QUARKONIUM AND EXTENDED CONSTITUENT QUARKS*

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Constituent quarks with electromagnetic and strong-interaction form-factors behaving as $1/\sqrt{Q^2}$ for $Q^2 \to \infty$ are considered. One-gluon-exchange potential is used to calculate the hfs and fs of quarkonium $q\bar{q}$ built of such extended quarks. No confining potential is included in the calculations but its possible form, compatible with the picture of extended quarks, is suggested. The constituent quark gets then a strong-interaction radius r_0 of large magnitude $\gtrsim 1$ GeV^{-1} being in contrast with its small electromagnetic radius $\lesssim 0.1$ GeV^{-1}. So it behaves as a composite object with electric charge concentrated in the middle and colour broadly extended around (with a range comparable to actual hadron radii). Thus, if the structure of a constituent quark is dominated by the parton configuration consisting of one current quark and one current gluon, the gluon must be effectively lighter than the quark. The above picture emerges if the confining potential does not play any essential role in the lowest states in charmonium. It is true in fact for the suggested confining potential which switches on only for $r \geqslant 2r_0$.

It has been recently conjectured that the constituent quarks inside hadrons may display in electromagnetic interactions an elastic form-factor $G(Q^2)$ which behaves as $1/\sqrt{Q^2}$ for $Q^2 \to \infty$ [1]. Some arguments have been presented that such a behaviour of $G(Q^2)$ should in fact appear if the constituent quarks are parton clusters consisting in the lowest-order parton approximation (i.e., in the valence approximation) of one current quark and one current gluon each (plus further current quarks and gluons appearing in higher parton approximations). In the Bjorken limit such clusters must be completely dissociated into their partons if exact scaling is to hold in this limit. Then, after this dissociation (which may be considered as a phase transition), the nucleon can be described as a parton cluster containing three valence quarks and three valence gluons (plus sea quarks and sea gluons). It has been argued that in this case the nucleon structure functions should behave as $(1-x)^4$ for $x \to 1$, while the behaviour of the nucleon elastic form-factor should be as $1/Q^{5/2}$ for $Q^2 \to \infty$ (if the Drell-Yan-West relation is true).

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If the quark electromagnetic structure described by $G(Q^2)$ really exists, it is likely that also in strong interactions the constituent quarks display a structure corresponding to an elastic form-factor $G_s(Q^2)$, possibly with a similar asymptotic dependence on Q^2 as $G(Q^2)$. Then, if $G_s(Q^2) \sim 1/\sqrt{Q^2}$ for $Q^2 \to \infty$, the one-gluon-exchange interaction between two constituent quarks can be described by the effective gluon propagator $G_s^2(Q^2)/Q^2 \sim 1/Q^4$ for $Q^2 \to \infty$. It leads to the asymptotic dependence $\sim 1/p_\perp^8$ of the inclusive cross-section for the process pp $\to$ hX, where h is a hadron [1]. From the experimental point of view, the quark-quark cross-section $d\sigma/dt \sim (s^2+u^2)/s^2t^4$ implied by the interaction $G_s^2(t)/t \sim 1/t^2$ considered here seems to be worse (being too peripheral) than the quark-quark cross-section $d\sigma/dt \sim 1/st^3$ introduced phenomenologically by Field and Feynman [2] (which gives also the asymptotic dependence $\sim 1/p_\perp^8$ for pp $\to$ hX). The difference, however, does not seem to be terrible (in the actual experimental situation). In the literature, different shapes of the form-factor $G_s(t)$ have been considered and reasonably fitted to the experimental data (cf. e.g. [3, 4]).

In the present note we discuss the one-gluon-exchange static interaction between two constituent quarks, each having the form-factor $G_s(Q^2) \sim 1/\sqrt{Q^2}$ for $Q^2 \to \infty$. In this case, the attractive potential for quark-antiquark interaction is given by the formula

$$V(r) = -\frac{f^2}{(2\pi)^3}\int d_3\vec{q}\,\frac{G_s^2(\vec{q}^2)}{\vec{q}^2}\,e^{i\vec{q}\cdot\vec{r}}$$

$$= -\frac{f^2}{4\pi}\int d_3\vec{r}'_1 d_3\vec{r}'_2\,\frac{\varrho_s(|\vec{r}_1-\vec{r}'_1|)\varrho_s(|\vec{r}'_2-\vec{r}_2|)}{|\vec{r}'_1-\vec{r}'_2|}, \tag{1}$$

where $Q^2 = -q^2 = \vec{q}^2 - q_0^2$, $\vec{r} = \vec{r}_1 - \vec{r}_2$ and

$$\varrho_s(r) = \frac{1}{(2\pi)^3}\int d_3\vec{q}\,G_s(\vec{q}^2)e^{i\vec{q}\cdot\vec{r}}, \tag{2}$$

and hence

$$\int d_3\vec{r}\,\varrho_s(r) = G_s(0) = 1. \tag{3}$$

Notice that

$$\Delta V(r) = \frac{f^2}{(2\pi)^3}\int d_3\vec{q}\,G_s^2(\vec{q}^2)e^{i\vec{q}\cdot\vec{r}} = f^2\int d_3\vec{r}'\,\varrho_s(|\vec{r}-\vec{r}'|)\varrho_s(r'). \tag{4}$$

The asymptotic behaviour of $G_s(Q^2)$ for $Q^2 \to \infty$ in Eq. (1) is not yet specified. If now $G_s(\vec{q}^2) \sim 1/q$ for $q \to \infty$ (where $q = |\vec{q}|$), we can conclude from the relation

$$G_s(\vec{q}^2) = \int d_3\vec{r}\,\varrho_s(r)e^{-i\vec{q}\cdot\vec{r}} = 4\pi\int_0^\infty dr r^2\varrho_s(r)\,\frac{\sin qr}{qr} \tag{5}$$

that $\varrho_s(r) \sim 1/r^2$ for $r \to 0$. We will assume that $\varrho_s(r) \sim 1/r^2$ for $r < r_0$. For simplicity we shall put $\varrho_s(r) = 0$ for $r > r_0$. Then r_0 is a strong-interaction radius of the constituent quark. In this case we obtain from formula (5) the quark form-factor

$$G_s(\vec{q}^2) = \frac{1}{qr_0} \int_0^{qr_0} d\xi \frac{\sin \xi}{\xi} = \frac{\mathrm{Si}(qr_0)}{qr_0} \tag{6}$$

(plotted in Fig. 1) and from formula (2) the quark-antiquark potential $V(r) = (\beta_s/r_0)U(x)$, where $\beta_s = f^2/4\pi$, $x = r/r_0$ and the function $U(x)$ (plotted in Fig. 2) is given as follows: (i) for $r \leqslant r_0$

$$U(x) = \frac{(1-x)(3x-1)}{4x} \ln(1-x) + \frac{7+\pi^2}{8} x - \frac{x}{2} \sum_{n=1}^{\infty} \frac{x^n}{n^2} - \frac{9}{4}, \tag{7}$$

(ii) for $r_0 \leqslant r \leqslant 2r_0$

$$U(x) = \frac{x}{4} (\ln x)^2 + \frac{(1-x)(3x-1)}{4x} \ln(x-1) + \frac{21-\pi^2}{24} x + \frac{x}{2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\frac{1}{x}\right)^n - \frac{9}{4}, \tag{8}$$

(iii) for $r \geqslant 2r_0$

$$U(x) = -\frac{1}{x}. \tag{9}$$

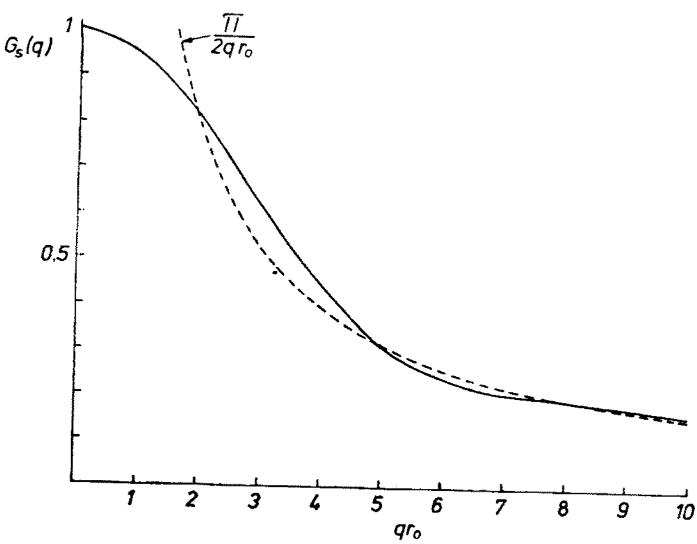

Fig. 1. The strong-interaction form-factor $G_s(\vec{q}^2)$ of the constituent quark as given by Eq. (6), compared with its asymptotic form $\pi/2qr_0$ valid for $q \to \infty$ ($q = |\vec{q}|$)

From Eq. (7) we get for $r \ll r_0$

$$U(x) \simeq \frac{\pi^2}{8}\,x-2+o(x^2) \quad \left(o(x^2) = -\frac{x^2}{6}\right). \tag{10}$$

Practically, the Coulomb form (9) is valid for $r \gtrsim 1.3r_0$, whereas the linear approximation (10) works for $r \lesssim 0.7r_0$. So the potential $V(r)$ has here the inverted behaviour to the well known phenomenological potential $-\alpha_s/r+r/\lambda^2+V_0$. However, if r_0 is very large, formula (10) simulates for moderately large r the linear confining potential r/λ^2+V_0 [5]. Of course,

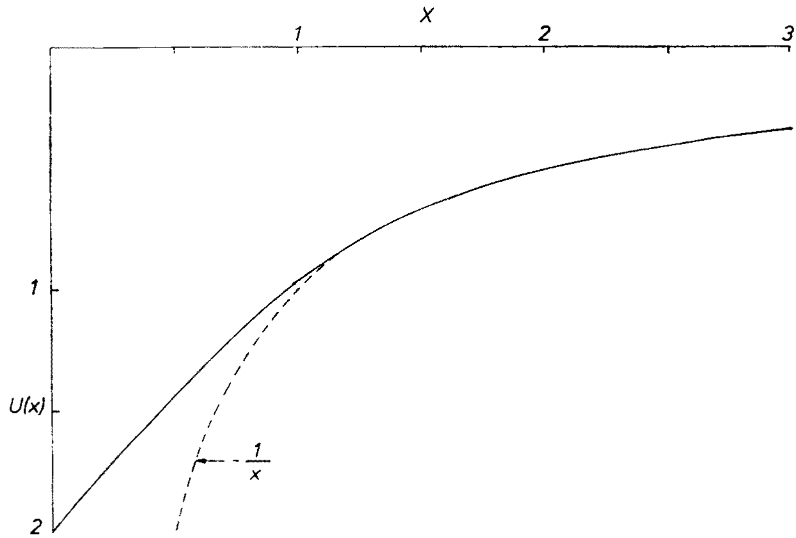

Fig. 2. The quark-antiquark potential $U(x)$ (in units of β_s/r_0) as given by Eqs. (7)–(9), compared with its Coulomb form $-1/x$ valid strictly for $x \geqslant 2\,(x = r/r_0)$

our potential $V(r)$ provides no real quark confinement. This confinement may be provided here by adding to $V(r)$ the confining potential of the linear form

$$V_c(r) = \frac{r-2r_0}{\lambda^2}\,\theta(r-2r_0) \tag{11}$$

which switches on at point $r = 2r_0$. This is reasonable since only for $r > 2r_0$ two spherically shaped constituent quarks are spatially separated and, therefore, should span a confining string (or tube) made of the gluon field. In this note, we will ignore in the calculations this confining potential.

Now, we solve numerically the Schrödinger equation for quarkonium $q\bar{q}$ with the potential $V(r)$ given by Eqs. (7)–(9), where β_s and r_0 as well as the quark mass m_q are free parameters (for each kind of quarks q). Subsequently, we calculate the first-order perturbation caused by spin-orbit, tensor and spin-spin couplings,

$$V_{LS}+V_T+V_{SS}, \tag{12}$$

where [6]

$$V_{LS} = \frac{3}{2m_q^2} \frac{1}{r} \frac{dV}{dr} \vec{L} \cdot \vec{S},$$

$$V_T = \frac{1}{6m_q^2} \left(\frac{1}{r} \frac{dV}{dr} - \frac{d^2 V}{dr^2} \right) \left[3 \left(\vec{S} \cdot \frac{\vec{r}}{r} \right)^2 - \vec{S}^2 \right],$$

$$V_{SS} = \frac{1}{6m_q^2} \Delta V (2\vec{S}^2 - 3), \tag{13}$$

while

$$\vec{L} \cdot \vec{S} = \tfrac{1}{2} (\vec{J}^2 - \vec{L}^2 - \vec{S}^2) = \tfrac{1}{2} [\vec{J}^2 - \vec{L}^2 - \tfrac{1}{2} (\vec{\sigma}_1 \cdot \vec{\sigma}_2 + 3)],$$

$$3 \left(\vec{S} \cdot \frac{\vec{r}}{r} \right)^2 - \vec{S}^2 = \tfrac{1}{2} \left[3 \left(\vec{\sigma}_1 \cdot \frac{\vec{r}}{r} \right) \left(\vec{\sigma}_2 \cdot \frac{\vec{r}}{r} \right) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right],$$

$$2\vec{S}^2 - 3 = \vec{\sigma}_1 \cdot \vec{\sigma}_2. \tag{14}$$

We neglect the relativistic orbital corrections,

$$V_{orb} = -\frac{\vec{p}^4}{4m_q^3} + \frac{1}{4m_q^2} \Delta V - \frac{1}{2m_q^2} V \left[\vec{p}^2 + \left(\frac{\vec{r}}{r} \cdot \vec{p} \right)^2 \right], \tag{15}$$

as they are spin-independent and give, therefore, no contributions to hfs splitting $^3S_1$–$^1S_0$ and fs splitting $^3P_2$–$^3P_1$–$^3P_0$ which at present are of main interest for quarkonium. The positronium-like annihilation potential

$$V_{ann} = \frac{1}{4m_q^2} \Delta V \vec{S}^2 \tag{16}$$

is here obviously absent as the virtual transition quarkonium ⇄ gluon is forbidden by the colour conservation. In Eqs. (13) (and Eqs. (15)) we have assumed that the one-gluon--exchange potential $V(r)$ is originated by vector gluons, so it transforms as the time component of a 4-vector.

In the case of charmonium $c\bar{c}$ we take as the input two masses $M(1^3S_1) = 3095$ MeV and $M(2^3S_1) = 3684$ MeV, and fit to them various values of three parameters β_s, r_0 and m_q. The results are presented in Table I, where $M(nl)$ are masses (in MeV) calculated from the Schrödinger equation, $M(n^{2s+1}l_j)$ — masses (in MeV) corrected by first-order perturbation and $\Gamma(n^3S_1 \to l^+l^-)$ — the leptonic widths (in keV) deduced from the non-relativistic formula [6]

$$\Gamma(n^3S_1 \to l^+l^-) = \frac{16\pi\alpha^2 Q_q^2}{M(n^3S_1)^2} |\psi_{nS}(0)|^2 \tag{17}$$

TABLE I

Comparison of calculations and data for charmonium. Explanation in the text

β_s r_0 (in GeV^{-1}) m_q (in GeV)	1.8 1.27 1.99	2.35 2.04 2.05	2.8 2.50 2.10	Data
$M(1S)$	input	input	input	3095 ± 3[a]
$M(2P)$	3600	3536	3508	3522 ± 5[b]
$M(2S)$	input	input	input	3684 ± 4[c]
$M(3D)$	3806	3796	3785	3772 ± 5[d]
$M(1^3S_1)-M(1^1S_0)$	216	123	98	$\simeq 270$[e]
$M(2^3S_1)-M(2^1S_0)$	38	30	29	$\simeq 230$[f]
$M(2^3P_2)-M(2^3P_1)$	118	108	99	$\simeq 44$
$M(2^3P_1)-M(2^3P_0)$	121	98	86	$\simeq 93$
$\Gamma(1^3S_1 \to l^+l^-)$	15.0	10.2	8.9	4.8 ± 0.6
$\Gamma(2^3S_1 \to l^+l^-)$	2.6	2.9	3.2	2.1 ± 0.3
$\Gamma(2^3S_1 \to 2^3P_2+\gamma)$	37	26	21	16 ± 9
$\Gamma(2^3S_1 \to 2^3P_1+\gamma)$	60	43	35	16 ± 8
$\Gamma(2^3S_1 \to 2^3P_0+\gamma)$	78	56	46	16 ± 9

[a] $M(1^3S_1)$; [b] $\frac{1}{9}\sum_{j=0}^{2}(2j+1)M(2^3P_j)$; one gets precisely the experimental value if $\beta_s = 2.5$ and $r_0 = 2.19$ (then $m_q = 2.07$); [c] $M(2^3S_1)$; [d] $M(3^3D_1)$; a better experimental value to compare with would be $\frac{1}{15}\sum_{j=1}^{3}(2j+1)M(3^3D_j)$; [e] The identification of X(2.82) as 1^1S_0 is, however, not certain; [f] The identification of $\chi(3.45)$ as 2^1S_0 is, however, doubtful.

with c-quark charge Q_q put equal to 2/3. In this Table we give also $\Gamma(2^3S_1 \to 2^3P_j+\gamma)$ — the electric dipole γ rates (in keV) calculated from formula

$$\Gamma(2^3S_1 \to 2^3P_j+\gamma) = \frac{4\alpha Q_q^2(2j+1)}{27}[M(2^3S_1)-M(2^3P_j)]^3|\langle 2P\|r\|2S\rangle|^2, \tag{18}$$

where $\langle 2P\|r\|2S\rangle$ is the radial reduced matrix element. In Eqs. (17) and (18) we use the experimental values of masses. The obtained relation between β_s and r_0 is plotted in Fig. 3.

In the case of Lederman's quarkonium we put [7] $M(1^3S_1) = 9.4$ GeV and $M(2^3S_1)-M(1^3S_1) = 0.6$ GeV, i.e., the same 2^3S_1–1^3S_1 splitting as for charmonium. Then the relation between β_s and r_0 is plotted again in Fig. 3. We can see that the possibility of equal β_s's and r_0's for both quarkonia is not excluded by the above mass input. This case corresponds to $\beta_s = 2.35$ and $r_0 = 2.04\ GeV^{-1}$ which imply that $m_q = 2.05$ GeV and $m_q = 5.36$ GeV for charmed quark and Lederman's quark, respectively. Masses $M(nl)$ calculated in this case for charmonium and Lederman's quarkonium are compared in Table II.

In conclusion, we would like to point out that in the case of charmonium the situation with masses of low lying states, leptonic widths and γ rates in our model (with extended quarks but without confining potential) is somewhat worse but roughly the same as in

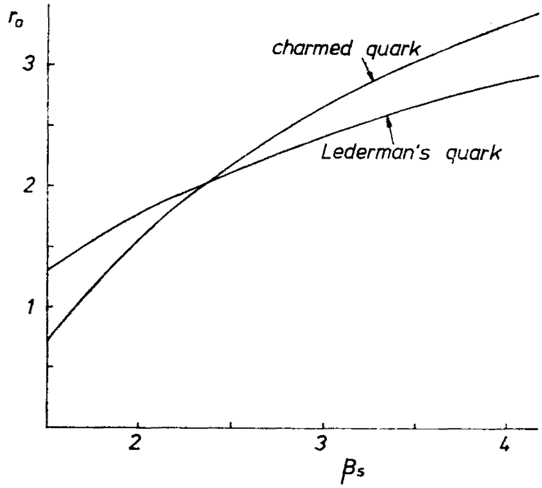

Fig. 3. The dependence $r_0 = r_0(\beta_s)$ for charmed quark and Lederman's quark. The intersection corresponds to $\beta_s = 2.35$ and $r_0 = 2.04\ \mathrm{GeV}^{-1}$ giving the quark mass $m_q = 2.05$ GeV in the first case and $m_q = 5.36$ GreV in the second

the popular (also nonrelativistic) model with pointlike quarks bound by the potential

$$V(r)+V_c(r) = -\frac{\alpha_s}{r}+\frac{r}{\lambda^2}+V_0 \tag{19}$$

plus perturbation terms (13), where our $V(r)$ is replaced by $V(r)+V_c(r)$ as given in Eq. (19). This is correct if not only $V(r)$ but also the confining potential $V_c(r)$ transforms as the time component of a 4-vector. It is known, however, that for scalar $V_c(r)$ the situation in the popular model changes and is rather improved [6, 7]. The fs in our model and in the popular model is of a good size and proper ordering but of a wrong proportion between $M(2^3P_2)$–$M(2^3P_1)$ and $M(2^3P_1)$–$M(2^3P_0)$. The hfs in both models is generally much too small if the leptonic widths have to be not too large. The present experimental identification

TABLE II

Masses of states (in GeV) and average distance between quarks (in GeV^{-1}) in charmonium and Lederman's quarkonium if $\beta_s = 2.35$ and $2r_0 = 4.08\ \mathrm{GeV}^{-1}$ (then $2m_q = 4.1$ GeV and $2m_q = 10.7$ GeV, respectively)

n	l	n_r	Charmonium		L's quarkonium	
			$M(l\,n)$	$\langle r\rangle$	$M(l\,n)$	$\langle r\rangle$
1	S	0	input	1.6	input	1.1
2	P	0	3.536	2.7	9.78	1.6
2	S	1	input	4.1	input	2.2
3	D	0	3.796	4.5	10.07	2.3
3	P	1	3.833	6.1	10.19	3.1
3	S	2	3.883	7.9	10.29	3.9

of 1^1S_0 and 2^1S_0 states is, however, by no means certain. The γ rates in both models are of good order. Notice that our coupling constant $\beta_s = f^2/4\pi \simeq 2$ for extended quarks without confining potential is one order of magnitude larger than the effective (running) coupling constant $\alpha_s = (4/3)g^2/4\pi$ for pointlike quarks with confining potential. We do not discuss in this note hadronic widths as the theoretical situation there is less clear, especially in the case of extended constituent quarks, being much more model-dependent.

One can think of a few improvements of our model:

(*i*) To include a confining potential $V_c(r)$, possibly in the form (11), which may be the time component of a 4-vector or alternatively a scalar or eventually a mixture of both. This can considerably change hfs splitting and improve the proportion in fs splitting.

(*ii*) To take into account in a more exact way the relativistic dynamics [8] in quarkonium and in its leptonic decays [9, 10]. This may hopefully relax the unwanted connection between hfs splitting and leptonic widths and also improve the proportion in fs splitting.

In the picture which emerges from fitting our model to the data, the constituent quark can be characterized by a *large strong-interaction radius* $\gtrsim 1\ \text{GeV}^{-1}$ (which is comparable to actual hadron radii) and *small electromagnetic radius* $\lesssim 0.1\ \text{GeV}^{-1}$ as the latter must be compatible with small bieaking of Bjorken scaling in deep inelastic lepton-nucleon scattering at large $x = Q^2/2M\nu$. Thus, if the constituent quark consists in the valence approximation of one current quark and one current gluon, the gluon must be effectively lighter than the quark, because only the latter carries the electric charge while the colour (which is the strong-interaction charge) is ascribed to both constituents. Then the gluon range in the quark-gluon wave function (which is practically equal to the strong-interaction radius of the constituent quark) is larger than the quark range (which is equal to the electromagnetic radius).

The above picture of the constituent quark may be drastically changed if the confining potential plays an essential role in quark interactions *even* in the lowest charmonium states. It is *not* the case, however, for the confining potential $V_c(r)$ of the form (11) which switches on only for $r \geqslant 2r_0$ (i.e., in the Coulomb region of the potential $V(r)$). This can be seen from Table II which presents the average interquark distance $\langle r \rangle$ in charmonium and Lederman's quarkonium calculated for $\beta_s = 2.35$ and $r_0 = 2.04\ \text{GeV}^{-1}$ (without the confining potential). Table II shows that in the lowest states $1S$ and $2P$ this distance in charmonium is distinctly smaller than $2r_0$. On the other hand, already in the state $3D$ it exceeds $2r_0$ so that the confining potential (11) begins here to be relevant. In Lederman's quarkonium the confining potential is less important if r_0 does not decrease much, which is reasonable as far as r_0 is practically equal to the gluon range. In Table II the "universal" values of parameters β_s and r_0 are used corresponding to the intersection in Fig. 3. Then, still in the state $3S$ the distance $\langle r \rangle$ in Lederman's quarkonium is smaller than $2r_0$.

Calculations including the confining potential of the form (11) are in progress.

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