STUDY OF THE PRODUCTION PROPERTIES OF $\Sigma(1385)$ BY MEANS OF THE STATISTICAL TENSOR METHOD

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The complete spin density matrix of $\Sigma^+(1385)$ has been determined by means of the statistical tensor method in the quasi two-body reactions $\pi^+p \to K^+ \ \Sigma^+(1385)$, $K^-p \to \pi^-\Sigma^+(1385)$ and in the inclusive reactions $\pi^+p \to \Sigma^+(1385)+X$, $K^-p \to \Sigma^+(1385)+X$, all at 16 GeV/$c$. It has been found that the experimental results are in good agreement with the predictions of the additive quark model (Q.M.) in the case of the inclusive reactions. In the two-body reactions the statistics is too small to make stronger quantitative statements, the results, however, are not inconsistent with Q.M.

1. Introduction

Recently there have been several papers on the determination of complete spin density matrix in two body reactions in which the information can be extracted from cascade decay. An obvious candidate for such analysis is $\Sigma^+(1385) \to \Lambda^0\pi^+$ for which such analysis has been done at 4.2 GeV/c K^-p [1, 2]. Another way of presenting the full information about the production of a particle with a given spin parity is provided by the statistical tensor formalism, and has been used by the ABCCLVW collaboration in π^+p and K^-p reactions at 8 and 10 GeV/c, respectively [3].

The statistical tensor method while being equivalent to the usually applied method of determining density matrix elements, is particularly convenient, as the values of the tensors are directly related to the moments of the decay angular distributions, and the predictions of the additive quark model are direct statements about these values.

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In Ref. [3] the analysis has been done on a comparatively small statistics of $\pi^+ p \to \Lambda+2$ charged prongs at 8 GeV/c and $K^- p \to \Lambda+2$ or 4 prongs at 10 GeV/c, fitted channels only.

Meanwhile more data have been obtained on the K^-p and π^-p reactions also at a higher energy value, i. e. 16 GeV/c.

The aim of this paper is to perform an analogous analysis on a much bigger data sample, and in particular to test whether the possibility indicated in the previous paper [3] that one of the statistical tensor components deviates from the predictions of the quark model is confirmed.

2. *Method of analysis*

The formalism of the method has been described in detail in Ref. [4] while the explicit formulae have been given in [3].

Here they are again summarized for the non-zero components ($T^J_M = 0$ for M odd, due to parity conservation in strong Σ (1385) decay).

$$T^1_0 = \frac{9\sqrt{5}}{8\alpha_\Lambda} \langle e^{\mp i\varphi_p} \sin \Theta_p \sin \Theta \rangle, \tag{1a}$$

$$T^1_0 = \frac{9\sqrt{5}}{2\alpha_\Lambda} \langle \cos \Theta_p \cos \Theta \rangle, \tag{1b}$$

$$T^2_2 = -\tfrac{5}{4}\sqrt{\tfrac{3}{2}}\langle e^{2i\varphi} \sin^2 \Theta \rangle, \tag{2}$$

$$T^2_0 = -\tfrac{5}{4}\langle 3 \cos^2 \Theta - 1 \rangle, \tag{3}$$

$$T^3_2 = -\frac{35}{16\alpha_\Lambda}\sqrt{\tfrac{3}{2}}\langle e^{\mp i\varphi_p + 2i\varphi} \sin \Theta_p \sin \Theta(3 \cos^2 \Theta \mp 2 \cos \Theta - 1)\rangle, \tag{4a}$$

$$T^3_2 = -\frac{35}{4\alpha_\Lambda}\sqrt{\tfrac{3}{2}}\langle \cos \Theta_p \sin^2 \Theta \cos \Theta e^{2i\varphi}\rangle, \tag{4b}$$

$$T^3_0 = \frac{21\sqrt{5}}{16\alpha_\Lambda}\langle e^{\mp i\varphi_p} \sin \Theta_p \sin \Theta(5 \cos^2 \Theta - 1)\rangle, \tag{5a}$$

$$T^3_0 = \frac{7\sqrt{5}}{4\alpha_\Lambda}\langle \cos \Theta_p(5 \cos^3 \Theta - 3 \cos \Theta)\rangle. \tag{5b}$$

The fact that some components can be determined in a different way (1a, b), (4a, b), (5a, b) comes from the possibility of using either transverse (a) or longitudinal (b) polarization of Λ^0 for odd J values. Θ and φ describe the Σ(1385) decay in its transversity frame, Θ being the angle between the Λ momentum $\mathbf{p}_\Lambda$ and the normal to the Σ(1385) production plane taken as z axis, φ the azimuthal angle between the projection of $\mathbf{p}_\Lambda$ on the x, y plane and the x axis, where the y axis is opposite to the direction of K^+(π^- or X) and x defined so as to obtain a right-handed coordinate system.

Similarly Θ_p and φ_p determine the direction of the momentum of the decay proton in the Λ rest frame, however in this case the z axis is opposite to the $\Sigma(1385)$ direction, and y is normal to the decay plane of the resonance.

The asymmetry parameter α_Λ has been accepted as 0.647 (Ref. [5]).

3. Additive Quark Model predictions

If the additivity assumption holds, the Quark Model gives complete predictions for all components of the statistical tensor in two-body reactions with definite spin parities J^P of the particles involved.

In particular, in the reactions

$$K^-p \to \pi^-\Sigma^+(1385)$$

and

$$\pi^+p \to K^+\Sigma^+(1385)$$

which are both of the same type:

$$0^-\tfrac{1}{2}^+ \to 0^-\tfrac{3}{2}^+$$

all T^J_M components should be equal to 0 except $T^2_0 = -0.5$. For the inclusive reactions:

$$\pi^+p \to \Sigma^+(1385)+X, \qquad K^-p \to \Sigma^+(1385)+X$$

the additive Q. M. gives some predictions only if $\Sigma(1385)$ is produced as the only particle in the baryon vertex, and both reactions are of the type $0^-\frac{1}{2}^+ \to X\frac{3}{2}^+$. In such a case additivity predicts

$$T^3_0 = T^3_2 = 0.$$

4. Data sample and results

For our analysis we use data from the K^-p and π^+p experiments at 16 GeV/c. More detailed information on these experiments has been published in Refs [6, 7]. We also perform the analysis for a bigger sample of interactions at 10 GeV/c than in the previous paper [3].

The experiments have been made at CERN in the 150 cm British and 200 cm CERN hydrogen bubble chambers. The following reactions have been analysed:

$$\pi^+p \to K^+\Sigma^+(1385), \tag{6}$$

$$\pi^+p \to \Sigma^+(1385)X, \tag{7}$$

$$K^-p \to \pi^-\Sigma^+(1385), \tag{8}$$

$$K^-p \to \Sigma^+(1385)X, \tag{9}$$

where X denotes a mesonic system with $S = Q = 1$ in the reaction (7) and $S = 0$, $Q = -1$ in the reaction (9).

In order to extract the $\Sigma(1385)$ production in the baryon vertex we have applied the following selection criteria:

$$1.32 < M_{\Lambda\pi} < 1.45 \text{ GeV},$$

$$|t| \text{ or } t' < 1 \text{ GeV}^2.$$

We have also excluded from the analysis the $\Lambda\bar{\Lambda}$ and $p\bar{p}$ channels, as baryon antibaryon pairs are likely to be produced in the central region. The $\Lambda\pi^+$ mass distributions for the reactions (6), (7), (9) are shown in Figs 1, 2, 3 respectively. Fig. 4 shows the mass distribution after the t-cut (1 GeV^2).

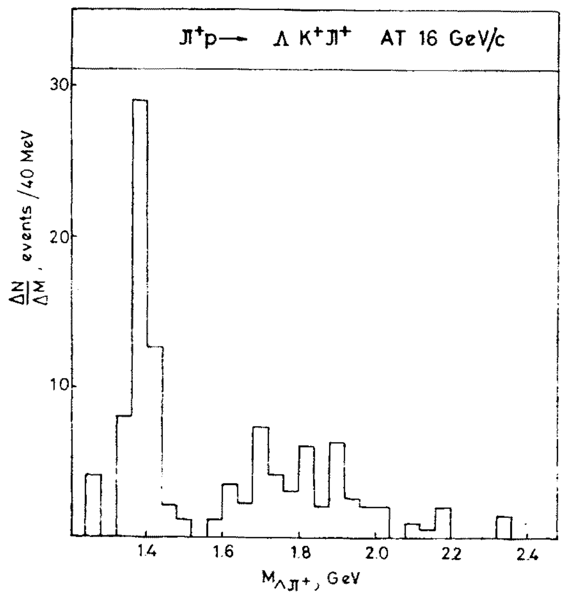

Fig. 1. Effective mass distribution of the $\Lambda\pi^+$ system in the reaction $\pi^+p \to \Lambda K^+\pi^+$ at 16 GeV/c

It seems that this selection cleans the sample from spurious mass combinations in the investigated $\Sigma(1385)$ mass region. A similar cut in t' is less efficient. It can be seen from Figs 1 and 4 that the influence of the background under the resonance (a part of which is the s-wave) on the values of the tensor components should be small (within the error limits). (The s-wave contribution is certainly less than 20% in the inclusive reaction and completely negligible in the exclusive channel). We have therefore neglected it in the evaluation of the tensors.

The final numbers of $\Sigma(1385)$ events, after all selections, taken for the determination of statistical tensor components are listed in Table I. The results obtained for the 2-body reaction (6) at 16 GeV/c are shown in Table II and are compared with the previous results

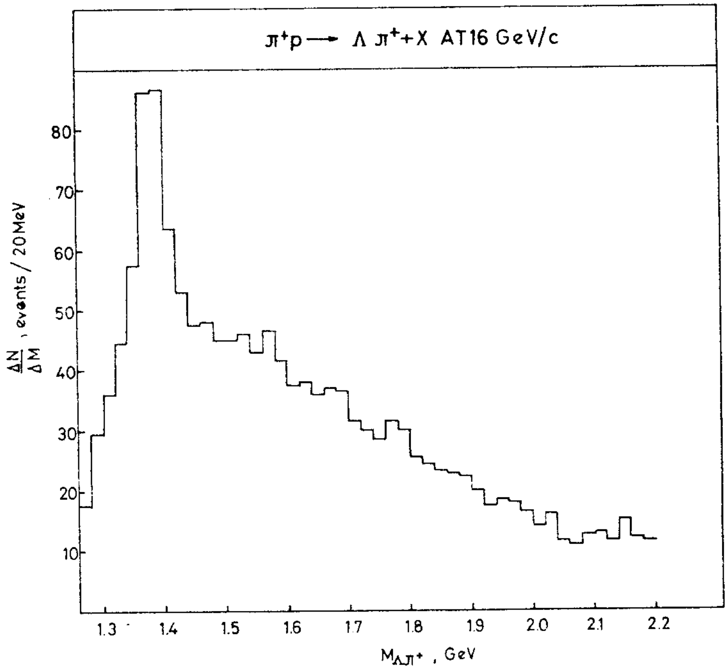

Fig. 2. Effective mass distribution of the $\Lambda\pi^+$ system in the reaction $\pi^+p \rightarrow \Lambda\pi^+ + X$ at 16 GeV/c, no cuts

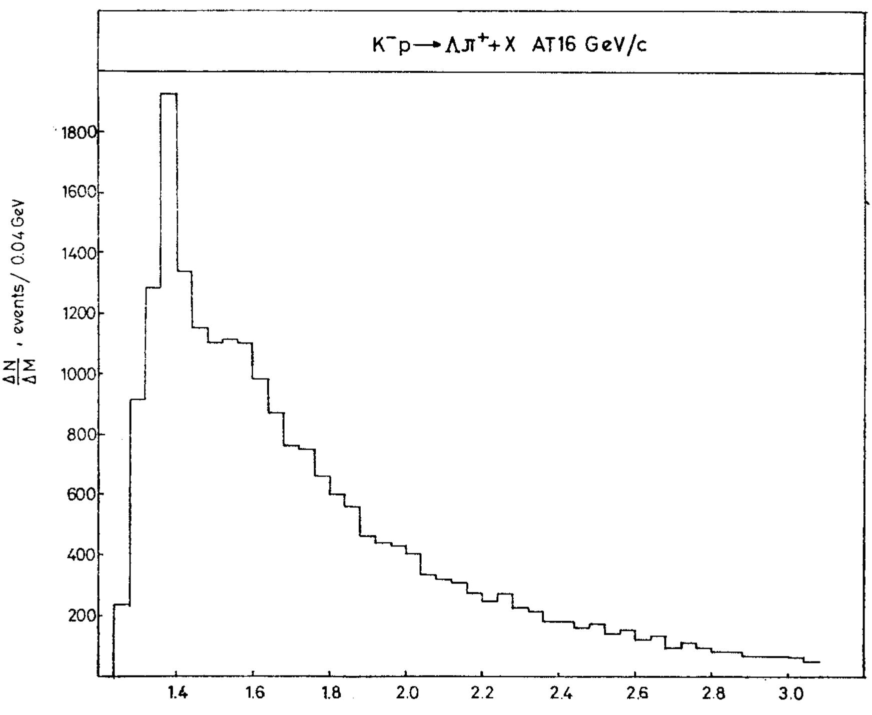

Fig. 3. Effective mass distribution of the $\Lambda\pi^+$ system in the reaction $K^-p \rightarrow \Lambda\pi^+ + X$ at 16 GeV/c, no cuts

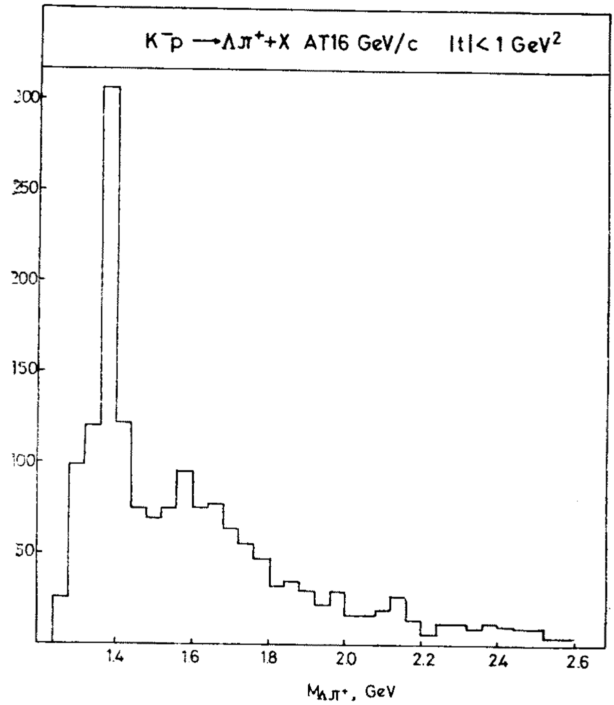

Fig. 4. Effective mass distribution of the $\Lambda\pi^+$ system in the reaction $K^-p \to \Lambda\pi^+ + X$ at 16 GeV/c with the t — cut: $|t_{p \to \Lambda\pi^+}| < 1$ GeV^2

TABLE I

Number of $\Lambda^0\pi^+$ combinations accepted as $\Sigma(1385)$ with $|t| < 1$ GeV^2

Reaction	Incident momentum	
	10 GeV/c	16 GeV/c
$\pi^+p \to \Sigma^+(1385)K^+$	—	49
$K^-p \to \Sigma^+(1385)\pi^-$	32 (15[1])	21
$\pi^+p \to \Sigma^+(1385)X$	—	775
$K^-p \to \Sigma^+(1385)X$	698 (282[1])	519

[1] Numbers in previous analysis (in parentheses).

(Ref. [3]). The values which could be determined independently from transverse and longitudinal polarization are denoted by (a) and (b), respectively. In the same way are given the tensor components for $\Sigma(1385)$ produced in the inclusive reaction (7) (Table III). For the $K^-p \to \pi^-\Sigma^+(1385)$ reaction (8) at 10 and 16 GeV/c the results are shown in Table IV.

As expected, for the small statistics in the 2-body reactions the error values are rather big, although they do not include possible systematic contributions from biases. Therefore the results provide only qualitative information, nevertheless they do not contradict the predictions of the Quark Model (see Fig. 5a). A much better situation is in the case of the inclusive reactions (7) and (9).

TABLE II

Statistical tensors for $\Sigma^+(1385)$ produced in the 2-body reaction $\pi^+p \to \Sigma^+(1385)K^+$ at 16 GeV/c. Quark model predictions: all $T^J_M = 0$, except Re $T^2_0 = -0.5$

T^J_M a) from $P_\perp$ b) from $P_\parallel$		Re: Present results 16 GeV/c π^+p	Re: Previous results 8 and 10 GeV/c π^+p and K^-p	Im: Present results 16 GeV/c π^+p	Im: Previous results 8 and 10 GeV/c π^+p and K^-p	Kinematical limits
T^1_0	a)	0.17 ± 0.23	-0.10 ± 0.22			±0.67
	b)	-0.95 ± 0.75				
T^2_2		0.23 ± 0.10	0.00 ± 0.10	-0.02 ± 0.11	0.14 ± 0.12	±0.35
T^2_0		-0.24 ± 0.19	-0.52 ± 0.18			±0.50
T^3_2	a)	0.39 ± 0.37		-0.34 ± 0.31		
		-0.17 ± 0.32	0.29 ± 0.27	-0.05 ± 0.34	-0.18 ± 0.23	±0.35
	b)	0.30 ± 0.19		-0.18 ± 0.19		
T^3_0	a)	0.64 ± 0.35	0.88 ± 0.31			±0.67
	b)	-0.43 ± 0.45				

TABLE III

Statistical tensors for $\Sigma^+(1385)$ produced in the reaction $\pi^+p \to \Sigma^+(1385)+X$ at 16 GeV/c, $|t_p| < 1$ GeV^2. Quark model: $T^3_2 = T^3_0 = 0$

T^J_M a) from $P_\perp$ b) from $P_\parallel$		Re: Present results 16 GeV/c π^+p	Re: Previous results 8 GeV/c π^+p	Im: Present results 16 GeV/c π^+p	Im: Previous results 8 GeV/c π^+p	Kinematical limits
T^1_0	a)	-0.06 ± 0.06	0.06 ± 0.12			±0.67
	b)	-0.04 ± 0.20				
T^2_2		0.03 ± 0.03	0.03 ± 0.09	-0.07 ± 0.03	0.26 ± 0.09	±0.35
T^2_0		0.01 ± 0.04	0.20 ± 0.11			±0.50
T^3_2	a)	0.01 ± 0.08		0.04 ± 0.08		
		-0.04 ± 0.09	0.13 ± 0.10	0.08 ± 0.08	-0.04 ± 0.10	±0.35
	b)	0.00 ± 0.07		0.06 ± 0.07		
T^3_0	a)	0.06 ± 0.08	-0.17 ± 0.12			±0.67
	b)	-0.27 ± 0.10				

We have verified that the use of the selection $t' < 1$ GeV^2 instead of $|t| < 1$ GeV^2 does not significantly influence the results. This is illustrated for the reaction (9) at 16 GeV/c in Table V, where the results are given for both selections. The t' selection is obviously

TABLE IV

Statistical tensors for $\Sigma^+(1385)$ produced in the reaction: $K^-p \to \pi^-\Sigma^+(1385)$ at 10 and 16 GeV/c, $|t| < 1$ GeV^2, $1.32 < M_{\Lambda\pi^+} < 1.45$ GeV

T^J_M a) from $P_\perp$ b) from $P_{\|\|}$		Re		Im		Kinematical limits
		10 GeV/c	16 GeV/c	10 GeV/c	16 GeV/c	
T^1_0	a)	0.36 ± 0.27	-0.15 ± 0.29			± 0.67
	b)	0.07 ± 1.00	1.43 ± 1.32			
T^2_2		0.06 ± 0.10	-0.08 ± 0.08	-0.03 ± 0.13	-0.27 ± 0.10	
T^2_0		-0.55 ± 0.19	-1.11 ± 0.18			± 0.50
T^3_2	a)	-0.47 ± 0.35	0.25 ± 0.44	-0.78 ± 0.31	-1.03 ± 0.53	± 0.35
	b)	0.65 ± 0.31	-0.26 ± 0.21	-0.18 ± 0.41	0.32 ± 0.45	
T^3_0	a)	-1.03 ± 0.44	0.77 ± 0.67			± 0.67
	b)	0.07 ± 0.49	-0.54 ± 0.48			

TABLE V

Statistical tensors for $\Sigma^+(1385)$ produced in the reaction $K^-p \to \Sigma^+(1385) + X$ at 16 GeV/c $1.32 < M_{\Lambda\pi^+} < 1.45$ GeV

T^J_M a) from $P_\perp$ b) from $P_{\|\|}$		Re		Im		Kinematical limits
		$\|t\| < 1$ GeV^2	$t' < 1$ GeV^2	$\|t\| < 1$ GeV^2	$t' < 1$ GeV^2	
T^1_0	a)	-0.02 ± 0.08	-0.04 ± 0.05			± 0.67
	b)	0.26 ± 0.22	0.30 ± 0.13			
T^2_2		0.12 ± 0.03	0.05 ± 0.02	-0.08 ± 0.03	-0.08 ± 0.02	± 0.35
T^2_0		0.00 ± 0.05	0.03 ± 0.03			± 0.50
T^3_2	a)	-0.02 ± 0.10	0.02 ± 0.06	-0.17 ± 0.10	-0.21 ± 0.06	± 0.35
	b)	-0.06 ± 0.08	-0.01 ± 0.05	-0.07 ± 0.08	0.03 ± 0.05	
T^3_0	a)	0.03 ± 0.11	0.04 ± 0.06			± 0.67
	b)	-0.02 ± 0.11	0.00 ± 0.06			

much less rigorous (final numbers of $\Lambda\pi^+$ combinations amounting to 1441 as compared to 519 for the t-selection). We believe, however, that for our purpose, i. e., isolation of $\Sigma(1385)$ in the baryon vertex, the t cut is more appropriate.

It follows from Tables III, V and VI that our results for the inclusive reactions (7) and (9) at 10 and 16 GeV/c are in good agreement with the predictions of the additive quark model. This is shown in Fig. 5b.

No significant deviation from the Quark Model predictions ($T_0^3 = T_2^3 = 0$) is seen for the reaction (9), in contrast to the previous results at 10 GeV/c in which Im T_2^3 was at the kinematical limit (see Table VI), however, only 3.5 standard deviations from 0. This deviation disappeared when the statistics has been more than doubled. It should be born in mind, however, that small but significant deviations from the Q. M. predictions have been observed in Ref. [1] in the exclusive reaction (8) at 4.2 GeV/c but only for

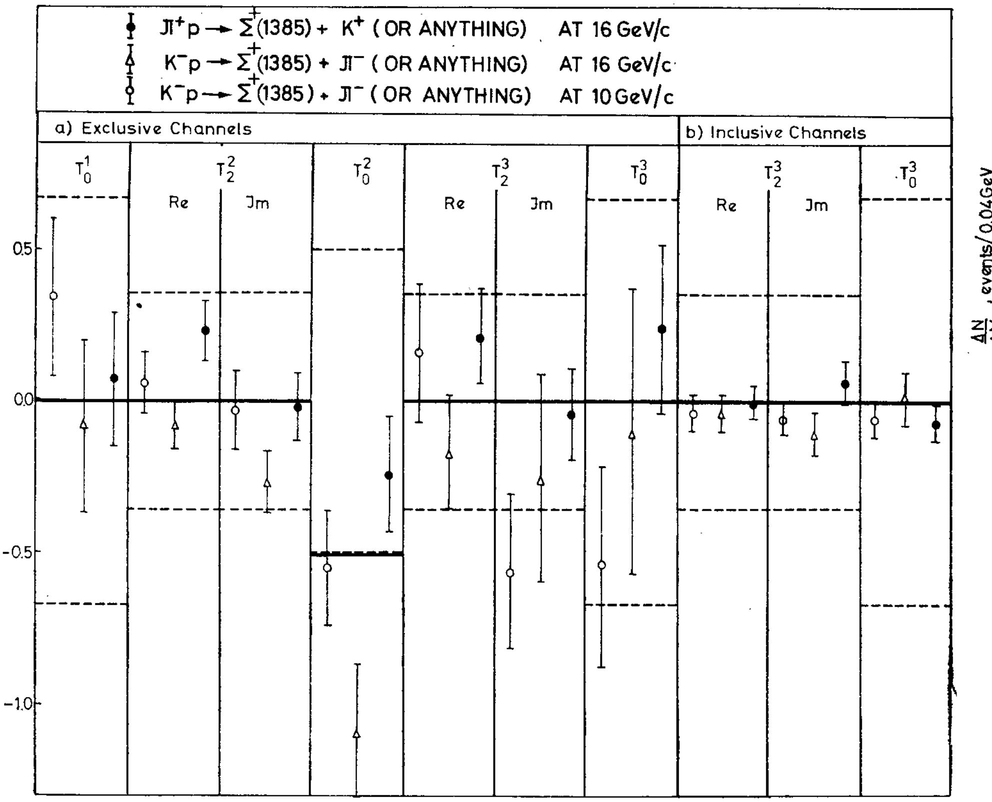

Fig. 5a. Statistical tensors for $\Sigma(1385)$ produced in the two-body reactions: 1) $\pi^+p \to \Sigma^+(1385)K^+$ at 16 GeV/c, 2) $K^-p \to \Sigma^+(1385)\pi^-$ at 16 GeV/c, 3) $K^-p \to \Sigma^+(1385)\pi^-$ at 10 GeV/c; ---- kinematical limits, ——— quark model predictions, b. Statistical tensors for $\Sigma^+(1385)$ produced in the inclusive reactions: 1) $\pi^+p \to \Sigma^+(1385)+X$ at 16 GeV/c, 2) $K^-p \to \Sigma^+(1385)+X$ at 16 GeV/c, 3) $K^-p \to \Sigma^+(1385)+X$ at 10 GeV/c; ---- kinematical limits, ——— quark model predictions

$t' < 0.25$ GeV^2, such deviations would be unobservable in our analysis as owing to the limited statistics we did not study the t-dependence.

To estimate the precision of our measurement and to check the compatibility with the positivity condition for the density matrix we have introduced two parameters λ and μ which have been proposed by Daumens et al. [8]. The positivity conditions for the ϱ matrix of the $\Sigma^+(1385)$ determine the so-called polarization domain D in 7-dimensional space of X_M^J which corresponds to the 7 non-vanishing components. Our experimental points ϱ_{exp} for the reactions (6)–(9) should be inside this domain, at least within the error limits. To have a feeling for the size of these errors one can define the ellipsoid

$$\sum_{J,M} \frac{(X^J_M - T^J_M)^2}{\Delta T^{J2}_M} \leqslant \chi^2$$

TABLE VI

Statistical tensors for $\Sigma^+(1385)$ produced in the reaction $K^-p \to \Sigma^+(1385)+X$ at 10 GeV/c, $1.32 < M_{\Lambda\pi^+} < 1.45$ GeV, $|t| < 1.0$ GeV^2

T^J_M a) from $P_\perp$ b) from $P_\parallel$		Re 10 GeV/c present results	Re 10 GeV/c previous results	Im 10 GeV/c present results	Im 10 GeV/c previous results	Kinematical limits
T^1_0	a)	-0.08 ± 0.07	-0.06 ± 0.15			±0.67
	b)	-0.01 ± 0.19				
T^2_2		0.11 ± 0.03	0.10 ± 0.07	-0.11 ± 0.03	-0.06 ± 0.07	±0.35
T^2_0		0.02 ± 0.04	0.03 ± 0.10			±0.50
T^3_2	a)	-0.10 ± 0.09	-0.03 ± 0.12	-0.11 ± 0.08	-0.43 ± 0.12	±0.35
	b)	-0.01 ± 0.07		-0.03 ± 0.07		
T^3_0	a)	-0.07 ± 0.08	0.06 ± 0.16			±0.67
	b)	-0.04 ± 0.10				

TABLE VII

The values of the parameters λ and μ defined in the text

Reaction	Incident momentum GeV/c	λ	μ
$\pi^+p \to K^+ + \Sigma^+(1385)$	16	2.25	0.029
$\pi^+p \to \Sigma^+(1385)+X$	16	3.53×10^{-4}	1.0
$K^-p \to \pi^- + \Sigma^+(1385)$	16	15.31	0.0
$K^-p \to \Sigma^+(1385)+X$	16	8.56×10^{-4}	1.0
$K^-p \to \pi^- + \Sigma^+(1385)$	10	9.47	0.005
$K^-p \to \Sigma^+(1385)+X$	10	3.51×10^{-4}	0.95

in the 7-dimensional space X^J_M; (T^J_M, ΔT^J_M are the experimental values of the statistical tensor components and their errors, respectively). We have calculated the volume of the ellipsoid $E_{1/2}$ containing points compatible with T^J_M at the confidence level 1/2 ($\chi^2 = 6.346$). Next we have estimated the ratio of the volume of this ellipsoid to that of the domain D

$$\lambda = \frac{V(E_{1/2})}{V(D)}.$$

For the exclusive reactions (6), (8) λ is greater than unity and therefore our test of the quark model predictions is rather not significant. However in the inclusive reactions (7), (9) the situation seems to be much better (see Table VII).

Moreover we have checked whether our experimental points ϱ_{exp} satisfy the positivity conditions. For this purpose we have calculated the ratio μ of the volume of the overlap region of the ellipsoid $E_{1/2}$ (computed by the Monte Carlo method) and the polarization domain to the volume of $E_{1/2}$ $\mu = \dfrac{V(E_{1/2} \cap D)}{V(E_{1/2})}$. For the inclusive reactions (7), (9) the experimental points lie inside the domain and $\mu = 1$ (Table VII).

5. *Conclusions*

The results of the present analysis can be summarized as follows. In the case of two body reactions the statistics available turned out to be still too small to make any stronger quantitative statements. This is due to cross section rapidly falling off with incident energy. However for the $\Sigma(1385)$ produced in the baryon vertex in the inclusive reactions the predictions of the Q. M. seem to work quite well for all the investigated reactions π^+p and K^-p in the energy range 8–16 GeV.

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