [14] that values of $1f_{7/2}$ matrix-elements depend on dimensions of configuration space. This effect is presented in Table III. "The bigger

TABLE III

Matrix elements $\langle f_{7/2}^2|V|f_{7/2}^2\rangle_J$ with $1f_{7/2}$ and $2p_{3/2}$ orbits from Ref. [14]

J	Pure $f_{7/2}$ shell [14]	$f_{7/2}$ and some $p_{3/2}$ nucleons [15]	All $f_{7/2}$-$p_{3/2}$ configuration [16]
0	−3110	−2800	−2110
2	−1520	−1290	−1110
4	−360	−170	−100
6	80	340	230

the shell model space the smaller the matrix elements (in absolute value) attributed to $1f_{7/2}^2$ states". This comment together with the odd-even effect allows one to conclude that an enlargement of configuration space is more important for even nuclei like ^{50}Ti, ^{52}Cr and ^{54}Fe than for odd nuclei like ^{51}V and ^{53}Mn. The differences between our parameters and the parameters obtained for the whole $f_{7/2}-p_{3/2}$ configuration space are smaller for odd isotones than for even ones.

A second comment concerns the work of Eisenstein and Kirson [17]. It has been shown that energy levels in $f_{7/2}$ region can be well reproduced by using a three-body interaction in addition to the two-body one. These calculations as well as ours were performed within the pure $f_{7/2}$ configuration space. Maybe, the odd-even effect reflects some contributions of the three-body interaction, if it was necessary to take them into account. It is well known that δ-force interaction can be taken as a zero order approximation of the two-body interaction. Such an interaction with a small spin-exchange term has the form

$$V_{12} = K\delta(\vec{r}_1 - \vec{r}_2)\,[(1-\alpha)+\alpha P(\vec{\sigma}_1 \cdot \vec{\sigma}_2)], \tag{17}$$

where coefficient α is equal to about 0.2. The matrix element which corresponds to such interaction can be calculated from the relations [18]

$$V_{J,0} = \frac{Q}{1+\delta(1,2)}(2j_1+1)(2j_2+1)\left\{\tfrac{1}{2}(1+S)\begin{pmatrix} j_1 & j_2 & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 + \begin{pmatrix} j_1 & j_2 & J \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix}^2\right\}, \quad (18)$$

$$V_{J,1} = \frac{Q(1-2\alpha)}{1+\delta(1,2)}(2j_1+1)(2j_2+1)\left\{\tfrac{1}{2}(1-S)\begin{pmatrix} j_1 & j_2 & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2\right\}, \quad (19)$$

where $S = (-1)^{2j-J}$ and Q are connected with the radial wave function. The mean interaction energy $\bar{E}(2)$ defined by

$$\bar{E}(2) = \sum_J (2J+1)V_J / \sum_J (2J+1) \quad (20)$$

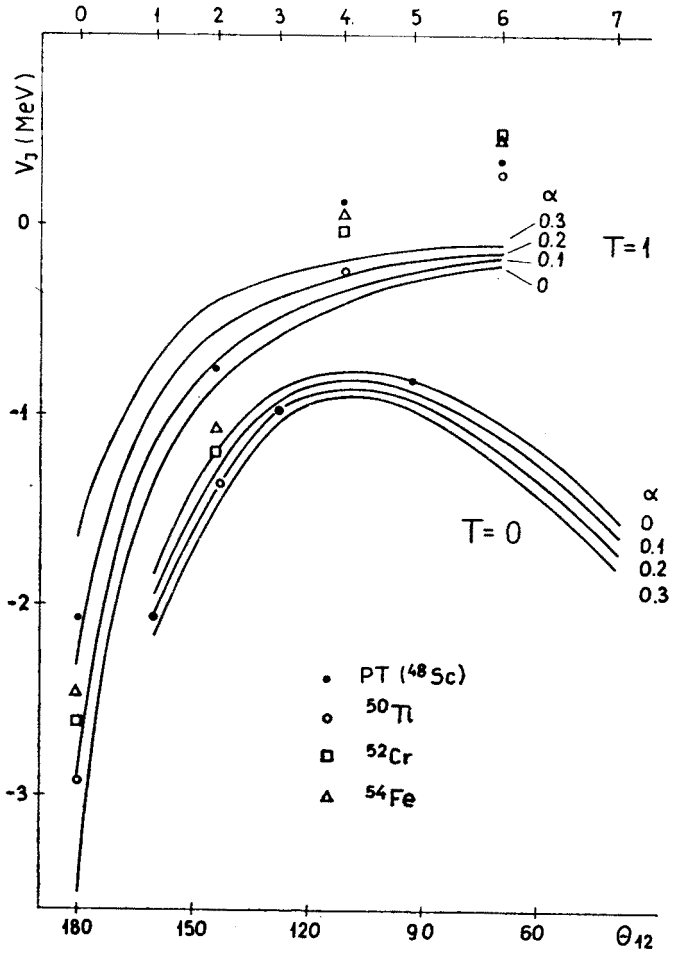

Fig. 2. Comparison of the observed and calculated two-body interaction energies as a function of the θ_{12} angle between the two orbits of identical nucleons. The δ-force with a small spin-exchange term was chosen as a zero-order approximation of the two-body interactions

satisfies the relation

$$\bar{E}(2) = Q(1-\alpha/2). \tag{21}$$

Normalization of δ-forces was chosen in such a way that relation (21) was fulfilled for two-body interactions obtained from ^{48}Sc emploing the Pandya transformation. Comparison between the obtained parameters and predictions for the δ-force interactions are shown in Fig. 2.

The two-body interaction energies are plotted as a function of the angle θ_{12} where

$$\theta_{12} = \arccos\left(\frac{J(J+1)}{2j(j+1)} - 1\right) \tag{22}$$

is the angle between the two orbits of identical nucleons [19]. The smooth curves correspond to the different contributions of the spin-exchange term. A proper separation of the V_0 and V_1 values can be obtained with $\alpha \approx 0.2$. This comparison shows that it is necessary to include some additional interaction to the δ-force. This interaction should be repulsive at the angles less than 120° and attractive for the angles close to 150°.

One can observe an interesting behaviour of the differences between V_J for different nuclei, e.g. $V_0(\mathrm{Fe}) - V_0(\mathrm{Ti}) < V_6(\mathrm{Fe}) - V_6(\mathrm{Ti})$. A similar behaviour is observed with the values predicted for the δ-forces with different contributions of the spin-exchange term.

4. Comparison with the experiment

The number of levels predicted for $f_{7/2}$ region within the pure $f_{7/2}$ configuration is small. Therefore it is difficult to compare the experimental positions of levels with those calculated with the help of parameters derived from the same nuclei. A general conclusion is that the positions of $3/2^-$ states do not agree with the experiment. Admixture of $2p_{3/2}$ configuration is important for these states. On the other hand, it is interesting to calculate the positions of levels for these nuclei which have a few active nucleons outside the closed core. As an illustration of this effect, Fig. 3 shows the experimental energies of ^{49}V [20] compared with the strong coupling model calculations, A, [21] and with the energies obtained using parameters derived from the present method, B. Quality of both calculated sets is similar.

In the Zr region of nuclei, energy levels of ^{91}Nb and ^{92}Mo were also calculated. Results are presented in Figs 4 and 5. The sets of energies denoted by A were calculated with the help of effective two-body parameters obtained by a fitting procedure for the whole $g_{9/2}$ region [13]. For these nuclei, the model predicts a lot of levels. The agreement between the experimental [7, 8] and calculated level positions obtained in these examples enables us to conclude that parameters derived from well known levels of some nucleus can be able to reproduce energies of other model levels not yet observed in this nucleus.

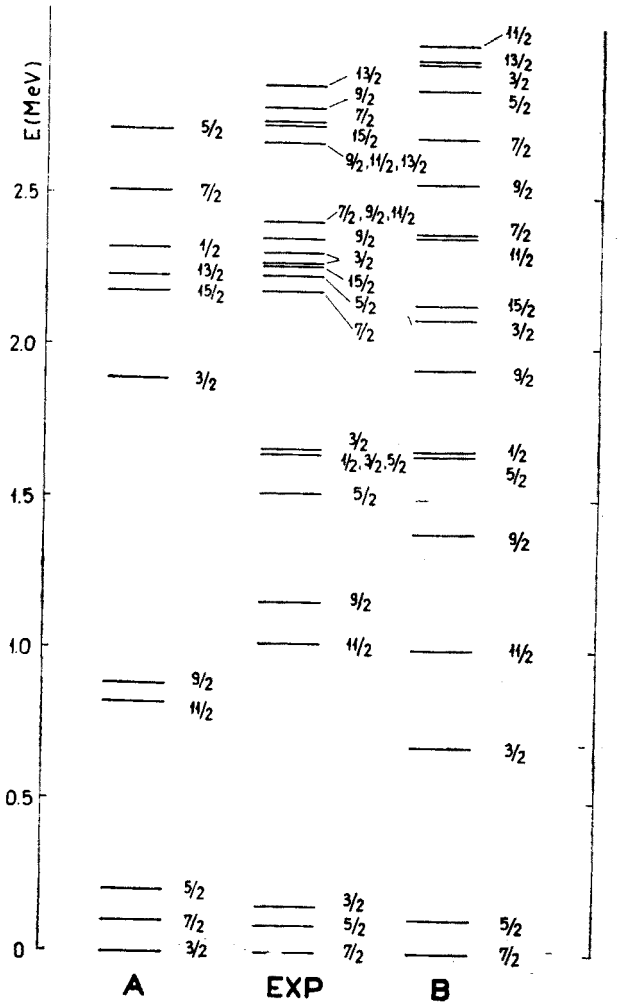

Fig. 3. Experimental and theoretical negative parity level scheme for the ^{49}V; A — strong coupling model [21], B — shell model and phenomenological parameters

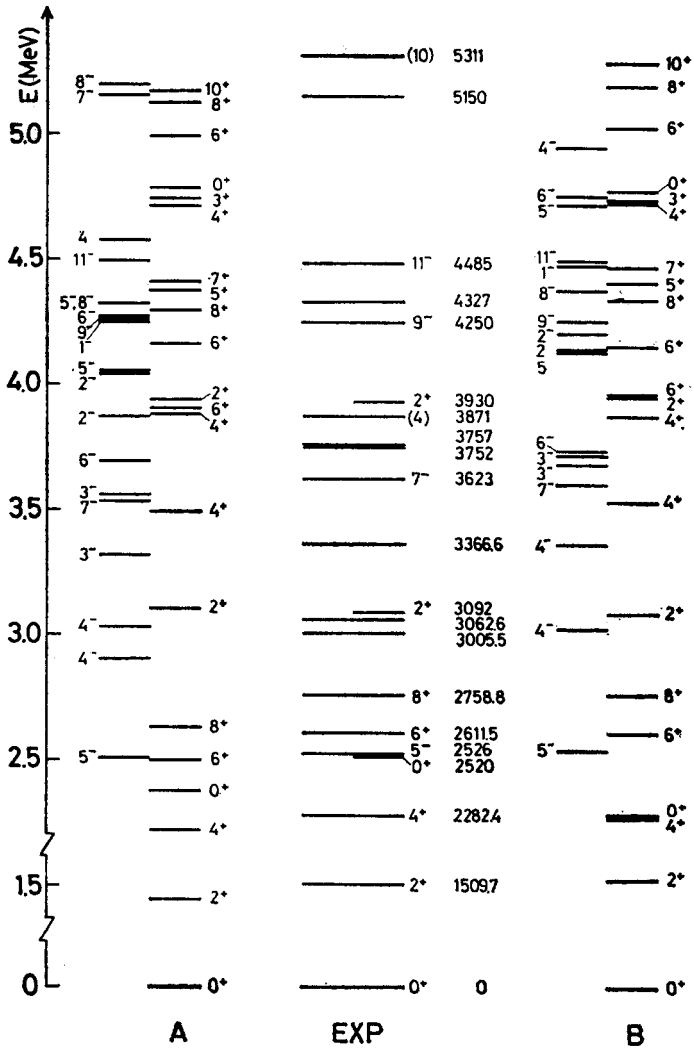

Fig. 4. Experimental and calculated positions of levels for ^{91}Nb. The version A was obtained with the parameters SLG from Table II [13] and B with our parameters

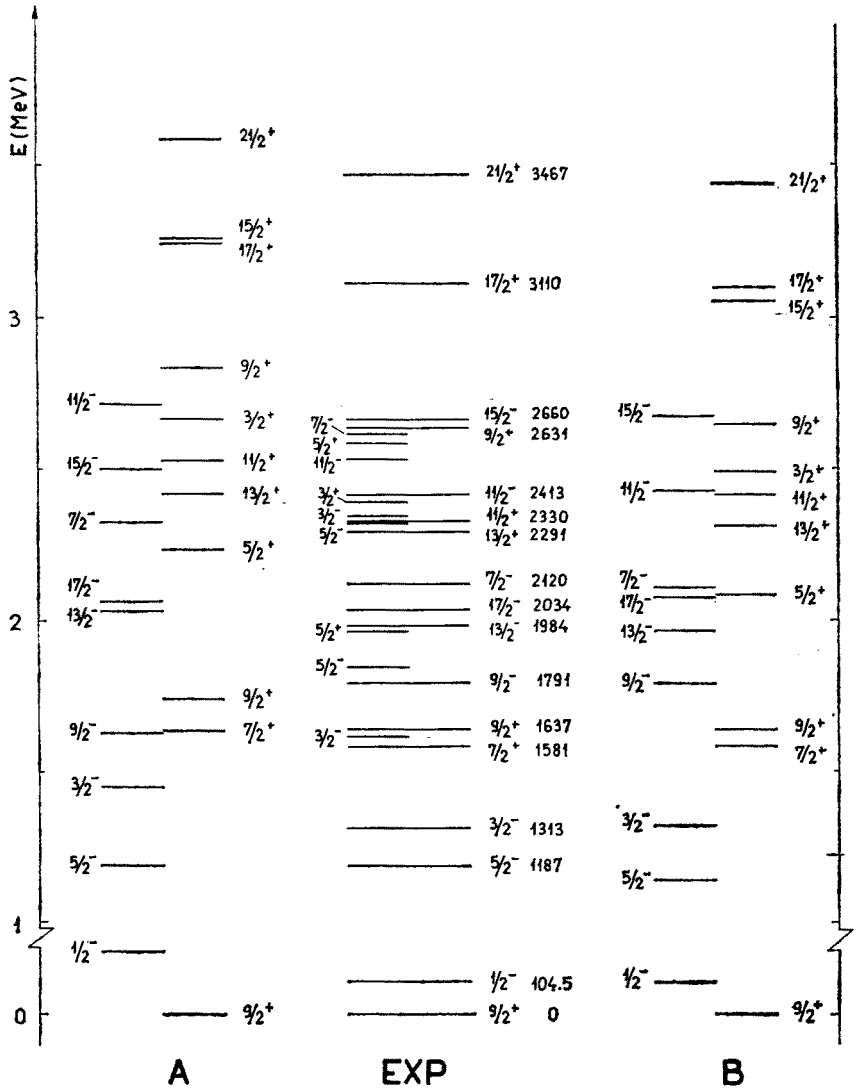

Fig. 5. Experimental and calculated positions of levels for ^{92}Mo. See comment to Fig. 4

REFERENCES

[1] J. D. Mc Cullen, B. F. Bayman, Larry Zamick, *Phys. Rev.* **134**, 515 (1964).
[2] D. H. Gloeckner, F. J. D. Serduke, *Nucl. Phys.* **A220**, 477 (1974).
[3] A. de Shalit, I. Talmi, *Nuclear Shell Theory*, Academic Press, New York 1963.
[4] A. Bałanda, Report IFJ No 986/PL (1977).
[5] A. Bałanda, *Acta Phys. Pol.* **B8**, 501 (1977).
[6] J. B. Ball, M. W. Johns, K. Way, *Nucl. Data Tables* **A8**, 407 (1970).
[7] A. Bałanda, R. Kulessa, W. Waluś, J. Sieniawski, *Acta Phys. Pol.* **B7**, 355 (1976).
[8] A. Bałanda, R. Kulessa, W. Waluś, Z. Stachura, IFJ Annual Report 1977, p. 63; in print.
[9] M. W. Johns, J. Y. Park, S. M. Shafrth, D. M. Van Patter, K. Way, *Nucl. Data Tables* **A8**, 373 (1970).
[10] N. Auerbach, I. Talmi, *Nucl. Phys.* **64**, 458 (1965).
[11] P. W. M. Glaudemans, G. Wiechers, P. J. Brussaard, *Nucl. Phys.* **56**, 529 (1964).
[12] R. Gross, A. Frenkel, *Nucl. Phys.* **A267**, 85 (1976).
[13] F. J. D. Serduke, R. D. Lawson, D. H. Gloeckner, *Nucl. Phys.* **A256**, 45 (1976).
[14] I. Talmi, in *Effective Interactions and Operators in Nuclei*, ed. by B. R. Barrett, Springer-Verlag, Berlin 1975, p. 64.
[15] P. Federman, S. Pittel, *Nucl. Phys.* **A155**, 161 (1970).
[16] J. B. Mc Grory, B. H. Widenthal, E. C. Halbert, *Phys. Rev.* **C2**, 186 (1970).
[17] I. Eisenstein, M. W. Kirson, *Phys. Lett.* **47B**, 315 (1973).
[18] M. Moinester, J. P. Schiffer, W. P. Alford, *Phys. Rev.* **179**, 984 (1969).
[19] J. P. Schiffer, *Ann. Phys.* (USA) **66**, 798 (1971).
[20] J. Styczeń, Report IFJ No 917/PL (1976).
[21] B. Haas, P. Taras, J. Styczeń, *Nucl. Phys.* **A246**, 141 (1975).