# WHAT SINGULARITIES GOVERN THE $\varrho$ REGGE TRAJECTORY BEHAVIOUR?

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A model of the $\varrho$ Regge trajectory with correct threshold behaviour and square-root asymptotics is developed. All the data available on charge-exchange $\pi$N cross-sections and on difference of the $\pi^+$p and $\pi^-$p total cross-sections at $p_l \geqslant 5$ GeV/$c$ are analyzed to determine the positions of the leading singularities. Light thresholds with mass up to $1-1.5$ GeV are shown to play an appreciable role in the $\varrho$-trajectory linearization. The conclusion is drawn from the data on cross-sections that the leading thresholds of the $\varrho$-trajectory are not too heavy and they are disposed in the region from 2 to 3 GeV.

Owing to the process $\pi^-p \to \pi^0n$, the data available on the $\varrho$ Regge trajectory always exceeded information concerning other trajectories except for, may be, the leading vacuum trajectory. The data on the $\varrho$-trajectory both in the scattering region and in the resonance region have demonstrated its considerable linearity. This, in particular, has led to a conclusion that the dynamics of states lying on Regge trajectories was to be determined by those channels where these states are more coupled states than resonances [1]. The estimation of the positions of leading singularities for the mesonic Regge trajectories may be exemplified by speculations [1] in terms of the quark model: with a quark mass $\gtrsim 5$ GeV the leading quark-antiquark threshold is located at $s \gtrsim 100$ GeV$^2$, that is, linear approximation to mesonic trajectories will be good enough in the range, say, $|s| \lesssim 20$—$30$ GeV$^2$.

As shown in [2, 3], the mass values of the leading thresholds can be significantly lowered. In this paper we want to construct a simple model of the $\varrho$-trajectory possessing both correct threshold behaviour and square-root asymptotics, and analyze the data on cross-sections which contain information on the $\varrho$-trajectory with a view of determining the leading-threshold positions. A model trajectory of this kind can also be a constituent of dual amplitudes with the finite number of resonances (for example, dual amplitudes with Mandelstam analyticity [4]).

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One needs to emphasize that the results derived in the present paper do not essentially depend on the parametrization of Regge trajectory. The parametrization used mostly reflects general analytic properties of the trajectory. The final result somewhat depends only on the supposed asymptotics of the trajectory. At this point we are guided by the requirements of the dual analytic models [4] and by the arguments given in [5].

### *The model*

When constructing a model for the Regge trajectory, we base on the approximation of a few stable thresholds. Moreover, we require the following properties of threshold contributions:

(*i*) additivity, i.e.

$$\alpha(s) = a + \sum_n \alpha_n(s), \qquad a = \alpha(0), \qquad \alpha_n(0) = 0,$$

where $\alpha_n(s)$ has only one threshold branch-point on the physical sheet;

(*ii*) positivity of the imaginary parts,

$$\operatorname{Im} \alpha_n(s) > 0 \quad \text{at} \quad s > s_n, \quad s = |s| + i\varepsilon;$$

(*iii*) threshold behaviour

$$\operatorname{Im} \alpha_n(s) \underset{s \to s_n}{\sim} (s - s_n)^{\operatorname{Re} \alpha(s_n) + 1/2};$$

(*iv*) square-root asymptotics on the physical sheet,

$$\alpha_n(s) \underset{|s| \to \infty}{\sim} (-s)^{1/2}.$$

It is convenient to use for $\alpha_n(s)$ a dispersion relation with one subtraction

$$\alpha_n(s) = \frac{s}{\pi} \int_{s_n}^{\infty} ds' \, \frac{\operatorname{Im} \alpha_n(s')}{s'(s' - s)}, \tag{1}$$

and to parametrize $\operatorname{Im} \alpha_n(s)$ is such a manner that the conditions (*i—iv*) should be satisfied. It is easy to be convinced that these conditions are met with the expression

$$\operatorname{Im} \alpha_n(s) = \gamma_n \sqrt{s/s_n - 1} \left[ \frac{\sqrt{s/s_n - 1}}{k_n + \sqrt{s/s_n - 1}} \right]^{d_n + f_n \sqrt{s/s_n - 1}}, \tag{2}$$

where $\gamma_n$, $k_n$, $d_n$, and $f_n$ are some positive parameters.

### *Choice of parameters*

In accordance with the requirement (*iii*) $d_n = 2 \operatorname{Re} \alpha(s_n)$. The parameters $k_n$ and $f_n$ rule how fast and in what manner $\alpha_n(s)$ go on the asymptotic regime, where

$$\alpha_n(s) \simeq \gamma_n e^{-k_n f_n} (-s/s_n)^{1/2}.$$

The parameter $k_n$ is responsible for the additional singularity that is present in $\alpha_n(s)$ and is located on unphysical sheets at real $s = s_n^{(2)} > s_n$, where $s_n^{(2)}$ is found from $k_n = \sqrt{s_n^{(2)}/s_n - 1}$. A branch point at $s = s_n^{(2)}$ will be soft when $\bar{f}_n \equiv f_n/k_n \geqslant d_n$.

As effective thresholds we took thresholds with

$$s_1 = 4m_\pi^2, \quad s_2 = 4m_K^2, \quad s_3 \geqslant 4m_N^2.$$

The first two thresholds define the behaviour of imaginary part of the trajectory in the resonance region, and the latter determines the resonance masses.

The following values have been chosen for the parameters $k_n$ and $f_n$:

$$k_n = \sqrt{s_n^{(2)}/s_n - 1}\,, \quad \sqrt{s_n^{(2)}} = \sqrt{s_n} + 2m_\pi,$$

$$f_n = k_n \bar{f}_n, \quad \bar{f}_n = 2d_n.$$

The parameters $a$, $\gamma_1$, $\gamma_2$ and $\gamma_3$ were deduced from masses and widths of the $\varrho$- and $g$-mesons [6].

*Results*

The $\varrho$-trajectory was calculated for different values of the parameter $s_3$. The dependence of $\varrho$-trajectory behaviour on the leading-threshold position is shown in Figs 1, 2.

One can see from Fig. 1 that the higher the mass of the leading threshold, the lower

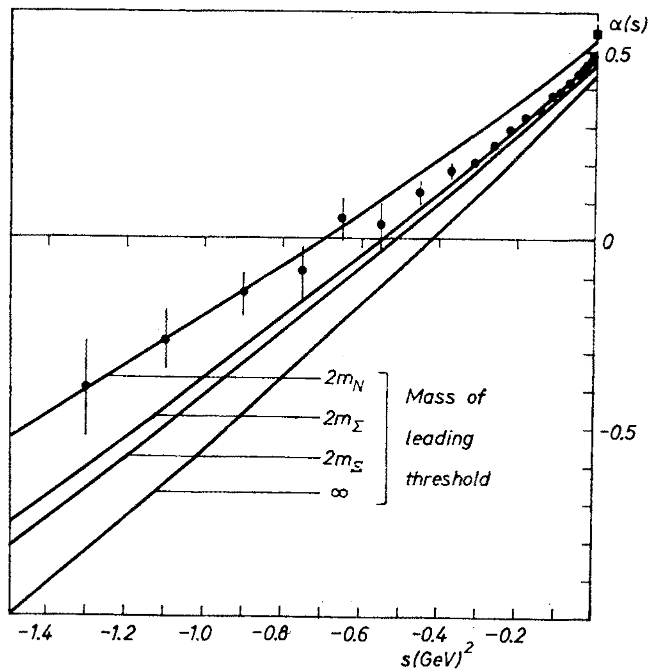


Fig. 1. The $\varrho$-trajectory in the scattering region at a few positions of the leading threshold. The case $s_3 = \infty$ corresponds to the presence of a linear term in the trajectory. Points are the data on effective $\varrho$-trajectory: ● from Ref. [7], ■ from Ref. [8]

is the trajectory; data on effective $\varrho$-trajectory [7] being situated between curves with $2m_N \leqslant \sqrt{s_3} \leqslant 2m_\Xi$, i.e. the leading threshold is situated within the range of the stable baryon-antibaryon thresholds. The lower curve in Fig. 1 corresponds to the case $s_3 \to \infty$ when the main term in the trajectory is linear,

$$\alpha(s) = 0.47+0.89s-0.145\sqrt{4m_\pi^2-s}\,. \tag{3}$$

Figs 1 and 2 give, to a certain extent, an answer to the question how to deduce the $\varrho$-trajectory in the range $s > 3$ GeV$^2$ when the data on $\varrho$ and $g$-resonances and the behaviour at negative $s$ are known. Such an analytic continuation of the $\varrho$-trajectory data to the

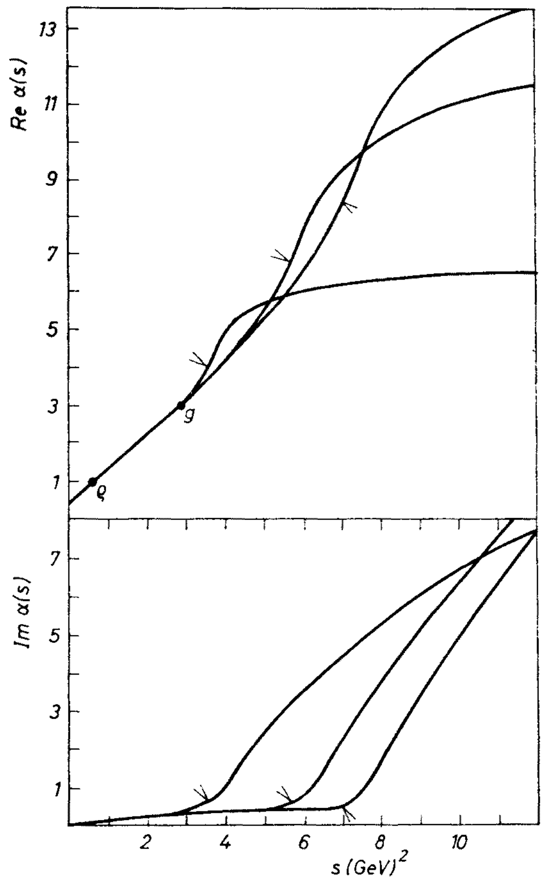


Fig. 2. The real and imaginary parts of the $\varrho$-trajectory in the positive — $s$ range at a few positions of the leading threshold (the threshold is shown by arrow)

region of large $s$ is, without saying, ambiguous. This ambiguity has been significantly decreased by the assumptions made above.

One can see from Fig. 2, that beyond the leading threshold the rise of a real part of the $\varrho$-trajectory slows down, the imaginary part increases more rapidly, and the resonances become superbroad.

It is commonly believed that the purely linear $\varrho$-trajectory — $\alpha_0(s) = 0.47+0.89s$ — reproducing masses of $\varrho$- and $g$-mesons, is rather consistent with the scattering-range data. This conclusion is, however, correct only when one does not take into account the resonance instability. With due regard for instability, for example, in the form (3),

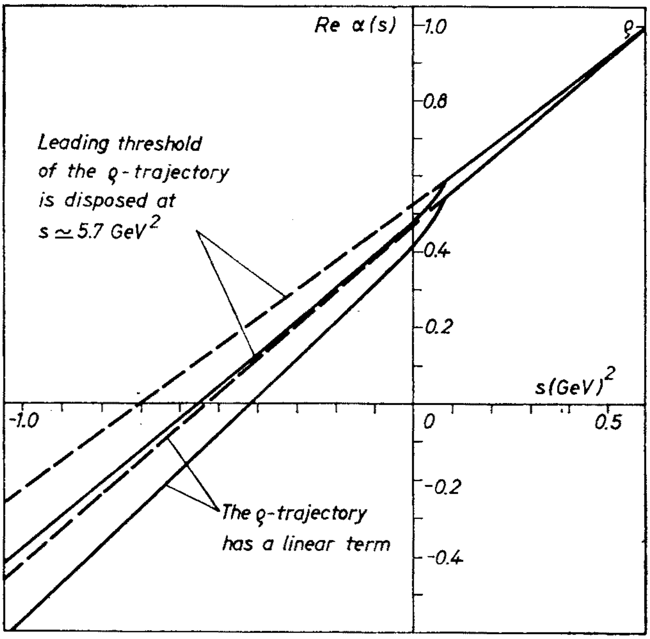


Fig. 3. An influence of the resonance instability on the behaviour of the $\varrho$-trajectory. The dashed lines are the model trajectories when $\pi\pi$-threshold is left aside: the upper line is the $\varrho$-trajectory model with the leading threshold at $s_3 = 4m_\Sigma^2$, the lower dashed line is the model with a linar term. The solid lines correspond to the cases when instability of the resonances is taken into account

the $\varrho$-trajectory is lowered down at negative $s$ (Fig. 3) and is poorly consistent with the data (lower curve on Fig. 1). One can estimate, following Ref. [9], that light thresholds lower the $\varrho$-trajectory at $s = 0$ by the value $\simeq 0.04$—$0.05$ resulting in $\alpha_\varrho(0) \simeq 0.43$, while from experiment

$$\alpha(0) = 0.48 \pm 0.01$$

(from $d\sigma/dt(\pi^- p \to \pi^0 n)$, [7]),

$$\alpha(0) = 0.55 \pm 0.03$$

(from $\sigma_t(\pi^+ p) - \sigma_t(\pi^- p)$, [8]).

In the model under treatment the $\varrho$-trajectory with the meson-meson thresholds not considered (upper dotted line in Fig. 3) goes above experimental points, and the $\varrho$- and $g$-mesons instability taken into account lowers it to the reasonable values. For more careful determination of the leading threshold mass let us analyze *all* the available data on the differential cross-sections of $\pi^- p \to \pi^0 n$ [10, 7] and on the difference of the $\pi^+ p$

and $\pi^- p$ total cross-sections at $p_l \geqslant 5$ GeV. We utilize the model $(\varrho+\varrho')$ and the following parametrization of the Regge-pole contribution to the invariant $\pi$N-amplitude [11, 12]:

$$A'_\nu = a_\nu(t)(1+\alpha_\nu)[i+\mathrm{tg}(\pi\alpha_\nu/2)](s/s_\nu)^{\alpha_\nu},$$

$$B_\nu = b_\nu\alpha_\nu(1+\alpha_\nu)[i+\mathrm{tg}(\pi\alpha_\nu/2)](s/s_\nu)^{\alpha_\nu-1},$$

$$\nu = \varrho, \varrho', \quad a_\varrho(t) = a_1+a_2 t, \quad a_{\varrho'}(t) = a_3, \quad \alpha_{\varrho'} = \alpha_{\varrho'}(0)+\alpha'_{\varrho'}t.$$

The observed quantities are connected with $A'_\nu$ and $B_\nu$ in such a manner

$$\frac{d\sigma}{dt} = \sum_\nu \frac{1}{8\pi p_l^2}\left[\left(1-\frac{t}{4m_N^2}\right)|A'_\nu|^2 - \frac{t}{4m_N^2}\,\frac{4m_N^2 p_l^2+st}{4m_N^2-t}\,|B|^2\right],$$

$$\sigma_t(\pi^+ p)-\sigma_t(\pi^- p) = 2\sum_\nu \mathrm{Im}\, A'_\nu(s,0)/p_l.$$

When the model was fitted to the data, the parameters $a_1, a_2, a_3, b_\varrho, b_{\varrho'}, s_\varrho, s_{\varrho'}, \alpha_{\varrho'}(0)$ and $\alpha'_{\varrho'}$ were free and the $\chi^2$ for different values of $s_3$ was calculated. In all cases $\chi^2$ per point exceeded 2 which is apparently due to the inconsistency of data from different

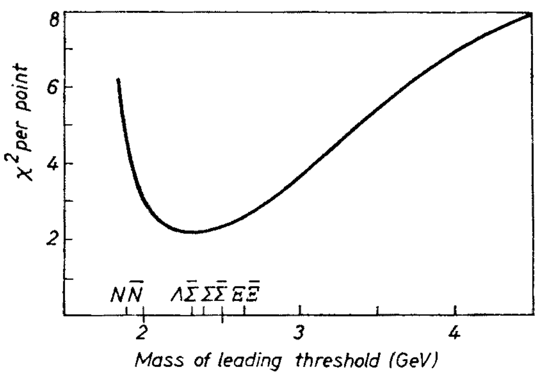


Fig. 4. The dependence of $\chi^2$ on the position of the $\varrho$-trajectory leading threshold when the model is fitted to the data on $d\sigma/dt(\pi^- p \to \pi^0 n)$ and $\sigma_t(\pi^+ p)-\sigma_t(\pi^- p)$. Stable baryon-antibaryon thresholds are also shown

teams (this point was discussed in Ref. [12]). Results of the fit are in Fig. 4, from which one can conclude that $2 \lesssim \sqrt{s_3} \lesssim 2.8$ GeV, i.e. the leading threshold in the $\varrho$-trajectory lies within the region of stable baryon-antibaryon thresholds. This conclusion was derived earlier in Refs [2, 3]. It is consistent also with findings made by Shirkov [13] and Bogdanova et al. [14]. Role of the baryon-antibaryon thresholds in hadronic processes was discussed by Chew and Koplik [15], and also by Bugrij and Kobylinsky [3].

The analysis carried out in this paper, and also in Refs [2, 3] stands for some connection between principal parameters of the $\varrho$-trajectory: its intercept, the first nonsense wrong--signature point, a mass of the leading singularity and maximal spins of resonances on the $\varrho$-trajectory. These correlations are summed up in Fig. 5. There the range is also picked out, which is preferred by data on $d\sigma/dt(\pi^- p \to \pi^0 n)$ and $\Delta\sigma_t$.

In conclusion we want to emphasize two findings following from the analysis. Firstly, light thresholds in the $\varrho$-trajectory with a mass up to 1—1.5 GeV play the sizable role

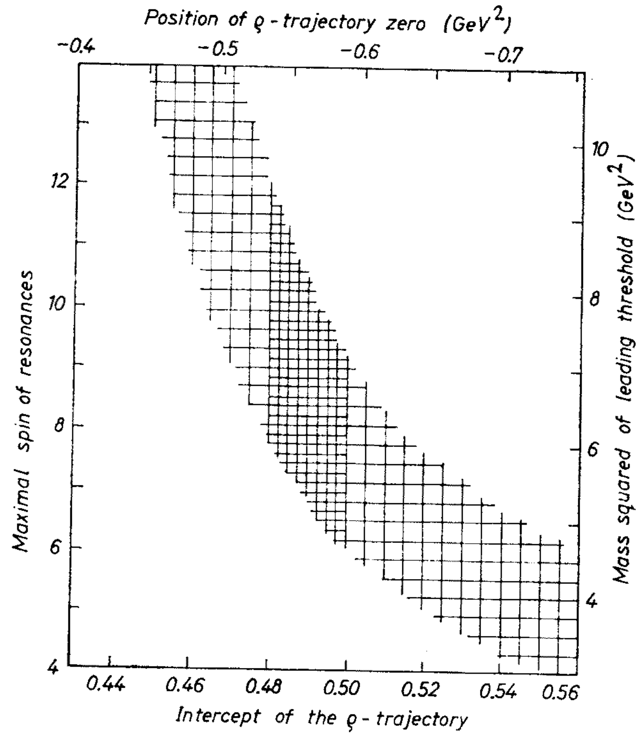


Fig. 5. Interrelation of the $\varrho$-trajectory principal parameters: intercept, zero position, leading-threshold mass and maximal spin of resonances. An ambiguity in parametrization of the trajectory results in a dispersion of parameters. There is also marked out the range preferable from the point of view of scattering data

in its linearization and, secondly, leading thresholds in this trajectory are not too heavy and are disposed in the region from 2 to 3 GeV.

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