DISCUSSION OF THE EXPERIMENTAL METHODS OF THE ESTIMATION OF THE REACTION IMPACT PARAMETER

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Two methods of determination of the reaction impact parameter, the one proposed by Webber and other by Henyey and Pumplin, are compared and discussed. It is shown that the lower limits of the impact parameter b_L obtained by means of these methods are comparable and are always very low ($\lesssim$ 0.5 fm). On the example of the Henyey-Pumplin method it is argued that the experimentally obtained values b_L may be very unreliable estimates of the reaction impact parameter and that any comparison of different reactions or reaction channels may be meaningless.

1. Introduction

The knowledge of the impact parameter structure of the high energy collision amplitude is of great importance both from the theoretical and experimental points of view. On the one hand it could serve us as the basis for an intuitive classical understanding of the high energy particle-particle collision, and on the other it could be a decisive test for many theoretical models. However, the impact parameter defined for a given final state [1] as

$$\vec{b} = \sum_{i=1}^{N} \vec{b}_i x_i \tag{1}$$

(where b_i's are the partial impact parameters and x_i's are the longitudinal Feynman variables) is not directly accessible in the experiment. All what one can do is to estimate the lower limits of the average impact parameter squared $b_L = \sqrt{\langle b^2 \rangle_L}$, for a given

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sample of the experimental events [2–7]. Such estimations are based on the generalized uncertainty principle and the final formula for b_L derived from it [6] reads:

$$b_L = \sqrt{\left\langle \sum_j x_j \frac{\partial A}{\partial \vec{k}_j} \right\rangle^2 \Big/ 2\langle A^2 \rangle}\,, \tag{2}$$

where A is a function of the final particle transverse momenta $\vec{k}_i (i = 1, \ldots, N, N$ is the number of particles in a given final state). Using various additional assumptions concerning the form and the properties of the function A we can obtain various estimates for b_L.

Two different approaches have been recently much discussed. These are: the one, proposed by Henyey and Pumplin [8] and the other, proposed by Webber [9, 10]. In Webber's approach the function A was assumed to be

$$A = \sum_{i=1}^{N-1} w_i \vec{k}_i. \tag{3}$$

In the Henyey-Pumplin method a broader class of functions is considered i. e. $A = f(Q)$ where $Q = \sum_i^{N-1} w_i \vec{k}_i$.

In each of these methods the set of arbitrary chosen weights w_i has to be defined in such a way as to maximize the left-hand side of the basic relation (2). In Webber's approach the maximum of b_L value dictates the choice of these weights as

$$w_i = \sum_{j=1}^{N-1} \langle \vec{k}_j \cdot \vec{k}_i \rangle^{-1} \cdot x_k. \tag{4}$$

In the first approximation one can assume the matrix $\langle \vec{k}_i \cdot \vec{k}_j \rangle$ to be diagonal i. e. $\langle \vec{k}_i \cdot \vec{k}_j \rangle = \alpha \delta_{ij}$ (lack of the $\vec{k}_i, \vec{k}_j$ correlations). With this additional assumption the formula (4) simplifies to

$$w_i = \frac{x_i}{\alpha}\,. \tag{5}$$

However, it is not clear a priori which among the N final particles should be removed[1]. In the Henyey-Pumplin approach no numerical prescription for the optimal choice of w_i's can be readily invented, but it is claimed by the authors that, if the strong absorption effects are present, their method is far superior to that proposed by Webber, i. e. it yields the values of b_L much closer to the real value. If this is really the case, the experimental data should give the answer. However, it should be pointed out that the whole problem of the experimental estimation of b_L using these methods rises some essential questions:

1) in general, the mean squared impact parameter $\langle b^2 \rangle$ can be written as a sum of two terms

$$\langle b^2 \rangle = \langle b^2 \rangle_{\text{MOD}} + \langle b^2 \rangle_{\text{PHASE}}, \tag{6}$$

[1] Note that in formulae (3) and (4) summations run from 1 to N–1 to take into account momentum conservation.

where $\langle b^2\rangle_{\mathrm{MOD}}$ and $\langle b^2\rangle_{\mathrm{PHASE}}$ are contributions coming from the modulus and the phase of the amplitude, respectively [1, 8]. It has to be stressed that in both approaches discussed the contribution coming from the amplitude phase is neglected. Clearly, by doing this we get the lower limit for $\langle b^2\rangle$, but is this phase contribution really unimportant [11, 14] ? E. g. cannot it be sizable and thus the b_{L} estimate us much too low? Or even worse, having quite different dependence on e. g. multiplicity cannot it reverse the whole experimentally found dependence of b_{L} on N?

2) In view of what was said above and since there exists freedom in the choice of w_i's for calculation of b_{L}, one can ask if the universal optimal choice of weights for various reactions or even reaction channels exists at all. We could easily imagine that the optimal choice for one reaction channel is not such for another one. If so, any comparison of different reactions or reaction channels is rather meaningless.

In an attempt to answer some of the above mentioned questions we have investigated several π^+p reactions at 16 GeV/c [12].

2. *Experimental results*

The data which we used in the analysis are the data of the Aachen–Berlin–Bonn–CERN–Cracow–Heidelberg–Warsaw Collaboration on π^+p interactions at 16 GeV/c, observed in the 2m HBC. For the analysis we have chosen the 4c, four, six, eight and ten prong reactions. Number of events in each reaction are given in Table I.

TABLE I

Average impact parameters b_{L} and numbers of events for different multiplicity channels of π^+p reaction at 16 GeV/c

π^+p 16 GeV/c			
Reaction $\pi^+\mathrm{p} \to$	Number of events	b_{L}[fm]	
		Henyey-Pumplin method	Webber method
$\pi^+\mathrm{p}\,\pi^+\pi^-$	28913	0.54 ± 0.01	0.489 ± 0.010
$\pi^+\mathrm{p}\; 2(\pi^+\pi^-)$	8505	0.35 ± 0.02	0.305 ± 0.015
$\pi^+\mathrm{p}\; 3(\pi^+\pi^-)$	1493	0.23 ± 0.04	0.192 ± 0.015
$\pi^+\mathrm{p}\; 4(\pi^+\pi^-)$	202	0.19 ± 0.05	0.139 ± 0.020

In Webber's approach one usually uses the formula:

$$b_{\mathrm{L}} = \sqrt{\langle(\sum_{i=1}^{N-1} w_i x_i)^2\rangle / \langle(\sum_{i=1}^{N-1} w_i \vec{k}_i)^2\rangle}\,, \tag{7}$$

where w_i's are calculated according to formula (5). The errors in this case are only statistical.

Much more complicated procedure is involved in the calculation of b_L following the Henyey-Pumplin method. Using the formula (2) and all assumptions discussed above, one can prove an alternative bound (see Ref. [8]).

$$b_L = \sqrt{\left\langle |\tau| \left(\frac{d}{d\tau} \ln \frac{d\sigma}{d\tau} \right)^2 \right\rangle}, \tag{8}$$

where arbitrary weights w_i's are included in the expression for the τ variable:

$$\tau = (\sum_i^{N-1} w_i \vec{k}_i)^2 / (\sum_i^{N-1} w_i x_i)^2. \tag{9}$$

We have tried a number of various sets of weights w_i's with the aim to maximize the value of $b_L{}^2$. We found that the optical choice is if we put

$$w_i = x_i \tag{10a}$$

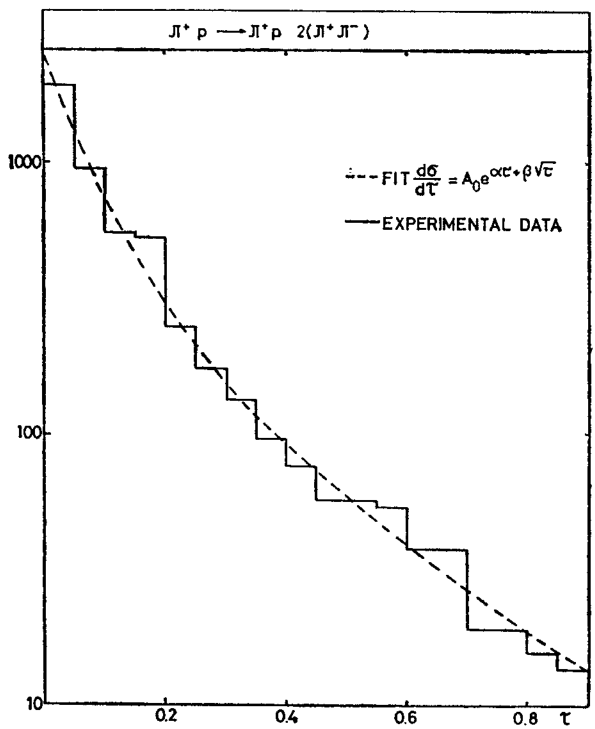


Fig. 1. $d\sigma/d\tau$ experimental distributions for $\pi^+p \to \pi^+p2(\pi^+\pi^-)$ channel (dotted line shows the $d\sigma/d\tau = A_0 e^{\alpha\tau+\beta\sqrt{\tau}}$fit)

for all final particles except two slowest in CMS for which we put

$$w_i = 0. \tag{10b}$$

[2] The weights w_i's cannot depend on the transverse momenta, see e. g. [6].

Having chosen the optimal set of weights, we calculated τ for each event and using the histogram of τ we made an analytical fit to the $d\sigma/d\tau$ distribution. The formula which gave a reasonable fit to our experimental distribution appeared to be of the form (see Fig. 1):

$$\frac{d\sigma}{d\tau} = A_0 e^{\alpha\tau + \beta\sqrt{\tau}}, \tag{11}$$

where A_0, α and β are the fitted parameters. Then the estimate of b_L was given using Eq. (8). The errors reflect now also the fitting uncertainties — and thus are larger than in the Webber method.

3. *Results of analysis*

The values of b_L calculated by means of both above mentioned methods for 4, 6, 8 and 10 particle states in π^+p reactions at 16 GeV/c are listed in Table I and plotted in Fig. 2. The Webber limit was calculated using the formula (7) where in summation we

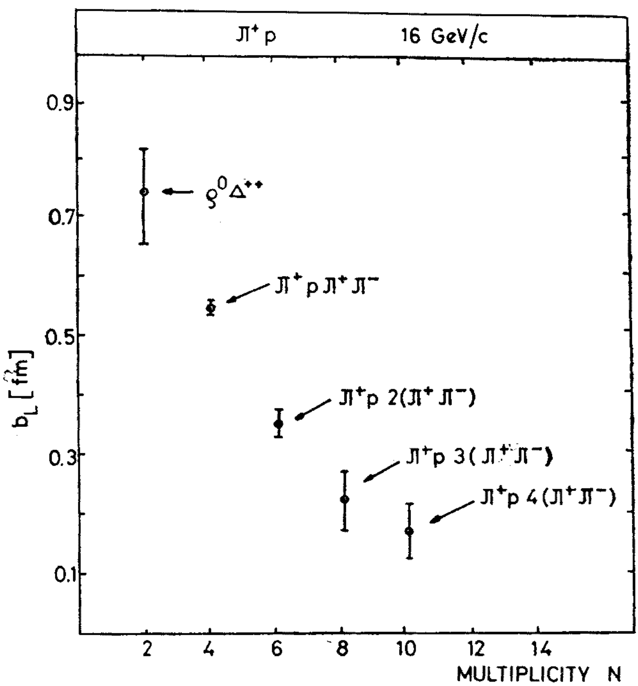


Fig. 2. Average impact parameters b_L for different multiplicity channels of π^+p reaction at 16 GeV/c as function of multiplicity (first choice of weights)

omitted the slowest particle in the c. m. s. while for the Henyey-Pumplin limit we used formula (8). We checked that in the Webber method calculated b_L is not sensitive to that which particle is omitted in summation. An obvious conclusion follows from the

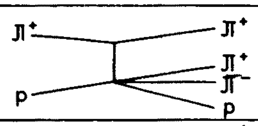
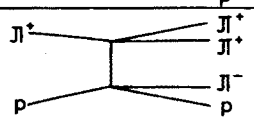
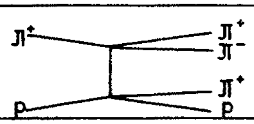
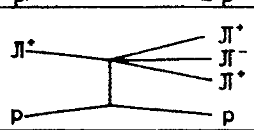
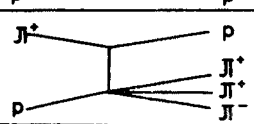
TABLE II

Average impact parameters b_L and numbers of events for quasi-two body $\rho^0\Delta^{++}$ reaction and for $p\bar{p}$ and $K\bar{K}$ pairs production channels

π^+p 16 GeV/c		
Reaction $\pi^+p \to$	Number of events	b_L[fm]
$\rho^0\Delta^{++}$	2929	0.73 ± 0.10
$\pi^+p\bar{p}p$	456	0.27 ± 0.05
$\pi^+p\,K^+K^-$	1884	0.40 ± 0.05

TABLE III

Average impact parameters b_L for the LPS selections

TYPE OF SELECTION	b_L[fm]	NUMBER OF EVENTS
	0.52 ± 0.01	5904
	0.52 ± 0.04	1796
	0.53 ± 0.02	9484
	0.55 ± 0.03	11004
	0.58 ± 0.02	222

comparison: no significant difference between b_L values calculated using the two methods is observed[3]. The errors in the Henyey-Pumplin method are of course larger. All the values of b_L are rather small, all $\lesssim 0.5$ fm. In addition b_L decreases with the multiplicity — the situation which intuitively is understandable — a high multiplicity collision is the more central one. To see further if this intuitive picture holds also in the other aspects of

[3] Similar results were reported by France–Soviet Union and CERN–Soviet Union Collaborations at the International High Energy Physics Conference in Budapest (*Impact parameter bounds for multiparticle exclusive K^+p reactions at 32 GeV/c*, I. V. Ajinenko et al.).

particle-particle interaction we have calculated b_L for the following reaction channels:

— production of baryon-antibaryon pairs;
— production of kaon pairs $\pi^+p \to K\bar{K}\pi^+p$;
— different sectors of LPS i. e.

- $\pi^+p \to (\pi^+\pi^+\pi^-)p$ dominated by the diffraction dissociation of π^+;
- $\pi^+p \to \pi^+(\pi^+\pi^-p)$ dominated by the diffraction dissociation of the proton;
- $\pi^+p \to (\pi^+\pi^-)(p\pi^+)$ dominated by quasi two-body reactions like e. g. $\pi^+p \to \varrho^0\Delta^{++}$;[4]
- background $\pi^+p \to (\pi^+\pi^+)(p\pi^-)$.

The results are listed in Tables II and III. Indeed some intuitively clear trends are observed. Namely baryon-antibaryon pair production is confined to rather small b_L values as compared to the total sample of 4c fit four prong reaction. Kaon production has b_L values larger (but also one expects that diffraction dissociation processes $\pi \to K\bar{K}\pi$ can contribute here sizably). However, it is remarkable that the different processes selected with LPS technique do not differ significantly in value of b_L.

4. *Discussion of results*

In view of the problems mentioned in the introduction there are few points which we would like to stress.

1. All b_L values are small irrespectively of the applied method ($\lesssim 0.5$ fm). We do expect from the overlap function analysis that the average impact parameter for all inelastic channels is of the order of 1 fm [5, 10, 13]. In our analysis even for diffraction dominated channels (certain sectors in the LPS technique) b_L is about 0.5 fm. Thus it comes out indeed very small.

2. We have calculated b_L value by means of the Henyey-Pumplin method outlined in Section 3, for a subset of the 4 prong 4c events, which represent (to a good approximation) quasi-two-body reaction $\pi^+p \to \varrho^0\Delta^{++}$, using the weights w_i's which were found to be optimal for all 4 prong 4c events. We got value of 0.73 fm. On the other hand, in this case $\langle b^2 \rangle$ can be reliably estimated from the $d\sigma/dt$ distribution and turns to be 1.32 fm.[5] Thus again the value obtained using the Henyey-Pumplin method is too low. One could try to improve it by changing the set of weights, but this would mean that the optimal choice of weights is dependent on the reaction mechanism.

3. We have changed the choice of w_i's defined by the relation (10) into the one defined as follows: we put $w_i = x_i$ for all final particles except two fastest in the CMS for which we put $w_i = 0$. With this set of weights we found that although our ten prong b_L value remains unchanged, the others went down and b_L become now almost independent of multiplicity, see Fig. 3. This example demonstrates e. g. that the 4 prong reaction is very

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[4] This reaction was selected from the $\pi^+p \to \pi_f^+p\pi_s^+\pi^-$ sample by the cut in effective mass distributions; for ρ^0: 0.6 GeV $< M(\pi^-\pi_f^+) <$ 0.9 GeV and for Δ^{++}: 1.14 GeV $< N(p\pi_s^+) <$ 1.34 GeV.

[5] $d\sigma/dt \sim \exp(-Bt)$, with $B \approx 15\ (\text{GeV}/c)^{-2}$.

sensitive to the changes of w_i's we made, while the ten prong reaction is not. We would like to conclude our observations made on the basis of the analysis of the π^+p 16 GeV/c sample of events in the following way:

(*i*) the lower limits on the average impact parameter obtained with Webber and Henyey-Pumplin methods are comparable and one does not observe a significant improvement when using the Henyey-Pumplin method;

(*ii*) the optimal choice of weights w_i's is crucial in the Henyey-Pumplin method and seems to be strongly dependent on the reaction considered. Thus it is rather difficult (if not impossible) to find a universal choice for different reactions. Two immediate consequences follow then from this observation:

a) b_L limits found for different reactions may not be equally good (equally close to the real value);

b) any comparison of b_L for different reactions may be meaningless.

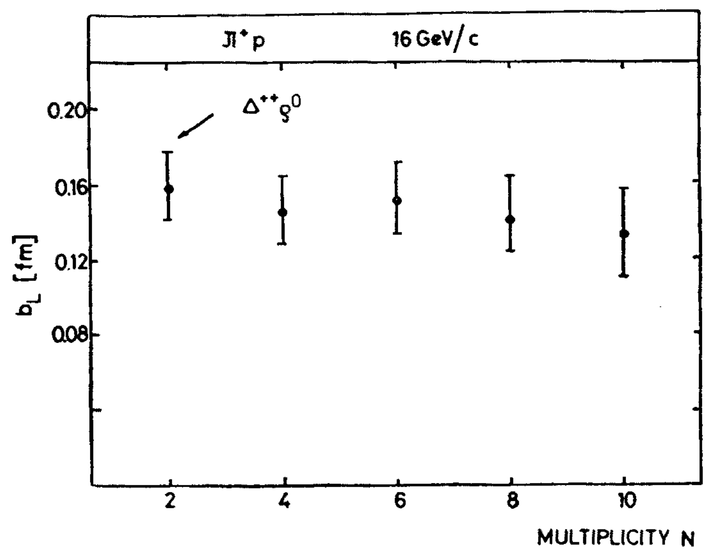

Fig. 3. Average impact parameters b_L for different multiplicity channels of π^+p reaction at 16 GeV/c as function of multiplicity (second choice of weights)

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