# $\overline{\rm N}$N ANNIHILATIONS AND THE STATISTICAL BOOTSTRAP MODEL

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(*Received July 29, 1977*)

We critically discuss the applicability of the statistical bootstrap model (SBM) to $\overline{\rm N}$N annihilation reactions where statistical properties are much pronounced compared to typical inelastic collisions. To this end we mainly consider multiplicity distributions, single particle spectra and the neutral excess. The available data show that for annihilations in flight ($p_{\rm lab} > 2$ GeV/$c$) the dynamics inherent in the SBM is not sufficient for a consistent understanding of the experimentally observed behaviour.

## 1. *Introduction*

Within the last few years much effort has been devoted to the refinement (exact energy-momentum and internal quantum-number conservation) of the statistical bootstrap model (SBM) [1] describing the decay of single hadronic fireballs [2]. The model does not include any assertion on fireball production in strong interactions. Therefore the question arose for reactions being reasonable candidates for single fireball formation processes. Per def. in $\overline{\rm N}$N annihilation reactions no final-state baryons occur, carrying as leading partices in inelastic nonannihilation processes typically 50% of the collision energy. As exlpected from this property the average multiplicity is larger than usually in inelastic collisions and the particle spectra are less anisotropic, see for instance [3]. Because of these properties of annihilation reactions statistical-type models have been applied mainly to explain multiplicity distributions for several years [4–12].

In this paper we critically discuss the applicability of the SBM to $\overline{\rm N}$N annihilations considering some main properties of these reactions. In Section 2 we consider the limiting case of $\overline{\rm N}$N annihilations at rest to fix the model parameter and to test the reliability of the model. Some aspects of the annihilation process in flight are discussed in Section 3: the $s$ dependence of the multiplicity distribution, the single-particle spectra and the kaon production rates. In Section 4 we turn in more detail to the neutral excess observed in $\overline{\rm N}$N

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(as well as in $e^+e^-$ annihilations). A summarizing discussion is given in Section 5. For definiteness the formulae relevant to our SBM calculations are summed up in the appendix.

## 2. $\bar{N}N$ *annihilations at rest*

In view of the isotropic spectra, relatively large multiplicity and the low angular momentum of the initial state (essentially $J = S = 0{,}1$) this reaction has often been used for testing statistical model predictions [5,8–12]. In a previous publication [11] we have obtained a good fit of the charged multiplicity distributions in $\bar{p}p$ and $\bar{p}n$ at rest by the

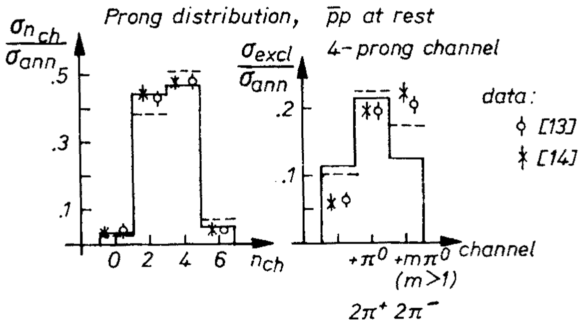


Fig. 1. Fit of the SBM calculations for the charged multiplicity distribution in $\bar{p}p$ annihilation at rest to data [11]: ——— SBM, $T_0 = 167$ MeV; ········ SBM, $T_0 = 152$ MeV

statistical bootstrap model (SBM), compare Fig. 1. This fit fixes the coupling constant for pion production $B_\pi = 1.09\ \mathrm{GeV}^{-2}$ corresponding to a maximum temperature $T_0 = 167$ MeV [11]. Now all quantities concerning pion production uniquely can be calculated within the model. Applying the SBM to the generation of neutral pions we find

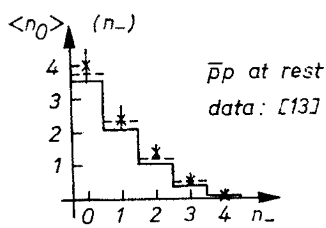


Fig. 2. $\bar{n}_{\pi^0}(n_{ch})$ — comparison of data from $\bar{p}p$ at rest [35] with SBM predictions; ——— $T_0 = 167$ MeV; ········ $T_0 = 152$ MeV

at fixed charged multiplicity a $\pi^0$-number $\langle n_{\pi^0}\rangle\ (n_{ch})$, which is systematically lower than the experimental figures (see Fig. 2). Therefore also the total $\pi^0$-multiplicity is underestimated in the model (compare Table I). Although these $\pi^0$-multiplicities are not directly-measured numbers, real deviations from the model predictions seem to be present.

TABLE I

Neutral and charged pion multiplicities in $\bar{N}N$ annihilation at rest; SBM compared with data

| | $\bar{p}p$ | | | $\bar{p}n$ | | |
|---|---|---|---|---|---|---|
| | exp. [35, 36] | SBM/$T_0$ (MeV) 167 | SBM/$T_0$ (MeV) 152 | exp. [37] | SBM/$T_0$ (MeV) 167 | SBM/$T_0$ (MeV) 152 |
| $\bar{n}_{ch}$ | $3.06 \pm 0.03$ | 3.06 | 3.26 | $3.18 \pm 0.05$<br>$3.15 \pm 0.03$ | 3.12 | 3.32 |
| $\bar{n}_0$ | $1.96 \pm 0.23$ | 1.56 | 1.66 | $1.95 \pm 0.20$ | 1.53 | 1.63 |

The experimental curves for inclusive as well as for semi-inclusive $\pi^{\pm}$ distributions essentially show a flat behaviour and scarcely reflect the existence of resonances. So the simple model used can roughly describe the shape of the data. But compared to the model predictions the measured momentum distributions

$$\frac{dN}{dp_\pi} = \frac{2\pi p_\pi^2}{E_\pi}\left[2E_\pi \frac{d^3N}{d^3p_\pi}\right] \tag{1}$$

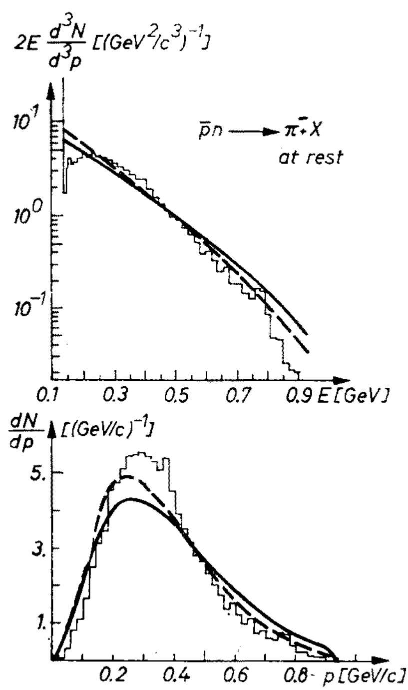


Fig. 3. Inclusive energy and momentum spectra for the reaction $\bar{p}n \to \pi^- + X$ at rest [37] compared with SBM predictions; ——— $T_0 = 167$ MeV; - - - - $T_0 = 152$ MeV

exhibit a more pronounced peak around the average $|\vec{p}|$-value connected with a somewhat steeper decrease of the invariant distributions. As demonstrated in Figs 3 and 4 this tendency is found in inclusive as well as in semi-inclusive spectra.

Remembering the neutral excess these deviations can be considered as a reflection of the too low number of neutral pions in the model — qualitatively a larger average particle

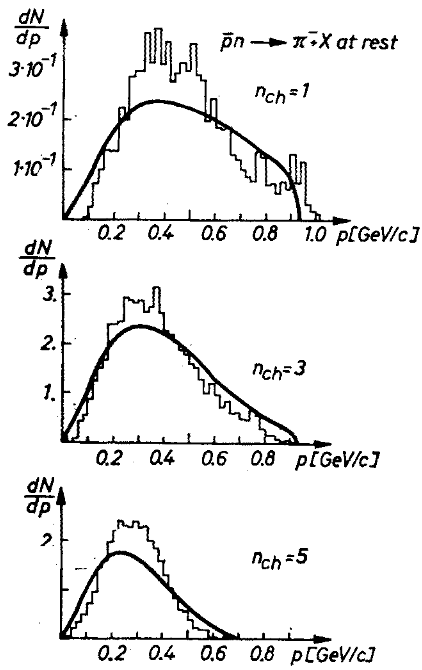


Fig. 4. Semi-inclusive momentum distributions in $\bar{p}n \to \pi^- + X$ at rest [37] compared with the SBM ($T_0$ = 167 MeV)

number would diminish the available phase space and thereby cause a steeper decrease of the $\pi^{\pm}$-spectra. Therefore the deviations of the predicted spectra from data are smaller if the pion coupling $B_\pi$ is chosen by fitting the total pion number as in [8, 9, 12]. This is shown in Fig. 3 for the inclusive $\pi^-$-spectra.

Of course by this procedure for fixing $B_\pi$ the discrepancies concerning the neutral excess are also diminished, as can be seen from Fig. 2 and Table I. On the other hand this procedure leads to clear deviations of the model calculations for the charged multiplicity distribution from the measured values, compare Fig. 1.

The charged multiplicity distribution, however, represents the best known experimental information. Since we are not interested in minimizing the discrepancies between SBM and data, but rather in discriminating between statistical and dynamical effects, we choose the parameter $B_\pi$ according to the best fit to the charged pion multiplicity distributions in $\bar{p}p$ and $\bar{p}n$ at rest. The differences arising in this case between SBM predictions and data have their main origin in the neutral excess. This effect is discussed further in Section 4.

### 3. *Annihilations in flight*

As already mentioned the SBM — like all statistical-type models — describes the decay of single fireballs (SF). In this section we critically discuss the direct applicability of the SBM to $\bar{p}p$ annihilation reactions in flight, e. g. the formation of single fireballs with masses $M = \sqrt{s}$ in these reactions. Characteristic properties of the fireball decay in the SBM are the linear increase of the average multiplicity with the fireball mass $M$,

$$\langle n \rangle \propto a(T_0) \cdot M \quad (M \gtrsim 2 \text{ GeV}), \tag{2}$$

and the isotropic single-particle spectra of nearly exponential shape at least for particle energies $E_\pi$ far from the kinematic boundary,

$$N_1 \equiv 2E \frac{d^3N}{d^3p} \propto M \exp(-E/T_0). \tag{3}$$

(Note the practically constant slope for $M \gtrsim 3$ GeV [13, 14].) By means of the temperature relation (see A. 9) the maximum temperature $T_0$ is unambiguously related to the coupling constants $B_i$ for particles of the kind $i$. Therefore these parameters $B_i$ also fix shape and energy (resp. mass) dependence of the multiplicity distributions as well as of single-particle spectra. Although the observed pion-multiplicity can be fitted by an expression linear in $\sqrt{s}$

$$\langle n_{\pi^\pm} \rangle_{\bar{p}p} \propto 0.86 \sqrt{s} \quad (p_{lab} \leqslant 12 \text{ GeV}/c), \tag{4}$$

the slope is not compatible with our SBM-result

$$\langle n_{\pi^\pm} \rangle_{SBM} \propto 1.5 \cdot M. \tag{5}$$

Consequently the model calculations show clear deviations from data on multiplicity distributions already for $p_{lab} > 2$ GeV/$c$ (compare Table II) whereas a fair description is

TABLE II

Prong distribution for $\bar{p}p$ annihilation into pions at $p_{lab} = 2.32$ GeV/$c$ ($\sqrt{s} = 2.54$ GeV) [38] compared to SBM predictions ($T_0 = 167$ MeV)

| $\bar{p}p$ annihilation $n_{ch}$ | Data (%) [38] | SBM (%) $T_0 = 167$ MeV |
|---|---|---|
| 0 | 2.7 ± 1.5 | 1.0 |
| 2 | 31.9 ± 2.3 | 21.3 |
| 4 | 51.8 ± 1.2 | 53.1 |
| 6 | 13.2 ± 1.0 | 22.9 |
| 8 | 0.34 ± 0.02 | 1.7 |
| $\bar{n}_{\pi^-}$ | 1.77 ± 0.08 | 2.03 |
| $\bar{n}_{\pi^0}$ | 2.22 ± 0.25 | 2.06 |
| $f_2^-$ | −1.38 ± 0.22 | −1.48 |

TABLE III

Multiplicity distribution for $\bar{p}p$ annihilation into pions at $p_{lab} = 0.945$ GeV/$c$ [21] compared to SBM results ($T_0 = 167$ MeV)

| Channel | Exp. [21] | SBM | $\bar{n}_0$ ($n_-$) exp. [21] | $\bar{n}_0$ ($n_-$) SBM |
|---|---|---|---|---|
| $n_{ch} = 0$ | 1.8 ±1.8 | 2.5 | 3.1 ±0.3 | 3.80 |
| $n_{ch} = 2$ | 36.0 ±5.0 | 37.1 | 2.3 ±0.3 | 2.39 |
| $\pi^+\pi^-$ | 0.7 ±0.1 | 0.8 | | |
| $\pi^+\pi^-\pi^0$ | 8.0 ±2.6 | 6.1 | | |
| $\pi^+\pi^- m\pi^0$ ($m > 1$) | 27.3 ±3.0 | 30.1 | | |
| $n_{ch} = 4$ | 56.0 ±4.0 | 51.6 | 1.5 ±0.15 | 1.30 |
| $2\pi^+2\pi^-$ | 6.0 ±0.5 | 9.8 | | |
| $2\pi^+2\pi^-\pi^0$ | 27.0 ±1.0 | 21.8 | | |
| $2\pi^+2\pi^- m\pi^0$ | 23.0 ±3.0 | 20.0 | | |
| $n_{ch} = 6$ | 6.0 ±0.6 | 8.6 | 0.77±0.08 | 0.58 |
| $3\pi^+3\pi^-$ | 2.0 ± 0.3 | 4.5 | | |
| $3\pi^+3\pi^-\pi^0$ | 3.9 ±0.4 | 3.4 | | |
| $3\pi^+3\pi^- m\pi^0$ | 0.1 ±0.06 | 0.8 | | |
| $n_{ch} = 8$ | | 0.13 | | |
| $4\pi^+4\pi^-$ | 0.1 ±0.06 | 0.11 | | |
| $\bar{n}_{\pi^-}$ | 1.67±0.06 | 1.67 | | |
| $\bar{n}_{\pi^0}$ | 1.77±0.13 | 1.70 | | |
| $f_2^{--}$ | −1.29±0.12 | −1.21 | | |
| $f_2^{0-}$ | −0.30±0.08 | −0.46 | | |
| $f_3^{---}$ | 2.14±0.30 | 1.99 | | |

achieved for $\bar{p}p$ annihilation around 1 GeV/$c$ (see Table III). In principle this problem arises in all similar treatments of the SBM [7–9]. Several attempts have been made to avoid these difficulties:

(*i*) Introduction of an $s$-dependent coupling $B_\pi$ [9], corresponding to an $s$-dependent fireball volume and via the temperature relation (A. 9) to an energy-dependent "highest temperature" $T_0$. Since the model cannot make statements on these dependences in this case it loses the predictive power.

(*ii*) In a recent paper [15] Margolis and coworkers show — at first view — a good fit to the data on multiplicity distributions in $\bar{p}p$ annihilations, taking into account resonance and kaon production. But first of all they give up the exact agreement between model calculations and the well established data for the charged multiplicity distribution in $\bar{p}p$ at rest which is the best candidate for SF-formation. Furthermore they do not care about the empirical reduction of $K\bar{K}$ pair production necessary in all statistical models

[4, 5]. This will give a strong overestimation of $\overline{K}K$ generation and, of course, reduce the average multiplicity.

We conclude that the observed $s$-dependence of multiplicity distributions in $\bar{p}p$ annihilations cannot be reproduced by the decay of single fireballs ($M = \sqrt{s}$) according to the SBM.

Because of the linear relation between the second multiplicity moments $f_2 = \langle n(n-1)\rangle - \langle n\rangle^2$ and the average multiplicity [3],

$$f_2^{--} \propto -0.75\langle n_-\rangle \tag{6}$$

predicted by the SBM [11] as well as by other statistical models the multiplicity distribution for $\bar{p}p$ annihilation in flight can be described by the SBM using a reduced effective cluster mass $M_{\text{eff}} < \sqrt{s}$ [7]. One possible explanation for this fact is to assume the formation of more than one cluster in typical annihilation events. Such assumption also naturally explains the observed $s$-dependent slope of single particle spectra in $\bar{p}p$ annihilation [16] without introducing an $s$-dependent temperature parameter $T = T(s)$, increasing with $s$. Assume for example that in $\bar{p}p$ annihilation reactions ($p_{\text{lab}} \leqslant 12$ GeV/$c$) two moving

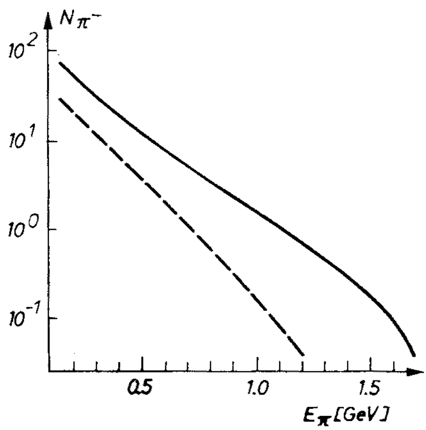


Fig. 5. Invariant $\pi^-$ spectrum for $\bar{p}p$ annihilation, $N_{\pi^-} = \dfrac{1}{2\pi p}\dfrac{dN}{dE}$ at $\sqrt{s} = 4.36$ GeV (arbitrary units); ——— from two moving fireballs ($M_{\text{fb}} = 1.55$ GeV); - - - - from single fireball decay ($M_{\text{fb}} = \sqrt{s} = 4.36$ GeV). (The curves are not normalized to the same multiplicity; only the shape should be compared)

clusters are produced with equal masses chosen to give the right multiplicity distribution. Then energy-momentum conservation fixes the cluster momenta. Ascribing nonvanishing momenta to the fireballs effectively reduces the slope of the resulting invariant distributions corresponding to an increasing "effective temperature". This is demonstrated in Fig. 5 for $p_{\text{lab}} = 9.1$ GeV/$c$ [17]. Unfortunately the accessible annihilation energies are too small for discussing $\bar{p}p$ annihilations more detailed within the framework of multi-cluster models.

There are further properties of the experimental spectra uncompatible with statistical model predictions. Although the — with $s$ increasing — anisotropy of the single-

-particle distributions qualitatively could be ascribed to the influence of angular momentum (neglected in the used version of the SBM), the $\pi^+/\pi^-$ asymmetry or "leading pion" effect cannot be understood in any phase space model [18–20]. Furthermore the known experimental results indicate an approximately constant rate $R_K$ of events containing $K$-mesons, $R_K \simeq 10\%$ [21, 22]. Within the SBM, the strongly $M$-dependent kaon production rate speaks against the formation of single clusters with masses $M = \sqrt{s}$, although at low energies — $p_{lab} \lesssim 2$ GeV/$c$ — the relative branching ratios for channels containing one $\overline{\mathrm{K}}\mathrm{K}$ pair and pions are rather well described by the SBM [17].

Closing this section we summarize that most of the main properties of $\bar{\mathrm{p}}\mathrm{N}$ annihilations in flight cannot be accounted for, assuming the formation of single fireballs in the sense of the statistical bootstrap model.

### 4. *The neutral excess in $\bar{p}p$ annihilation reactions*

For annihilations the part $E^{tot}_{neutr}$ of the total energy $\sqrt{s}$ going into the production of neutral particles is larger than expected from an approximate equidistribution of the available energy over the different charge states of the final state particles, mainly pions. In the following this behaviour — often denoted as neutral excess — is discussed by means of the quantities

$$R_{ch} = \frac{E^{tot}_{ch}}{\sqrt{s}} \quad \text{and} \quad R_r = \frac{2E^{tot}_{neutr}}{E^{tot}_{ch}} \tag{7}$$

which are related by

$$R_{ch} = \frac{2}{2+R_r}\,. \tag{8}$$

Generally, two simplifying assumptions are used for the data analysis:
(A) All charged particles are pions. With this assumption the measured momenta of the charged particles determine their contribution $E^{tot}_{ch}$ to the total energy and because of

$$E^{tot}_{ch} + E^{tot}_{neutr} = \sqrt{s} \tag{9}$$

also the contribution of neutrals.

An isospin invariant production mechanism will give $R_{ch} = 2/3$ and $R_r = 1.0$, whereas for kaons and nucleons $R_{ch} = 1/2$ and $R_r = 2.0$. Therefore the presence of kaons and nucleons in the final state would cause a decrease of $R_{ch}$, $R_{ch} < 2/3$ ($R_r < 1.0$). But furthermore the assumption (A) implies a systematic underestimation of the charged energy $E^{tot}_{ch}$ since the particle momentum is the primarily measured quantity. Because of their larger masses the heavy particles carry — at fixed momentum — a larger energy than pions.
(B) Also the neutral particles are mainly pions with the same average energy as the charged ones, e. g.

$$\frac{\langle n_{\pi^0}\rangle}{\langle n_{\pi^\pm}\rangle} = \frac{E^{tot}_{neutr}}{E^{tot}_{ch}}\,. \tag{10}$$

For $\bar{p}p$ annihilation this procedure results in [3]

$$R_{ch} = 0.625 \pm 0.02, \qquad R_r = 1.2 \pm 0.08 \tag{11}$$

essentially independent of the collision energy. With the statistical assumption (10) immediately follows from (11)

$$\langle n_{\pi^0} \rangle \simeq 0.6 \langle n_{\pi^\pm} \rangle. \tag{12}$$

The direct measurement of the $\pi^0$-production at $p_{lab} = 1.61$ GeV/$c$ confirms the conclusion (12) [23] whereas at $p_{lab} = 2$ GeV/$c$ [24]

$$\langle n_{\pi^0} \rangle \simeq \langle n_{\pi^-} \rangle.$$

In the following we accept the result (11) as representative for $\bar{p}p$ annihilations. But remembering the discussion concerning the misidentification of kaons as pions we give a quantitative estimate: Assuming the presence of nonidentified $K\bar{K}$ pairs in 10% of the annihilation events reduces the neutral excess as given in (11) by about 20%.

### 4.1. Discussion of the neutral excess in the framework of the SBM

Because of the practically equal masses of the different pions all phase space models result in the same average energy $\langle E_\pi \rangle$ for $\pi^\pm$ and $\pi^0$. Accordingly the total energy fraction delivered to $\pi^0$-production can be enlarged only by an increase of $\langle n_{\pi^0} \rangle$ compared to $\langle n_{\pi^\pm} \rangle$. But the inclusion of isospin conservation in the SBM causes for all model versions treated up to now the behaviour [25–28]

$$\frac{2\langle n_{\pi^0} \rangle}{\langle n_{\pi^\pm} \rangle} \underset{M \to \infty}{=} 1. \tag{13}$$

Also at nonasymptotic fireball masses $M \gtrsim 2$ GeV the deviations from (13) are small [17, 27, 28]. Neglecting the isospin conservation, however, can result in a nonvanishing excess of neutral pions, compared to (13) [6, 8, 9, 26]. The linear cascade model with charge conservation by Orfanidis and Rittenberg [6] for example leads to $\langle n_{\pi^0} \rangle = \sqrt{2} \langle n_{\pi^-} \rangle$ in the case of a neutral initial state. But for $\bar{p}p$ annihilation this result is excluded by general isospin bounds [29]

$$\tfrac{1}{2} \langle n_{\pi^-} \rangle_{\bar{p}p} \leqslant \langle n_{\pi^0} \rangle_{\bar{p}p} \leqslant \tfrac{4}{3} \langle n_{\pi^-} \rangle_{\bar{p}p}. \tag{14}$$

The SBM version used by us is in better agreement with data for $\bar{p}p$ annihilation at rest if the isospin conservation is neglected [8] (larger neutral pion yield, compare the discussion in Section 2). In spite of this fact it seems to us not justified to drop the demand for isospin invariance since it is an essential property of the strong interaction. In [30] an isospin invariant linear cascade model has been considered with additional free parameters. However, a consistent description of the charged multiplicity distributions in $\bar{p}p$ and $\bar{p}n$ annihilations at rest is only possible without any essential excess of $\pi^0$'s.

We conclude that an excess of neutral pions, alone sufficient for explaining the experimental numbers (11), cannot be established in any isospin conserving phase space model. However, the possibly enhanced production of particles heavier than pions (mainly $K$-pairs) as discussed above, would not contradict the model because of the freedom in choosing the coupling constants for different kinds of particles.

### 4.2. Influence of dynamical effects not included in the model

An approximation in the used SBM version consists in the neglect of quantum-statistical effects. An exact treatment of the pions according to Bose statistics with inclusion of isospin conservation [28] reproduces the result

$$\frac{2\langle n_{\pi^0}\rangle}{\langle n_{\pi^\pm}\rangle} \underset{M\to\infty}{=} 1 \tag{13}$$

which also is obtained in the case of Bolzmann statistics. The nonasymptotic deviations for Bose statistics are slightly larger; at $M = 2\,m_N = 1.88$ GeV one obtains for this ratio the value $(1.06 \pm 0.01)$, fastly decreasing to one [28]. Accordingly the nonasymptotic deviation from the result (13) enlarged by the exact quantumstatistical treatment of pions can explain about 25–30% of the neutral excess in $\bar{\mathrm{N}}\mathrm{N}$ annihilation at rest. Additional $\pi^0$-mesons (or $\gamma$'s resp.) can originate from resonances with electromagnetic decays, mainly the $\eta$-meson. Because of their small mass the $\eta$'s are often generated within statistical models together with a relatively large number of other particles. In channels with further neutral particles they generally cannot be identified. Therefore the actual production rate could be larger than indicated by the data. In the SBM we roughly estimated the possible contribution of electromagnetically decaying resonances to the observed neutral excess in $\bar{\mathrm{p}}\mathrm{p}$ annihilation to about $(30 \pm 10)\%$, e. g.

$$R_{\mathrm{ch}}^{\mathrm{res}} = 0.65 \pm 0.1, \qquad R_{\mathrm{r}}^{\mathrm{res}} = 1.07 \pm 0.02. \tag{16}$$

In the above discussion we have found several different effects explaining the neutral excess in $\bar{\mathrm{p}}\mathrm{N}$ annihilations at rest without additional dynamical assumptions. Since the discussed nonasymptotic effect rapidly decreases with increasing energy this explanation does not hold at higher energies.

## 5. *Summary*

Our aim was to look for the extent to which statistical models especially the statistical bootstrap model can account for $\bar{\mathrm{p}}\mathrm{N}$ annihilation data. To this end we discussed a simple version of the SBM. Although the model version used includes resonance production only in an average way it well describes the charged multiplicity distributions and single particle spectra in $\bar{\mathrm{p}}\mathrm{p}$ and $\bar{\mathrm{p}}\mathrm{n}$ annihilation reactions at rest and low energies, $p_{\mathrm{lab}} \lesssim 2\,\mathrm{GeV}/c$. Small systematic deviations from data seem to be caused by the neutral excess. The observed magnitude of this effect can be understood in $\bar{\mathrm{p}}\mathrm{N}$ annihilations at rest without additional dynamics from known effects — some kaon contamination ($\sim 10\%$), electromagnetically decaying resonances (mainly $\eta$, $\eta'$) and nonasymptotically (small mass of the $\bar{\mathrm{p}}\mathrm{N}$ system) enhanced $\pi^0$ yield as compared to $1/2\ \langle n_{\mathrm{ch}}\rangle$.

With increasing energy most of the known characteristic features of annihilations in flight — like for example average multiplicities, spectra, nearly $s$-independent neutral excess or $\pi^+/\pi^-$ asymmetry — cannot be reproduced further within the SBM assuming the formation of single fireballs. By introducing an effective $s$-dependent coupling constant the model loses its predictive power and — furthermore — cannot consistently reproduce multiplicity distributions and behaviour of the single particle spectra [16]. Because of the limited accessible annihilation energies the fact that multiplicity distributions in $\bar{p}N$ annihilations can be described by single SBM clusters with an effective mass $M_{eff} < \sqrt{s}$, does not allow powerful conclusions on the annihilation dynamics.

We conclude that the dynamics inherent in the SBM is not sufficient for the description of the essential $\bar{p}N$ annihilation properties for primary momenta larger than 2 GeV/$c$; one has to look for a dynamical scheme going essentially beyond the framework of statistical-type models.

The author thanks Professor J. Ranft for stimulating discussions and a careful reading of the manuscript. Furthermore he acknowledges helpful discussions with Dr J. Kripfganz and Dr E. M. Ilgenfritz.

## APPENDIX

### *Relevant formulae for the used SBM version*

Generally we use the invariant phase space formulation of the statistical bootstrap model [31] which can be solved exactly by Laplace transformation techniques and correctly ensures energy-momentum conservation. In a previous publication [11] we fitted various SBM versions to the charged multiplicity distributions in $\bar{p}p$ and $\bar{p}n$ annihilations at rest. Only minor differences between the SBM versions considered have been found. From this result we take the motivation (see also [26]) for choosing as characteristic version the full SBM in which the multiplicity of decay products in each generation of the fireball decay is not restricted. Exotic states are allowed in the decay chain. Resonances are only treated in an average sense by assuming a continuous hadron spectrum down to the two-particle threshold.

This simple version exactly leads to the following expression for the multiplicity distribution in the decay of an excited hadronic state (= fireball) with 4-momentum $k$ ($\sqrt{k^2} = M$)

$$P_n^{\{n_i\}}(k^2, \alpha) = \tilde{\varrho}(k^2, \alpha)^{-1} \cdot g_n \cdot n! C_{\{n_i\}}^{\alpha} \prod_{i=1}^{l} \left( \frac{B_i^{n_i}}{n_i!} \right) \Omega_n(k^2; m_1, \ldots, m_n), \qquad n = \sum_{i=1}^{l} n_i. \tag{A.1}$$

The coupling constants $B_i (i = 1, \ldots, l)$ for the $l$ considered particle species ($\pi$, $K$, N ...) are treated as free parameters to be fixed by comparison with experiment.

The invariant phase space integrals $\Omega_n(k^2; m_1, \ldots, m_n)$ are calculated numerically according to Lurcat and Mazur [32]. The normalization condition

$$\sum_{\{n_i\}} P_n^{\{n_i\}}(k^2, \alpha) = 1 \tag{A.2}$$

determines the hadronic mass spectrum $\tilde{\varrho}(k^2, \alpha)$ and the coefficients $g_n$ are given recursively [33]. $\alpha$ denotes the quantum numbers of the decaying fireball and the $C^{\alpha}_{\{n_i\}}$ ensure the conservation of strangeness $s$, baryon number $b$ and isospin $(T, t)$ [25]

$$C^{\alpha}_{\{n_i\}} \equiv C^{b,s,T,t}_{\{n_i\}} = \delta(b - \sum_{i=1}^{l} n_i b_i)\delta(s - \sum_{i=1}^{l} n_i s_i) G^{Tt}_{\{n_i\}}. \tag{A.3}$$

The statistical isospin weight factors can be calculated according to [34].

The invariant single-particle distributions

$$N_1 = 2E\,\frac{d^3N}{d^3p} \tag{A.4}$$

for a particle with momentum $\vec{p}$ and quantum numbers $\delta$ are given as sum over the exclusive (phase space like) ones [14],

$$N_1(k\alpha|\vec{p}\delta) = \sum_{\{n_i\}} P^{\{n_i\}}_n(k^2, \alpha) \cdot n_\delta \cdot \frac{\Omega_{n-1}((k-p)^2)}{\Omega_4(k^2)}. \tag{A.5}$$

The model predictions for asymptotic fireball masses $(M \to \infty)$ are governed by the maximum temperature $T_0$ [31], for instance

$$\langle n \rangle \propto \text{const}\,(T_0) \cdot M, \tag{A.6}$$

$$f_2^{--} \propto -\text{const}'\,(T_0) \cdot M, \tag{A.7}$$

$$N_1 \propto M \exp\,(-E/T_0). \tag{A.8}$$

This temperature parameter is, however, uniquely determined by the coupling constants $B_i$ by the relation [11]

$$\sum_{i=1}^{l} n_i 2\pi m_i B_i T_0 K_1\left(\frac{m_i}{T_0}\right) = 2 \ln 2 - 1. \tag{A.9}$$

So in the case of $\bar{\text{N}}$N annihilation the coupling for kaons and pions, $B_\text{K}$ and $B_\pi$ resp., are the only free parameters.

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