ON THE PROPERTIES OF NUCLEAR MATTER WITH AN EXCESS OF NEUTRONS, SPIN-UP NEUTRONS AND SPIN-UP PROTONS USING EFFECTIVE NUCLEON-NUCLEON POTENTIAL

By M. Y. M. Hassan*

International Centre for Theoretical Physics, Trieste, Italy

and S. Ramadan

Physics Department, Faculty of Science, Cairo University, Cairo, Egypt

(Received October 18, 1977; final version received March 18, 1978)

The binding energy of nuclear matter with an excess of neutrons, with spin-up neutrons and spin-up protons (characterized by the corresponding parameters $\alpha_\tau = (N-Z)/A$, $\alpha_n = (N\uparrow - N\downarrow)/A$, and $\alpha_p = (Z\uparrow - Z\downarrow)/A$) contains three symmetry energies: the isospin symmetry energy ε_τ, the spin symmetry energy ε_σ and the spin-isospin symmetry energy $\varepsilon_{\sigma\tau}$. These energies are calculated using velocity-dependent effective potential of s-wave interaction, which was developed by Dzhibuti and Mamasakhlisov. The spin, isospin and spin-isospin dependent parts of the single-particle potential in nuclear matter are also calculated using the same effective nucleon-nucleon potentials. The spin-spin part of the optical model potential is estimated. The results are compared with those of Dąbrowski and Haensel, Krewald et al. and Khanna and Barhai.

1. Introduction

Recently, much interest has been devoted to the calculation of the properties of nuclear matter using effective interactions. Lassey [1] has calculated the properties of nuclear matter and semi-infinite nuclear matter using several delta function interactions of the Skyrme type. Khanna and Barhai [2] have calculated the symmetry coefficient ε_τ and the isospin dependent part of the single-particle potential of nuclear matter using two effective interactions determined by them. Krewald et al. [3] have used a modified delta function interaction determined by Moszkowski [4] to calculate the properties of nuclear matter.

---

* On leave of absence from Physics Department, Faculty of Science, Cairo University, Cairo, Egypt.

The ground-state energy of nuclear matter with an excess of neutrons, spin-up neutrons and spin-up protons was considered by Dąbrowski and Haensel (DH) [5, 6], and Dąbrowski [7], using the K-matrix method and applying the Brueckner-Gammel, Thaler, Hamada-Johnston and Reid soft-core nucleon-nucleon potentials.

A velocity dependent effective potential of s-wave interaction with one free parameter was proposed by Dzhibuti and Mamasakhlisov [8]. Dzhibuti and Sallam [9] have used a modified version of this interaction to calculate the binding energy and radii of a large group of nuclei from ^{4}He to ^{208}Pb, and obtained good results. The two versions of this potential were applied to nuclear matter by Hassan et al. [10] and reasonable results were obtained. Therefore we use these two versions also in this work. In Section 2 we give a description of the potential, Section 3 contains the theory, and Section 4 contains the results and discussion.

2. *Description of the potential*

The velocity-dependent effective potential of s-wave interaction is defined by

$$V_{\rm eff}(\vec{r}) = \tfrac{1}{2}\left\{V_{\rm real}(\vec{r})\exp\left[-\vec{a}\frac{\partial}{\partial\vec{r}}\right]+\exp\left[\vec{a}\frac{\partial}{\partial\vec{r}}\right]V_{\rm real}(\vec{r})\right\}_{\vec{a}\to\vec{r}}$$

$$-\lambda(A)\frac{\hbar^2}{M}\{\delta(\vec{r})\nabla^2+\nabla^2\delta(\vec{r})\}, \tag{1}$$

where $V_{\rm real}(\vec{r})$ is the initial realistic potential parametrized in accordance with the two-nucleon problem in vacuum, while $\vec{a}\to\vec{r}$ is the substitution that must be made in the two-particle matrix elements after acting with the operator $\exp[-\vec{a}(\partial/\partial\vec{r})]$ on the wave function of the pair from the right, and with the operator $\exp[\vec{a}(\partial/\partial\vec{r})]$ on the analogous function from the left. The second term on the right-hand side of (1), which contains only the additional parameter λ, represents phenomenologically the multiparticle effects. In the first version of this potential, $V_{\rm real}(\vec{r})$ was taken in the Yukawa form [3], the effective interaction in this case will be referred to from now on as VY (with identical parameters for singlet and triplet central forces)

$$V_{\rm real}(\vec{r}) = -V_0\frac{e^{-\mu r}}{\mu r}, \tag{2}$$

with parameters from the free nucleon-nucleon scattering at low energies,

$$V_0 = 48.1 \text{ MeV}, \qquad \mu = 0.86 \text{ F}^{-1}.$$

In the second version of this potential, $V_{\rm real}(\vec{r})$ was taken in the Gaussian form, the effective interaction in this case will be referred to as VG,

$$V_{\rm real}(\vec{r}) = (a_\tau(\vec{\tau}_1\cdot\vec{\tau}_2)+a_{\sigma\tau}(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2))e^{-r^2/r_0^2}, \tag{3}$$

where σ_i and τ_i are the spin and isospin parameters,

$$a_\tau = 2.096 \text{ MeV}, \qquad a_{\sigma\tau} = 7.767 \text{ MeV}, \qquad r_c = 2.18 \text{ F}.$$

In the case of the VY interaction, Hassan et al. [10] obtained saturation of nuclear matter at $k_f = 1.7\ \mathrm{F}^{-1}$ with B. E./A $= -13.9$ MeV and $\lambda = 1.3\ \mathrm{F}^3$. In the case of the VG interaction, they obtained saturation of nuclear matter at $k_f = 1.35\ \mathrm{F}^{-1}$ with B. E./A = $= -18.1$ MeV and $\lambda = 3.9\ \mathrm{F}^3$.

3. *Theory*

The expression of the binding energy of nuclear matter composed of $N\!\uparrow$ ($N\!\downarrow$) neutrons with spin-up (down) and $Z\!\uparrow$ ($Z\!\downarrow$) protons with spin-up (down), with corresponding Fermi momenta $k_n(\lambda_n)$ and $k_p(\lambda_p)$ can be written in the form [5, 6]

$$E/A = \varepsilon_{\mathrm{vol}} + \tfrac{1}{2}\,\varepsilon_\tau \alpha_\tau^2 + \tfrac{1}{2}\,\varepsilon_\sigma(\alpha_n+\alpha_p)^2 + \tfrac{1}{2}\,\varepsilon_{\sigma\tau}(\alpha_n-\alpha_p)^2,$$

where

$$\alpha_\tau = (N\!\uparrow + N\!\downarrow - Z\!\uparrow - Z\!\downarrow)/A = (N-Z)/A,$$

$$\alpha_n = (N\!\uparrow - N\!\downarrow)/A, \qquad \alpha_p = (Z\!\uparrow - Z\!\downarrow)/A.$$

$\varepsilon_{\mathrm{vol}}$ is the volume energy, ε_τ is the usual isospin symmetry energy, ε_σ is the measure of additional energy necessary to maintain a spin excess in the system, characterized by the spin-excess parameter, $\alpha_\sigma = \alpha_n + \alpha_p$. The quantity ε_σ is referred to here as the spin symmetry energy. $\varepsilon_{\sigma\tau}$ is the measure of the additional energy necessary to maintain in the system an excess of spin-up neutrons and spin-down protons, characterized by the spin-isospin excess parameter $\alpha_{\sigma\tau} = \alpha_n - \alpha_p$. The quantity $\varepsilon_{\sigma\tau}$ is referred to here as the spin-isospin symmetry energy.

Similarly, the single-particle potential $\overline{U}$ (for a neutron (proton) with momentum $\hbar m$ and with spin-up) can be written in the form

$$\overline{U}(m\!\uparrow^{n}_{p}) = U_0(m) \pm \tfrac{1}{4}\,\alpha_\tau U_\tau(m) + \tfrac{1}{4}\,(\alpha_n+\alpha_p)U_\sigma(m)$$
$$\pm \tfrac{1}{4}\,(\alpha_n-\alpha_p)U_{\sigma\tau}(m),$$

and the single-particle potential $\overline{U}$ (for a neutron (proton) with momentum $\hbar m$ and with spin-down) can be written in the form:

$$\overline{U}(m\!\downarrow^{n}_{p}) = U_0(m) \pm \tfrac{1}{4}\,\alpha_\tau U_\tau(m) - \tfrac{1}{4}\,(\alpha_n+\alpha_p)U_\sigma(m)$$
$$\mp \tfrac{1}{4}\,(\alpha_n-\alpha_p)U_{\sigma\tau}(m),$$

where only linear terms are retained.

4. *Numerical results and discussion*

The results of the calculation of ε_τ, ε_σ and $\varepsilon_{\sigma\tau}$ are presented in Table I. The results of the VG interaction are in fair agreement with the values calculated by Dąbrowski and Haensel [5, 7] using Reid soft core potentials (RSC) and by using Landau parameters.

TABLE I

Potential	VG	VY	RSC(1)	RSC(2)	Landau
k_f	1.35	1.7	1.36	1.43	1.36
ε_τ	83.3	95.5	53	60.5	54
ε_σ	66.9	95.5	65	74.1	69
$\varepsilon_{\sigma\tau}$	75.1	95.5	76	86.6	78
U_τ	41.2	111.04		120.8	
U_σ	16.08	111.04		177.8	
$U_{\sigma\tau}$	49.50	111.04		225.3	

The results of VG and VY interactions are given in columns 1 and 2, respectively. Dąbrowski's [7] results are given in columns 3, 4 and 5, respectively.

The value of ε_τ is slightly high since both VY and VG interactions are s-state interactions. Brueckner and Gammel [12] have found that the symmetry energy is very sensitive to the nature of the potential used. They concluded that the potentials which do not contain odd state forces give higher symmetry energy. The experimental values of ε_τ obtained by Green [13] and Cameron [14] are 61 and 63 MeV, respectively. Khanna and Barhai [2] have calculated ε_τ using two effective interactions developed by them and obtained the values 89.78 and 86.32 MeV respectively. Krewald et al. [3] calculated ε_τ and ε_σ and obtained equal values for both of them which is 62 MeV using spin independent modified delta interaction which was developed by Moszkowski [15].

The quantity $\sqrt{\varepsilon_{\sigma\tau}/\varepsilon_\tau}$ can be estimated experimentally. According to Raphael, Uberall and Werntz the energies of the τ and $\sigma\tau$ dipole 1^{-1} levels in ^{16}O are 22.0 and 24.5 MeV, and thus their ratio is 1.1, which is nearly equal to the ratio 1.2 obtained from the theoretical values of ε_τ and $\varepsilon_{\sigma\tau}$ obtained by Dąbrowski [7]. The value of $\sqrt{\varepsilon_{\sigma\tau}/\varepsilon_\tau}$ for the VG interaction is 0.95 and for the VY interaction it is 1. It is clear that both the VG and VY interactions give a smaller value than the experimental one.

The values of the spin, isospin and spin-isospin dependent parts of the single-particle potential in nuclear matter are shown in Table I for VG and VY interactions as well as those calculated by Dąbrowski [7]. The VG interaction gives much smaller values, while the VY interaction gives comparable values to those obtained by Dąbrowski [7]. The spin-spin part of the optical model potential was estimated using the relation [6]:

$$U_{ss} = (U_\sigma \pm U_{\sigma\tau}) \begin{cases} (2I)^{-1} & \text{for } I = j = l+1/2, \\ -2(I+1)^{-1} & \text{for } I = j = l-1/2, \end{cases}$$

where the case in which the total spin I of the nucleus in its ground state is equal to the spin j of a valence nucleon is considered. The positive sign applies to the case when the scattered and the valence nucleon are like nucleons (both neutrons or both protons) and the negative sign to the opposite case. Using VY and VG interactions we get U_{ss} = zero and -6.37 MeV for n-^{59}Co scattering (valence proton configuration $(1f_{7/2})^{-1}$), which are smaller than the value -7 MeV obtained by Dąbrowski. Satchler [16] has estimated

U_{ss} to be -12 MeV. The measurements by Heeringa and Postma [17] of the total spin-spin cross section σ_{ss} for elastic scattering of polarized neutrons from polarized ^{59}Co in the neutron energy range $E_n = 0.39$–2.88 MeV have confirmed earlier data measured up to 8 MeV by Fischer et al. [18]. Their analysis gave $U_{ss} = 47$–84 MeV for $E_n = 0.3$–1 MeV and $U_{ss} = -34$–90 MeV for $E_n = 1$–8 MeV. However, Thompson [19] has demonstrated that for ^{59}Co(n, n)^{59}Co in the low-energy region the spin-spin cross section is probably dominated by compound-elastic scattering and concluded that the extraction of spin-spin terms in an optical-model potential from such data is very questionable.

We may also mention that our results for U_x ($x = \tau, \sigma, \sigma\tau$) are not conclusive since the effective interactions we have used do not depend on the density and U_x depends very much on the rearrangement contribution.

One of the authors (M. Y. M. H.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

Editorial note. This article was proofread by the editors only, not by the author.

REFERENCES

[1] K. R. Lassey, *Nucl. Phys.* **A192**, 177 (1972).
[2] K. M. Khanna, P. K. Barhai, *Phys. Rev.* **C11**, 264 (1975).
[3] S. Krewald, S. A. Moszkowski, V. Klemt, *Z. Phys.* **A276**, 339 (1976).
[4] S. A. Moszkowski, *Phys. Rev.* **C2**, 402 (1970).
[5] J. Dąbrowski, P. Haensel, *Phys. Rev.* **C7**, 916 (1972).
[6] J. Dąbrowski, P. Haensel, *Phys. Lett.* **42**, 163 (1972).
[7] J. Dąbrowski, *Acta Phys. Pol.* **B7**, 657 (1976).
[8] R. I. Dzhibuti, V. I. Mamasakhlisov, *Soobshchenie (Communications) Akad. Nauk Gruz. SSR* **54**, 57 (1969).
[9] R. I. Dzhibuti, H. M. Sallam, *Sov. J. Nucl. Phys.* **19**, 40 (1974).
[10] M. Y. M. Hassan, A. M. A. Ghazal, K. M. Mahmoud, to appear in *Z. Phys.*
[11] R. I. Dzhibuti, N. B. Krupennikova, *Sov. J. Nucl. Phys.* **11**, 199 (1970).
[12] K. Brueckner, G. L. Gammel, *Phys. Rev.* **109**, 1023 (1958).
[13] A. E. S. Green, *Rev. Mod. Phys.* **30**, 569 (1958); *Phys. Rev.* **95**, 1006 (1954).
[14] A. G. W. Cameron, *Can. J. Phys.* **35**, 1021 (1957).
[15] S. A. Moszkowski, *Phys. Rev.* **C2**, 402 (1970).
[16] G. R. Satchler, *Phys. Lett.* **B34**, 37 (1971); *Particles and Nuclei* **1**, 397 (1971).
[17] W. Heeringa, H. Postma, *Phys. Lett.* **B61**, 350 (1976).
[18] T. R. Fisher, H. A. Grench, D. C. Healey, J. S. McCarthy, D. Parks, R. Whitney, *Nucl. Phys.* **A179**, 241 (1972).