Vol. B9 (1978) ACTA PHYSICA POLONICA No 11

LETTERS TO THE EDITOR

ZEROS OF THE SPECTRAL FUNCTIONS FOR ELECTROMAGNETIC FORM--FACTORS*

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(Received July 29, 1978)

We show that if the electromagnetic form-factor decreases fast enough, then the spectral function has at least one zero.

The asymptotic behaviour of the various electromagnetic form-factors of hadrons is of great interest. The nucleon electric and magnetic form-factors have been measured over the interval, $0 \leqslant -t \leqslant 25$ $(\mathrm{GeV}/c)^2$. Fits to this data indicate a t^{-2} behaviour for large values of four-momentum transfer [1]. The corresponding fits to the electric form factor of the pion suggest an asymptotic t^{-1} decrease [2].

Various theoretical models have been proposed to determine the electromagnetic structure of both the pion and the nucleon [3–6]. However, depending upon what initial assumptions are made, almost any type of decreasing power-law behaviour may be obtained. In principle, if the form-factor, $F(t)$, satisfies a non-subtracted dispersion relation, i. e., [6],

$$F(t) = \int_{t_0}^{\infty} \frac{\varrho(x)dx}{x-t}, \tag{1}$$

then knowledge of the spectral function, $\varrho(x)$, will allow a determination of the exact asymptotic behaviour of $F(t)$.

In this paper, we investigate a limited aspect of this general problem. We show, within the context of a dispersion relation representation for the form-factor, $F(t)$, that

* Research supported in part by a grant from the National Aeronautic and Space Administration.

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if $F(t)$ decreases sufficiently fast, then the spectral function $\varrho(x)$ has at least one zero[1] Our major assumption is that $F(t)$ satisfies a dispersion relation with no subtractions.

We now prove the following theorem: if

$$\lim_{|t|\to\infty} |F(t)| \leqslant \frac{\text{const}}{|t|^{1+\varepsilon}}, \qquad \varepsilon > 0, \tag{2}$$

then, the spectral function $\varrho(x)$ has at least one zero.

Note that if $\varrho(x) > 0$, then from equation (1) it is easy to show that $F(t)$ is a Herglotz function [8, 9]. If $F(t)$ is a Herglotz function, then it has the following lower bounds on its asymptotic behaviour, [8],

$$\lim_{|t|\to\infty} |F(t)| \begin{cases} > \dfrac{C_1}{|t|}, & \alpha < \operatorname{Arg} t < \pi-\alpha, \\ > \dfrac{C_2}{|t \log t|}, & \operatorname{Arg} t = 0, \pi, \end{cases} \tag{3}$$

where C_1 and C_2 are constants, and $\alpha > 0$. The theorem stated above is the contrapositive [7] of this result, consequently, the theorem is true.

At present, we are investigating the question of whether there is any possible connection between the number of zeros of $\varrho(x)$ and the exact asymptotic behaviour of $F(t)$.

REFERENCES

[1] P. N. Kirk et al., *Phys. Rev.* **D8**, 63 (1973).
[2] C. N. Brown et al., *Phys. Rev.* **D8**, 92 (1973).
[3] See Table VIII of Ref. [1] and the references summarized in it.
[4] V. A. Matveef, R. M. Muradyan, A. V. Tavkhelidze, *Lett. Nuovo Cimento* **7**, 719 (1973).
[5] S. J. Brodsky, G. R. Farrar, *Phys. Rev. Lett.* **31**, 1153 (1973).
[6] R. Hofstadter, *Electron Scattering and Nuclear and Nucleon Structure*, Benjamin 1963.
[7] C. B. Allendoerfer, C. O. Oakley, *Principles of Mathematics*, McGraw–Hill 1963, 2nd edition; Theorem 19, page 39.
[8] Y. S. Jin, A. Martin, *Phys. Rev.* **135B**, 1369 (1964).
[9] J. A. Shohat, J. D. Tamarkin, *The Problem of Moments*, American Mathematical Society, New York 1943, cf., especially Chapter II.

[1] We do not include a possible zero of $\varrho(x)$ at $x = t_0$.