# SOME ASPECTS OF LARGE $p_T$ PHYSICS

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An elementary survey is given of the application of the parton model to large $p_T$ processes. Topics dealt with are: multiple scattering processes, logarithmic scale breaking, the Drell-Yan process, hard scattering mechanisms, parton $k_T$'s, critical phenomenological questions, downstream calorimetric triggers. The emphasis throughout is on basic ideas and techniques.

## 1. Introduction

In this brief course of lectures I cannot cover the whole range of large $p_T$ theory and phenomenology. Instead what I shall try to do is to point to certain features which I think are of special importance.

It will be very much one man's view and I shall not aim for completeness of coverage or referencing. I shall suppose that we are dealing with a hard scattering process like that of Fig. 1. The basic interaction in which the large $p_T$ is generated is the wide angle scattering

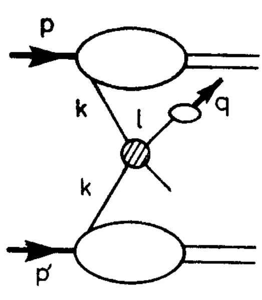


Fig. 1. A hard scattering process

of two constituents, one drawn from each incident hadron. The observed outgoing particle is a fragment of one of the products of this collision, or may be one of the products itself in appropriate circumstances. Such a picture implies that the final state will consist of four

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approximately coplanar jets of particles: the two longitudinal jets corresponding to the residual beam constituents and two sideways jets with balancing transverse momentum corresponding to the fragmentation of the products of the hard scatter. I shall not summarize again the well known evidence from correlation experiments which supports such a view [1]. The cross-section is found to take the scaling form

$$E\frac{d\sigma}{d^3p} \sim \frac{1}{p_T^n} f(x_T, \theta), \qquad x_T = \frac{2p_T}{\sqrt{s}}, \tag{1.1}$$

for the production of a particle of transverse momentum $p_T$ at an angle $\theta$ in the centre of mass of the primary collision. Calculating this cross-section will involve the probabilities of finding constituents within hadrons, the probability of finding the observed hadron within its parent constituent, and the cross-section for the hard scattering process. Only the latter has dimensions so that the power of $n$ in (1.1) is determined by the nature of the hard scatter.

## 2. *How to calculate*

The first thing to do is to get some idea of how to calculate such a process. The basic technique is simple. The momentum $k$ of a constituent is expected to be approximately parallel to that of its parent $p$. It is possible to choose a frame of reference in which we can write the latter as

$$p = (\tfrac{1}{2}\sqrt{s}, 0, 0, \tfrac{1}{2}\sqrt{s}), \tag{2.1}$$

neglecting masses at high energy $s$. Then a suitable representation of $k$ is

$$k = \left(\tfrac{1}{2}x\sqrt{s} + \frac{y}{\sqrt{s}}, \kappa_1, \kappa_2, \tfrac{1}{2}x\sqrt{s}\right). \tag{2.2}$$

Clearly this corresponds to $k$ carrying a fraction $x$ of the large longitudinal momentum of its parent, while $y$ and $\boldsymbol{\kappa}$ measure the deviations from exact parallelism. The form (2.2) is chosen to ensure that

$$k^2 = xy - \boldsymbol{\kappa}^2 \tag{2.3}$$

is finite for finite values of the parameters. It is an important assumption of parton theory [2] that the significant regions of integration correspond to finite values of the virtual parton masses $k^2$; that is the parton amplitudes are taken to decrease rapidly when the $k^2$ become large (see also Section 4 below). Instead of integrating over the four components of $k$ we integrate over $x, y, \boldsymbol{\kappa}$:

$$d^4k \to \tfrac{1}{2}dxdyd^2\boldsymbol{\kappa}. \tag{2.4}$$

Notice that (2.4) does not contain any power of $\sqrt{s}$.

Exactly similar parametrisation can be introduced for the other parton vectors $k'$, $l$. Of course their associated hadronic momenta $p'$ and $q$ have a different form from (2.1) in the overall centre of mass, chosen as the frame for $p$, but it is only necessary to make the appropriate rotation of forms like (2.1) and (2.2) to deal with this. The essential point is that none of these transformations will introduce powers of $\sqrt{s}$.

### 3. *The Landshoff process*

We can put these ideas to immediate use by calculating a process first discussed by Landshoff [3]. It pictures (for example) wide angle π-π scattering occurring in the way illustrated by Fig. 2. The q and $\bar{q}$ from one pion each scatters off q or $\bar{q}$ from the other

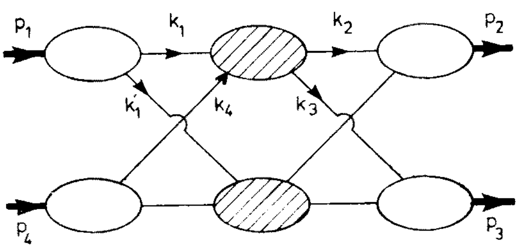


Fig. 2. The Landshoff process for ππ scattering

pion. The outgoing quarks are cleverly aligned so that they can recombine to form outgoing pions. One might suppose that such a mechanism is fantastically improbable but in fact it proves not to be so. The calculation goes as follows.

We approximately line up each $k_i$ with its $p_i$ using (2.1) and (2.2). By momentum conservation the associated $k_i' = p_i - k_i$ is then also in the same form. Section 2 teaches us that this costs no powers of $\sqrt{s}$. If the quark hard scatters are scale-free then their matrix elements also contain no powers of $\sqrt{s}$. The only dependence on $\sqrt{s}$ comes from requiring momentum conservation for the hard scatters:

$$k_1 + k_4 - k_2 - k_3 = 0, \qquad k_1' + k_4' - k_2' - k_3' = 0. \tag{3.1}$$

Since three of the components of the vectors of (3.1) are large (corresponding to the energy and the momenta in the scattering plane) the δ-functions enforcing the first equation of (3.1) produce a net factor of $(\sqrt{s})^{-3}$. The second equation then follows without further cost from overall energy-momentum conservation. Thus the high energy behaviour of the matrix element associated with Fig. 2 is $s^{-3/2}$ so that the differential cross-section behaves like

$$\frac{d\sigma^{\pi\pi}}{dt} \sim \frac{1}{s^5} f(\cos\theta), \qquad s \to \infty, \qquad \theta \text{ fixed}. \tag{3.2}$$

This is to be compared with the $s^{-6}$ behaviour expected from dimensional counting arguments.

For the physically more interesting case of proton-proton scattering the discussion is similar. There are now three quark hard scatters and enforcing momentum conservation for them costs $(s^{-3/2})^2$ in the matrix element. Thus we expect

$$\frac{d\sigma^{pp}}{dt} \sim \frac{1}{s^8} f(\cos\theta), \tag{3.3}$$

compared with the $s^{-10}$ of dimensional counting. The latter appears to be dominant at moderately high energies and it is of the highest interest to see if (3.3) manifests itself as the energy increases. In the fixed angle regime both invariants $s$ and $t$ are large. A detailed calculation using spin $\frac{1}{2}$ quarks interacting by vector gluon exchange shows that in the moderate angle regime

$$m^2 \ll t \ll s, \tag{3.4}$$

(3.3) takes the form

$$\frac{d\sigma^{pp}}{dt} \sim \frac{1}{t^8}, \tag{3.5}$$

a behaviour which appears consistent with ISR data [4].

### 4. *Logarithmic modifications*

The parton model as described so far gives exact Bjorken scaling in electroproduction. In fact quantum field theories like QCD give logarithmic violations of scaling and experimentally scale-breaking of at least qualitatively the right character seems to be observed. It is possible to modify the parton model to take this into account [5].

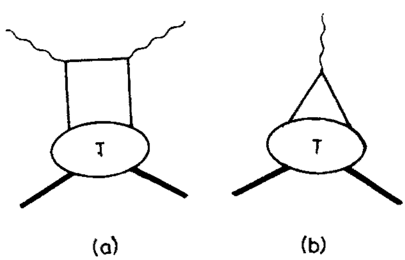


Fig. 3. The handbag diagram

A discussion of deep inelastic electroproduction starts with the celebrated "handbag" diagram of Fig. 3a. Large momentum has to be transferred through the diagram and this is most economically done by carrying it across the "handle" of the bag, that is the line joining the two current vertices. This produces an $s^{-1}$ behaviour multiplied by a coefficient which is proportional to the contribution of Fig. 3b. This vertex part is obtained by contracting out the line which carried the large momentum. Such a result only gives the true asymptotic behaviour if this vertex part integral is convergent. The original covariant formulation of the parton model assumed sufficient damping of the parton amplitudes at large values of $k^2$ to make this so. In renormalizable field theories such vertex parts are divergent. If the integrals are logarithmically divergent it means that the true asymptotic behaviour is enhanced by corresponding factors of $\log s$. When moments of the structure functions are calculated these logarithms exponentiate to give power-law scale-breaking. The logarithmic scale-breaking of asymptotically free gauge theories will be simulated by log log divergent vertex parts.

The sort of structure necessary to describe this is illustrated in Fig. 4a. The amplitude $A$ contains all the sources of divergent integrals in Fig. 3b, and $\tilde{T}$ is a reduced amplitude free from such divergences. This decomposition corresponds to the Operator Product Ex-

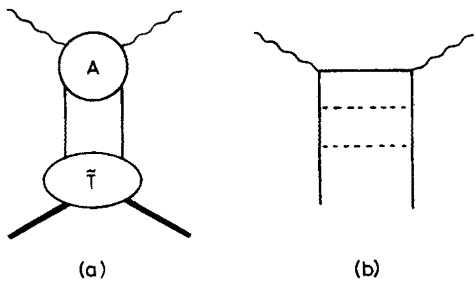


Fig. 4. The Asymptotically Free Parton Model Structure

pansion decomposition into singular functions ($= A$) and hadronic matrix elements of operators ($= \tilde{T}$). A very simple model for $A$ is shown in Fig. 4b. The broken lines represent gluon exchange. If the gluons are point coupled and have a propagator

$$\sim \frac{1}{k^2}, \quad k^2 \to \infty, \tag{4.1}$$

then they give logarithmic divergences in Fig. 3b. To get the log log divergences of asymptotically free theory we must replace (4.1) by

$$\sim \frac{1}{k^2 \ln k^2}, \quad k^2 \to \infty. \tag{4.2}$$

The extra factor of $(\ln k^2)^{-1}$ clearly simulates the effect of a running coupling constant in asymptotically free theories. Equation (4.2) corresponds to the Asymptotically Free Parton Model (AFPM) as originally formulated [5]. A simple ladder structure like Fig. 4b is easy to calculate. It leads simply to the Drell-Yan conjecture that the structure function for massive lepton pair production ($F$) is given by a convolution of quark ($f$) and anti-quark ($\bar{f}$) structure functions:

$$F = f * \bar{f}. \tag{4.3}$$

However more realistic theories involving vector gluons have a much more complex structure and the DY conjecture is correspondingly more difficult to discuss. Important progress has been made recently by Stirling [6].

Fig. 5 illustrates the sort of contributions to electroproduction and the DY process that must be considered. There are three categories:

(*i*) Effects, represented by black blobs, associated with the infinite renormalisations necessary for vertex ($V$) and propagators ($P$). At first sight these do not appear to match

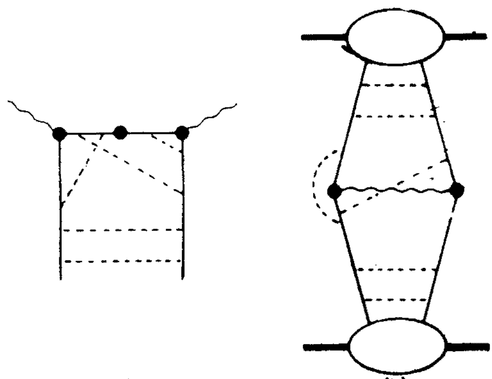


Fig. 5. The Structure of electroproduction and Drell-Yan contributions in a theory with vector gluons

in the way (4.3) would require, for Fig. 5a has $2V+P$ for $f$ while Fig. 5b has $2V$ for $F$. However the Ward identity for electromagnetic interactions implies that

$$V+P = 0, \tag{4.4}$$

so that in fact the correspondence is exact.

(*ii*) The ladders of exchanged gluons. Clearly they match diagrammatically in a way that appears to agree with (4.3). The AFPM calculation [7] shows that this is indeed the case.

(*iii*) The finite contributions from vertex gluons which loop about the current vertices. The hard part of verifying the DY conjecture lies here. Diagrammatically it is far from obvious that they respect (4.3). In Fig. 5a they are attached to the parton propagator joining the two current vertices. In Fig. 5b there is no such propagator, but instead a photon propagator to which such gluon lines can not be attached but which they can cross in various ways. Nevertheless Stirling has given a way of associating these contributions to Fig. 5b with the convolution of contributions to Fig. 5a and verified to second non-trivial order (two-gluon exchange) that it indeed works. For a description of his prescription the reader should refer to the original paper [6]. However I would like to underline one essential ingredient in the analysis.

This is the idea of *independent scaling sets* in the extraction of the high energy behaviour of Feynman integrals. The basic notion is simple. The dominant behaviour always comes from where the coefficient of the large variable ($s$) is small. Suppose, for example, that this is

$$\alpha_1\alpha_2 s, \tag{4.5}$$

with the $\alpha$'s constrained by

$$\alpha_1+\alpha_2 = 1. \tag{4.6}$$

Then the asymptotic behaviour comes from either $\alpha_1 = 0$ or $\alpha_2 = 0$, but of course (4.6) prevents both vanishing simultaneously. There are thus two independent contributions to the asymptotic behaviour which can be labelled by $\alpha_1$ or $\alpha_2$ and which must be added together. These are the two independent scaling sets in this simple case. More elaborate examples are discussed in Chapter 3 of reference 8.

The integrals associated with Fig. 5b contain many such independent scaling sets. The identification of the contributions to different terms in the convolution (4.3) is shown by Stirling to depend upon separating these sets according to the lines finally scaled over. If these all lie on one side of the photon line the contribution corresponds to one in which the vertex gluon structure is associated wholly with one of the $f$'s while the other $f$ corresponds to a simple propagator joining the current vertices. If the lines of the final scaling lie on both sides of the photon line then the vertex gluon lines are shared between the two $f$'s according to rules stated by Stirling.

## 5. *Large $p_T$ mechanisms*

Current data suggest that (1.1) holds in the regime investigated with $n = 8$. Two groups of hard scattering mechanisms have been considered as possible source of interaction:

(*i*) *Natural mechanisms*

$$\mathrm{q+q\to M+M}\ \text{(quark fusion)},\quad \mathrm{q+M\to q+M}\ \text{(CIM)},\quad \mathrm{q+q\to(qq)+M}\ \text{(diquark)}, \tag{5.1}$$

In each of these processes M denotes a hadronic system with meson quantum numbers ($q\bar{q}$) but which only exceptionally is expected to materialise as a single meson or resonance. The mechanisms (5.1) are called natural because dimensional counting ideas applied to them give $n = 8$ in (1.1)

(*ii*) *Fundamental mechanisms*

$$\mathrm{q+q \to q+q}, \qquad \mathrm{q+g \to q+g}, \qquad \mathrm{g+g \to g+g}. \tag{5.2}$$

g denotes a gluon and it is understood that the analogous processes with antiquarks are also included. The mechanisms (5.2) are termed fundamental for obvious reasons.

Dimensional counting applied to (5.2) would give (1.1) with $n = 4$ in flagrant disagreement with experiment. However these are various effects which could modify the dimensional counting prediction in the presently observed regime. They are:

(*i*) Scale breaking in the quark distribution and fragmentation functions. Although at truly asymptotic energies this is only a logarithmic effect it can be important for current phenomenology.

(*ii*) Scattering by the mechanism of gluon exchange must take account of the running coupling constant $\alpha(Q^2) \sim (\ln Q^2)^{-1}$. Again this asymptotically logarithmic effect can have sizable consequence in a subasymptotic regime.

(*iii*) The partons themselves possess transverse momentum. If the two initial state partons are in a configuration that lines up their $k_T$ in the direction of the observed large $p_T$ final state hadron then less of that hadron's $p_T$ has to be generated by the hard scatter itself. Since the cross section for the latter decreases steeply with $p_T$ generated a significant enhancement of the overall cross-section occurs in this favoured configuration. Of course against this must be set a diminished cross-section for unfavoured configurations in which the parton $k_T$'s line up in the wrong direction, but it turns out that a net enhancement results from this $k_T$ effect. Naturally the enhancement becomes less significant as the observed $p_T$ becomes large compared with the available parton $k_T$. Thus an enhancement decreasing with $p_T$ is produced which has the effect of steepening the $p_T$ dependence of the observed inclusive cross-section, that is increasing the effective value of $n$.

Field [9] has estimated that a combination of these effects (*i*) to (*iii*) can enable the fundamental mechanisms to explain present data. However he needs to choose a very large value for the parton $k_T$,

$$\langle k_T \rangle \sim 850 \text{ MeV}/c. \tag{5.3}$$

The results obtained depend on the details of the $k_T$ distributions – exponential or power law or what? (Theory suggests power law as the most likely). Big contributions then come from the tails of the $k_T$ distributions for which the central "hard scatter" is in fact rather low energy and soft. In that circumstance a further uncertainty attaches to what form the central scattering cross-section should be like in this soft regime. It will contain asymptotically factors of $s^{-1}$ and $t^{-1}$ which certainly can not be extrapolated to small values of $s$ and $t$!

## 6. *Parton $k_T$'s*

The subject of parton $k_T$'s deserves further consideration. It is reasonable to ask if there is supporting evidence for so large an effect as (5.3). The parton $k_T$'s produce the average $p_T$ of $\mu^+\mu^-$ pairs created by the Drell-Yan process of quark-antiquark fusion. They also manifest themselves in the average $\langle p_{out} \rangle$ of away side particles, where $p_{out}$ is the momentum component perpendicular to the plane defined by the incident beams and the trigger particle. In both these effects values are observed which appear consistent with (5.3).

A more subtle effect is associated with the residual longitudinally moving fragments of the incident hadrons. These have to recoil in the away side direction to compensate for the trigger-side $k_T$ of the interacting partons. This affects the observed distribution of moderate $p_T$ particles in the away-side hemisphere and this must be taken into account in calculating such distributions.

On a more fundamental level one can distinguish two sources of parton $k_T$, corresponding to the two parts of Fig. 4a. $\tilde{T}$ will give a bounded, but possibly numerically significant, contribution. This is sometimes called the "primordial" term. The QCD processes in $A$, on the other hand, will give a contribution to $\langle k_T \rangle$ which, to within logarithmic factors, must grow like $\sqrt{Q^2}$. This is because there is no other scale than $Q^2$ in QCD, or alternatively, because the log log divergent integrals over $k_T$ which give the scale breaking effects become linearly divergent when one calculates $\langle k_T^2 \rangle$ and are thus proportional to $Q^2$. Theoretical attempts to calculate the dependence of $\langle k_T \rangle$ on $x$, the longitudinal fraction of the parton, have been many and varied but no universally agreed form has emerged. The fact that the general picture described in this paragraph seems to accord quite well with data will be discussed in detail in Professor Yan's lectures.

## 7. *Aspects of large $p_T$ phenomenology*

Detailed reviews of large $p_T$ phenomenology are available [1] and the paper of Chase and Stirling [10] gives a good comparative discussion of the various mechanisms (5.1) and (5.2). In this brief section we draw attention to some points that seem to be of particular interest.

(a) *dN/dy*. The distribution of away side particles opposite to trigger is principally controlled by the form of the hard scattering cross-section. Analysis [11] of the observed rather flat distributions in rapidity $y$ indicate cross-sections behaving like

$$\frac{d\sigma}{dt} \sim \frac{1}{st^3}, \tag{7.1}$$

which is exactly what would result from the spin $\frac{1}{2}$ exchange of the natural mechanisms (5.1).

(b) $\langle z \rangle$, where

$$\langle z \rangle = \frac{p_{\text{trigger}}}{\langle p_{\text{jet}} \rangle}. \tag{7.2}$$

The average fraction of the trigger side $p_T$ taken by the trigger particle appears to be $\langle z \rangle \gtrsim 0.9$ [12]. It is very important to know this quantity as accurately as possible since it provides an important clue to the nature of the outgoing large $p_T$ system. So large a value of $\langle z \rangle$ is difficult to reconcile with a quark fragmenting into a pion, even if the fragmentation function is assumed to go to a constant as $z \to 1$, rather than vanishing like $(1-z)$. (In principle this behaviour can be checked in $e^+e^-$ annihilation). However for the natural mechanisms it is always possible for M to materialise as a single pion. Even if this happens only at the percent level it easily gives a value of $\langle z \rangle \sim 0.9$. This is because of *trigger bias*. When one triggers on a large $p_T$ particle the (presumably rare) fragmentation modes in which $z$ is close to or equal to 1 are greatly enhanced in effect. This is because the steeply falling hard-scattering cross-sections make them the most economic way of producing large $p_T$ trigger. (It is easier to get a 3 GeV/$c$ $\pi$ by a rare mode in which an outgoing system of $3\frac{1}{2}$ GeV/$c$ (say) decays into 3 GeV/$c$+$\frac{1}{2}$ GeV/$c$ rather than by a common mode in which 6 GeV/$c$ outgoing systems decays into 3 GeV/$c$+3 GeV/$c$.) Hence large $z$ fragmentation modes dominate.

An important consequence of trigger bias is that calorimetric detectors, which are free from bias and just trigger on the total $p_T$ in the outgoing jet, are expected to find cross-sections enhanced by two orders of magnitude over the case of a single particle trigger, since they count all the fragmentation modes equally. Experiments at FNAL have shown that this is indeed the case.

(c) *Beam ratios*. The QF and CIM mechanisms are much more effective with pion beams than proton beams since the former contains $\bar{q}$'s or M's more readily. The experimental ratios do not show such a striking effect. DQ does not suffer from the same problem. However too much of it can not be present without spoiling the $\pi^+/\pi^-$ production ratio for proton beams; see the discussion of Chase and Stirling [10]. This is the principal phenomenological difficulty for the natural mechanisms.

(d) *Gluons and photons*. The addition of gluons to the fundamental mechanism (5.2) is helpful in fitting the low $x_T$ data (the gluons are, of course, expected to be rather low $x$ partons in hadrons, being associated with the "sea" rather than "valence" distributions). In the outgoing state the substitution

$$g \to \gamma \tag{7.3}$$

will produce processes which can account for the production of the $\gtrsim 10\%$ level direct large $p_T$ photons which appears to be observed at the ISR. For the natural mechanisms

$$M \to \gamma \tag{7.4}$$

will play a similar role [13]. There is however an important distinction between the two processes (7.3) and (7.4). In (7.3) an elementary g is replaced by an elementary $\gamma$; in (7.4) a composite M is replaced by an elementary $\gamma$. Thus for (7.3) the $\gamma/\pi$ ratio is $p_T$ independent but for (7.4) it increases with $p_T$ like $p_T^2$. It clearly would be of the highest interest to determine how the $\gamma/\pi$ ratio behaves experimentally.

The rather large $\gamma/\pi$ ratios ($\sim 10\%$) obtained from calculations based on (7.3) or (7.4) are due to the absence of fragmentation effects for a photon. Thus the fine structure constant $\alpha$ should be multiplied into the hadronic *jet* cross-section to get an order of magnitude estimate for $\gamma$ production.

(e) *Baryons*. Baryons are copiously produced at large $p_T$ but their cross section corresponds to (1.1) with $n = 12$ rather than 8. This fact in itself is significant for it clearly indicates that these must be a variety of underlying mechanisms operating in large $p_T$ hadronic production. Possible mechanisms for baryon production are

$$q+B \to q+B, \tag{7.5}$$

which according to dimensional counting include has $n = 12$, and a multiple scattering mechanism [14] analogous to the Landshoff mechanism for exclusive scattering. The latter gives $n = 14$.

Escobar [15] has recently discussed these mechanisms in relation to data. He concludes that in the presently accessible regime (7.5) is dominant in the leading particle mode (that is with the initial B the incident proton itself rather than a baryonic constituent) and that the final B must be interpreted as a baryonic jet, just as M is interpreted as a mesonic jet. It is important to note that in the leading particle mode (7.5) gives a narrowly peaked away-side distribution, since the integration over incident longitudinal function $x$, which produces the "fan"-like $dN/dy$ for meson mechanisms, is in this case absent for B.

The multiple scattering mechanism may be important for explaining why large $p_T$ cross-sections on nuclei manifest on $A$-dependence greater than $A^1$. It is known that conventional final state interactions, which would give such an $A$-dependence, cancel out in the cancelation of cross-sections for hard scattering processes.

## *8. Downstream calorimeter triggers*

In a normal longitudinal event a downstream calorimeter in the direction of one of the beams will receive energy $E' = \frac{1}{2}\sqrt{s}$ deposited in it. Ochs and Stodolsky [16] pointed out that a significant deviation of the energy deposit $E$ from this value $E'$ would be a signal that some sizeable transverse component has been generated in the interaction. Clearly such a trigger is freeer from bias about the nature of the $p_T$-generating interaction than is a transverse calorimeter which looks for a jet in a particular direction.

Ochs and Stodolsky (OS) went on to introduce an important scaling law. Define

$$\tau = 2E/\sqrt{s}. \tag{8.1}$$

Transverse events correspond to $\tau$ significantly different from 1. In fact $\tau$ can be either greater or less than 1 but it will be convenient in this simple discussion to concentrate on the case $\tau < 1$. According to a hard scattering picture the deficit $(1-\tau)$ has been caused by a constituent carrying fraction $(1-\tau)$ of its parent's energy being scattered outside the calorimeter (whose half-angle acceptances is taken to be $\alpha$). If this scattering is scale-free OS found the cross-section took the form

$$\frac{d\sigma}{d\tau} = \frac{1}{s}\frac{f(1-\tau)}{1-\tau}G(\xi), \qquad \xi = (1-\tau)\tan^2\tfrac{1}{2}\alpha. \tag{8.2}$$

In (8.2) the factor $s^{-1}$arises from obvious dimensional considerations, and $f(1-\tau)$ is the probability of finding the parton with longitudinal fraction $(1-\tau)$ in the beam hadron. The interesting term is $G$. Instead of being a function of the dimensionless variables $\tau$ and $\alpha$ separately, according to OS it depends upon them only in the combination given by $\xi$.

Landshoff and I [17] have verified (8.2) in a parton model calculation and given the expression for $G$ in terms of an integral of the hard scattering cross-section and constituent function associated with the second beam particle. We have also considered (8.2) in the limit $\tau$ small, corresponding to finding energy deposit $E = E_0$. If

$$f(1-\tau) \sim \tau^m, \qquad \tau \to 0, \tag{8.3}$$

and we take $m = 3$ for a proton beam, one finds

$$\frac{d\sigma}{dE_0} \sim s^{-3}E_0^3 \tag{8.4}$$

or calculating the cross-section for energy deposit $E \leqslant E_0$,

$$\sigma(E_0) \sim s^{-3}E_0^4. \tag{8.5}$$

There is a multiple scattering mechanism which could also contribute with

$$\frac{d\sigma_{\text{mult}}}{d\tau} \sim s^{-3}M^4\delta(\tau), \tag{8.6}$$

with $M$ a fixed parameter determined by quark masses and the proton's wavefunction. It will give a contribution similar to (8.5)

$$\sigma_{\text{mult}}(E_0) \sim s^{-3}M^4. \tag{8.7}$$

I our paper we make various estimates of these processes. For details see reference [17].

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