THE UNIVERSALITY OF THE CABIBBO ANGLE

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It is shown that the Cabibbo angle is associated not only with the pion and kaon masses but also with the D meson mass. The universality of the Cabibbo angle is further established.

Motivated by the non conservation of strangeness in weak interactions Cabibbo had proposed that the relative strength of $\Delta S = 0$ and $|\Delta S| = 1$ transitions could be expressed in terms of a single parameter θ close to 15° [1]. An estimate of this angle was also made by Oakes who assumed that the realization of the physical mass of pions is due to the same mechanism that introduces non conservation of the strangeness preserving axial vector currents and the strangeness non conservation in weak processes [2]. By rotating the Hamiltonian, satisfying the condition that pions are Goldstone boson, around the 7th axis in SU(3) space, and then imposing strangeness conservation on the result, Oakes had postulated that this transformed Hamiltonian represents the world Hamiltonian of the hadrons. The following relation was derived:

$$\sin^2\theta \approx \frac{\sqrt{2}}{3}(c+\sqrt{2}),$$

(where if $c = -\sqrt{2}$, pions are massless) from which θ was found to be in very good agreement with Cabibbo's estimate.

The main objective of this paper is to look for a connection between chiral $SU(4)\times SU(4)$ breaking and the Cabibbo angle θ. Therefore, we begin with the following form of the hadronic part of the weak interaction Hamiltonian [3]:

$$H_w \equiv (\cos\theta \quad \Delta S = 0+\sin\theta \quad \Delta S = 1)_{\Delta C=0}+(-\sin\theta \quad \Delta S = 0+\cos\theta \quad \Delta S = 1)_{\Delta C=1}, \tag{1}$$

which may be written

$$H_w \equiv (\cos\theta \quad \Delta S = 0+\sin\theta \quad \Delta S = 1)_{\Delta C=0}+(-\cos\alpha \quad \Delta S = 0+\sin\alpha \quad \Delta S = 1), \tag{2}$$

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where

$$\alpha = \frac{\pi}{2} - \theta. \tag{3}$$

This is done while keeping in mind that just as θ was connected with the realization of the pion mass, α can be connected with the D meson in a manner as described below.

The hadron energy density using a simple generalization of the GMOR model [4] can be written as

$$\mathscr{H} = u_0 + pu_8 + tu_{15}, \tag{4}$$

where p and t are symmetry breaking parameters and u_i ($i = 0, 1, 2, \ldots, 15$) are scalars that belong to the $(4, \bar{4})+(\bar{4}, 4)$ representation of the chiral SU(4)×SU(4). We will now consider the Hamiltonian density $\mathscr{H}$, which satisfies the condition that pions are Goldstone bosons viz.

$$\frac{1}{\sqrt{2}} + \frac{p}{\sqrt{3}} + \frac{t}{\sqrt{6}} = 0, \tag{5}$$

rotate it by an angle 2θ around the 7th axis in SU(4) space and impose strangeness conservation on the result. Let $\mathscr{H}_{T_1}$ be the transformed $\mathscr{H}$.

The condition for the D mesons to be Goldstone bosons, on the other hand is

$$\frac{1}{\sqrt{2}} - \frac{p}{2\sqrt{3}} - \frac{t}{\sqrt{6}} = 0. \tag{6}$$

We next rotate $\mathscr{H}$ which satisfies the above condition (6) by an angle 2α around the 14th axis in SU(4) space and impose, as before, strangeness conservation on the result. The transformed $\mathscr{H}$ is called $\mathscr{H}_{T_2}$. Note that when there is no rotation around the 14th axis, i. e., if $\alpha = 0$, the ($\Delta S = 1$, $\Delta C = 1$) part of H_w vanishes.

Taking into account the fact that (5) and (6), when solved, give $p = -2\sqrt{\frac{2}{3}}$ and $t = \frac{1}{\sqrt{3}}$, $\mathscr{H}_{T_1}$ and $\mathscr{H}_{T_2}$ are of the form

$$\mathscr{H}_{T_1} = u_0 - \sqrt{2}\sin^2\theta u_3 - 2\sqrt{\tfrac{2}{3}}(1-\tfrac{3}{2}\sin^2\theta)u_8 + \frac{1}{\sqrt{3}}u_{15}, \tag{7}$$

$$\mathscr{H}_{T_2} = u_0 - 2\sqrt{\tfrac{2}{3}}(1-\sin^2\alpha)u_8 + \frac{1}{\sqrt{3}}(1-4\sin^2\alpha)u_{15}. \tag{8}$$

Next we assert that the hadron Hamiltonian density is a linear combination of $\mathscr{H}_{T_1}$ and $\mathscr{H}_{T_2}$, i. e. $\mathscr{H} = a\,\mathscr{H}_{T_1} + b\mathscr{H}_{T_2}$, where a and b are constants. Comparing the coefficients we arrive at

$$\begin{aligned} a+b &= 1, \\ -2\sqrt{\tfrac{2}{3}}\{a(1-\tfrac{3}{2}\sin^2\theta)+b(1-\sin^2\alpha)\} &= p, \\ \frac{a}{\sqrt{3}} + \frac{b}{\sqrt{3}}(1-4\sin^2\alpha) &= t. \end{aligned} \tag{9}$$

TABLE I

Estimates of α

Symmetry breaking parameters	α	Reference
$p = -0.065, \quad t = -1.63$	77°	[5]
$p = -0.099, \quad t = -1.577$	77.5°	[6]
$p = -0.099, \quad t = -1.57$	79°	[7]

The parameters p and t were estimated previously [5, 6, 7]. In Table I we have tabulated α against various estimates of p and t for $\theta \sim 13°$ [8]. Thus we get a range for the angle $\alpha \sim 77°$–$79°$ which satisfy Eq. (3) remarkably well for $\theta = 13°$.

An interesting feature that we observed deserves special consideration. The Cabibbo angle generates the mass of the pion and kaon [2, 9]. Now our analysis also shows that the Cabibbo angle is associated with the generation of the physical mass of the D meson. Thus the universal nature of the Cabibbo angle appears to be one of much higher degree.

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