DIPS, ZEROS AND LARGE $|t|$ BEHAVIOUR OF THE ELASTIC AMPLITUDE

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We discuss the Geometrical Scaling (GS) real part effects in the pp elastic differential cross-section, in particular around the dip and at large $|t|$. The forward ratio $\varrho = \mathrm{Re}\, F/\mathrm{Im}\, F$ is accurately predicted from the shape of $d\sigma/dt$ in the dip region. The real part is shown to tend to flatten and to weaken the energy dependence of the large $|t|$ differential cross-section. $\mathrm{Im}\, F(s, t)$ and $\mathrm{Re}\, F(s, t)$ were constructed numerically in the measured t region from the GS differential equation relating $d\sigma/dt$ to the amplitude. We claim, based on our constructed amplitude and on forward dispersion calculations, that the dip must disappear at higher energies, of the order $\sqrt{s} \simeq 300$ GeV.

1. *Introduction*

Geometrical Scaling (GS) relates in a specific way the elastic differential cross-section to the imaginary and real parts of the amplitude. This relation takes the form of a non-linear first order differential equation. In this paper we shall concentrate on the discussion of this equation.

Most of the results and predictions we obtain can be experimentally checked and as better and higher energy $d\sigma/dt$ data accumulate they may become very stringent tests of GS. In particular we show that from the shape of the pp differential cross-section around the moving dip position t_D the value of ϱ, the ratio of the real to imaginary part of the amplitude at $t = 0$ can be computed. Further, our GS analysis leads to the conclusion, depending only on the reliability of the forward dispersion relation calculation of $\varrho(s, 0)$, that the dip will disappear at energies of the order $\sqrt{s} \simeq 300$ GeV to reappear again at

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higher energies. Concerning the large $|t|$, $|t| \gtrsim 3$ GeV^2, behaviour of $d\sigma/dt$ we show that the real part tends to flatten $d\sigma/dt$ and as energy increases to weaken the GS shrinkage effect.

We start by briefly describing our main equation, Eq. (4) below. Geometrical Scaling as an high energy property of the hadronic elastic amplitude means

$$\text{Im}\, F(s, t) = \text{Im}\, F(s, 0)\, \phi(\tau) \tag{1}$$

and

$$\text{Re}\, F(s, t) = \text{Re}\, F(s, 0) \frac{d}{d\tau}(\tau\phi(\tau)), \tag{2}$$

where τ is an high energy scaling variable

$$\tau = -t\sigma_{\text{tot}}(s) \tag{3}$$

and the normalization is such that $\text{Im}\, F(s, 0) = s\sigma_{\text{tot}}(s)$, $\phi(0) = 1$. Relation (2) is an asymptotic relation and it follows from (1) using general arguments of analyticity and crossing [1, 2].

Combining (1) and (2) we obtain for the differential cross-section

$$\frac{d\sigma}{dt}(s, t) = \frac{d\sigma}{dt}(s, 0) \left\{\phi^2(\tau) + \varrho^2(s, 0)\left[\frac{d}{d\tau}(\tau\phi(\tau))\right]^2\right\} \frac{1}{1+\varrho^2(s, 0)}, \tag{4}$$

where

$$\varrho(s, 0) = \text{Re}\, F(s, 0)/\text{Im}\, F(s, 0).$$

The discussion of Eq. (4) will be the content of the paper. In Sections 2 and 3 we consider two special points of phenomenological interest, the behaviour in the dip region (Sect. 2) and the behaviour at large $|t|$ (Sect. 3) of $d\sigma/dt$. In Section 4 we present the result of the full numerical integration of Eq. (4). In the last Section we draw our conclusions.

2. *The dip*

At high energy and away from dips the real part is small and the first term on the right hand side of (4) dominates. GS should then be directly observed in the differential cross-section. At the dip in principle two possibilities may happen. Either one observes a minimum of the imaginary part of the amplitude $d\phi/d\tau(\tau_D) = 0$, or the imaginary part has a zero, $\phi(\tau = \tau_D) = 0$, and the zero is filled in by the real part. In the first case one sees from Eq. (4) that the second term, the real part, gives no contribution to the normalized differential cross-section, and GS should still be observed at the dip. Experimentally $d\sigma/dt$ at the dip has a dependence not in agreement with GS for the imaginary part. This then necessarily implies that it is the second possibility that takes place, $\phi(\tau = \tau_D) = 0$ and the magnitude and the s dependence of $d\sigma/dt$ at the minimum should be given by the real part contributions. This is the possibility we discuss next.

At the minimum, $\tau = \tau_D$, GS requires, as we have just mentioned,

$$\phi(\tau = \tau_D) = 0 \tag{5}$$

and Eq. (4) gives then the following relation:

$$\frac{d\sigma}{dt}(s, \tau_D) \Big/ \frac{d\sigma}{dt}(s, 0) = K^2 \frac{\varrho^2(s, 0)}{1+\varrho^2(s, 0)} \tag{6}$$

with

$$K = \left[\tau \frac{d\phi(\tau)}{d\tau}\right]_{\tau=\tau_D}. \tag{7}$$

The GS equation (6) relates two measurable quantities $d\sigma/dt$ at the dip (normalized to $d\sigma/dt$ at $t = 0$) and ϱ^2. The constant K is energy independent.

It is known for some time that relation (6) is in rough agreement with data [2]. At low energies, below the region of validity of GS for the elastic amplitude, $P_{LAB} \simeq 100$ GeV/c, no more than a qualitative agreement can be expected. However, at higher energies and to the extent that GS is correct, relation (6) should be quite well satisfied. At present there are accurate ISR data on $\varrho(s, 0)$ [3] and $d\sigma/dt$ at the dip [4] and a more detailed check of (5) should be attempted.

We would like to point out that the constant K is not a simple adjustable normalization parameter but can be computed from the measurements of $d\sigma/dt$ around the position of the dip. In fact from the good data which now exists in the dip region Eq. (4) allows not only the determination of K but as well predicts $\varrho(s, 0)$.

Expanding $\phi(\tau)$ around the dip position τ_D we have

$$\phi(\tau) = \phi(\tau_D) + (\tau - \tau_D) \left.\frac{d\phi}{d\tau}\right|_{\tau_D} + \ldots .$$

Because of (5) $\phi(\tau)$ is then, in first approximation, given by

$$\phi(\tau) \approx (\tau - \tau_D) \left.\frac{d\phi}{d\tau}\right|_{\tau_D}. \tag{8}$$

Using (7) and (8) we write (4) in the form

$$\frac{d\sigma}{dt}(s, t) \Big/ \frac{d\sigma}{dt}(s, 0) = K^2 \left\{\left(\frac{\tau - \tau_D}{\tau_D}\right)^2 + \frac{\varrho^2}{1+\varrho^2}\right\}. \tag{9}$$

The validity of Eq. (9) depends on the validity of the linear approximation (8) for $\phi(\tau)$ and it also requires $|\varrho| \ll 1$.

Using published data at $\sqrt{s} = 53$ GeV and Eq. (9) we made a three parameter fit (τ_D, ϱ, K) to the normalized $d\sigma/dt$ in the dip region. The result is shown in Fig. 1 (solid line). The values obtained for the parameters were: $t_D = -(1.364 \pm 0.016)$ GeV^2, $\varrho = 0.079 \pm 0.017$ and $K = (6.10 \pm 1.0)\, 10^{-3}$ with $\chi^2 = 0.94$ per degree of freedom. The experimental value at 53 GeV for ϱ is 0.078 ± 0.010 [3].

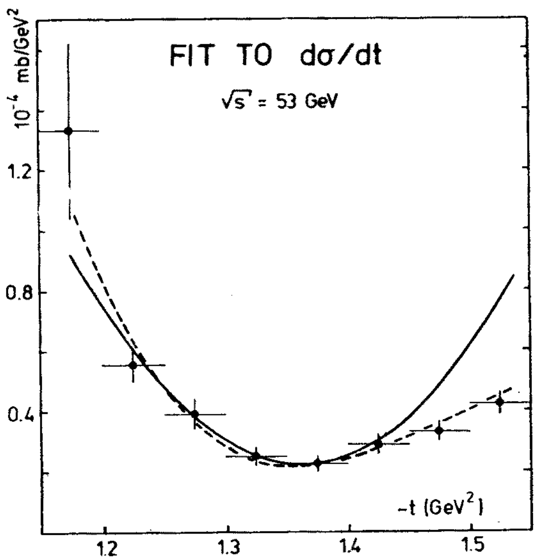

Fig. 1. The shape of $d\sigma/dt$ around the dip ($\sqrt{s}$ = 53 GeV). Data taken from Ref. [4]. Solid line: fit according to Eq. (9) in the t region from -1.175 to -1.425 GeV^2. Dashed line: fit according to an approximation to order $(\Delta\tau/\tau_D)^3$ and $\varrho^2(\Delta\tau/\tau_D)$, in the t region from -1.175 to -1.575 GeV^2 (see text)

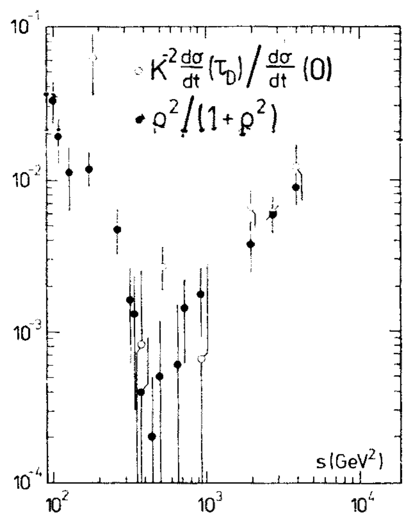

Fig. 2. Test of the energy dependence of $d\sigma/dt$ at the position of the dip. ($K = 6.10\times 10^{-3}$). Data taken from Refs [3] and [4]. Further references to the data are given in [2]

The parametrization (9) shows a symmetric behaviour around τ_D. This is due to the linear approximation (8). In a better approximation, to order $(\Delta\tau/\tau_D)^3$ and $\varrho^2(\Delta\tau/\tau_D)$ and in agreement with data, asymmetry occurs in $d\sigma/dt$ and we show in Fig. 1 (dashed line) the result of such a fit.

Having determined $K(= 6.10 \times 10^{-3})$ one may next check Eq. (5) without free normalization. The comparison with data is shown in Fig. 2. At ISR energies the agreement is good, below it is only qualitative. There the corrections to GS cannot be ignored [5] in particular for the real parts and and at large $|t|$. In general more accurate data of the dip position and of $d\sigma/dt$ at the dip, similar to the existing data at $\sqrt{s} = 53$ GeV, are still missing.

3. *Large* $|t|$ *behaviour*

Both the t and the s dependence of $d\sigma/dt$ at large $|t|$ may be of interest to test the extent of validity of GS. At large $|t|$, say $|t| \gtrsim 3.0$ GeV^2 at $\sqrt{s} = 53$ GeV, $d\sigma/dt$ decreases with $|t|$ showing no particular structure. From inspection of data (see Fig. 3) $d\sigma/dt$ in good approximation behaves as an exponential in $\sqrt{\tau}$. Let us then simply assume that

$$\phi(\tau) \sim \exp(-B\sqrt{\tau}), \qquad (10)$$

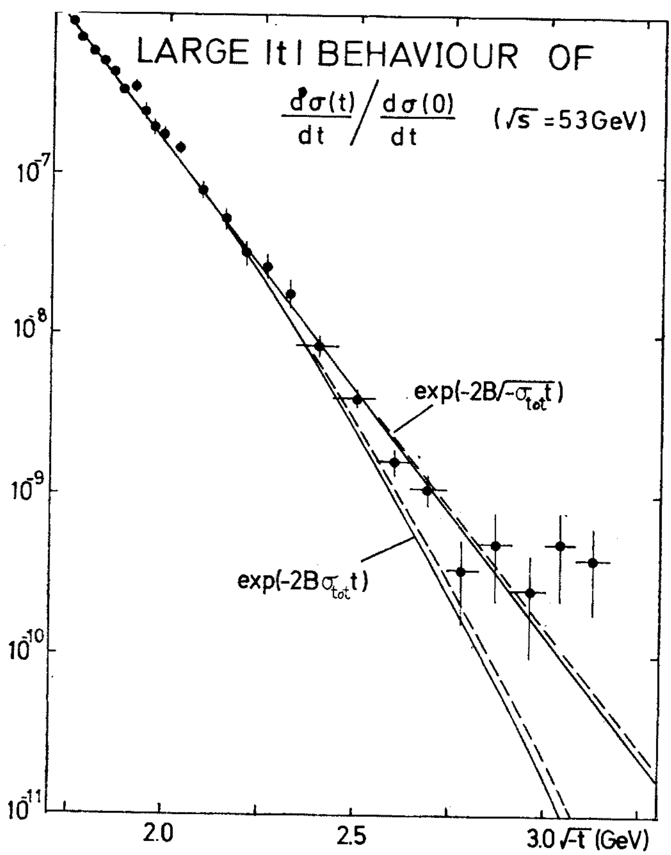

Fig. 3. The large $|t|$ behaviour of the normalized differential cross-section at $\sqrt{s} = 53$ GeV. Data taken from Refs [4] and [8]. Solid lines: contributions of Im F for two different parametrizations ($B\sqrt{\sigma_{tot}} = 3.47$ GeV^{-1} and $B\sigma_{tot} = 0.92$ GeV^{-2}). Dashed lines: $d\sigma/dt$ reconstructed from the full amplitudes corresponding to both of the parametrizations

where B is the effective (energy independent) slope. Hence using Eq. (2), we obtain

$$\varrho(s,t) = \varrho(s,0)\,(1 - B\sqrt{\tau}/2) \tag{11}$$

and

$$\frac{d\sigma}{dt}(s,t) \sim e^{-2B\sqrt{\tau}}\{1+\varrho^2(s,0)\,(1-B\sqrt{\tau}/2)^2\}. \tag{12}$$

The first thing one notes in Eq. (12) is that at a given energy the real part correction increases with $|t|$ roughly as $(\varrho B\sqrt{\tau}/2)^2$, thus making $d\sigma/dt$ flatter. Second, as the energy increases the shrinkage effect associated with (10), $\exp(-B\sqrt{\tau}) = \exp(-B\sqrt{\sigma_{tot}}\sqrt{-t})$, tends to be cancelled by the real part correction due to the rapid rise of $\varrho(s, 0)$ throughout the ISR energy range. Therefore, $d\sigma/dt$ becomes less energy dependent than what is expected from Eq. (10) alone. Flattening at large $|t|$ and weak energy dependence are in fact features seen in the data [7].

Using Eq. (12) and taking B and the overall normalization as free parameters, we made a fit to the large $|t|$ data of Ref. [8]. The result is shown in Fig. 3. The agreement is reasonable apart from the last two points.

In the measured t range the effect of the real part amounts to a correction of only 12%.

4. *The differential equation*

Having discussed in some detail the two regions that presently seem to be of major interest, the dip and the large $|t|$ region, we turn back again to Eq. (4). Using as input the 53 GeV $d\sigma/dt$ data [4, 8, 9] and the experimental value of $\varrho(s, 0)$ [3] and choosing as boundary value $\phi(0) = 1$, the equation was solved numerically. The results are presented in Fig. 4. As expected, Im F dominates apart from the dip region and at large $|t|$. Re F, being positive at $t = 0$, has a zero at around $t = -0.16$ GeV^2, followed by a minimum at $t = -0.36$ GeV^2 and a second zero at $t = -2.05$ GeV^2 shortly after the position of the secondary maximum of $d\sigma/dt$ (values are given for $\sqrt{s} = 53$ GeV). Compared with the results of Chapter 2 and 3, where fits over certain t ranges were made, the errors of the results as found by the numerically solution of Eq. (4) are much larger. Obviously the reason is the numerical determination of a derivative from two adjacent data points.

Our plots can be used to predict the energy dependence of the differential cross-section. For example as $\varrho(s, 0)$ is still rising with s one may ask the question if the dip will not disappear at higher energies. Using our solution we show in Fig. 5 how the shape of the differential cross-section around the dip is expected to vary as a function of ϱ. For $\varrho \simeq 0.13$ the dip will completely disappear. This value of ϱ should occur, according to the dispersion relation calculation of Grein [6], at $\sqrt{s} \simeq 300$ GeV. At much higher energies as $\varrho(s, 0)$ is bound to vanish the dip shall again reappear.

One should realize that Eq. (4) is a fixed s equation and the solution to it does not involve comparing data at different energies. This means that the constructed solution for the amplitude is much more stable than the results obtained by the usual fixed t s-derivative method [10].

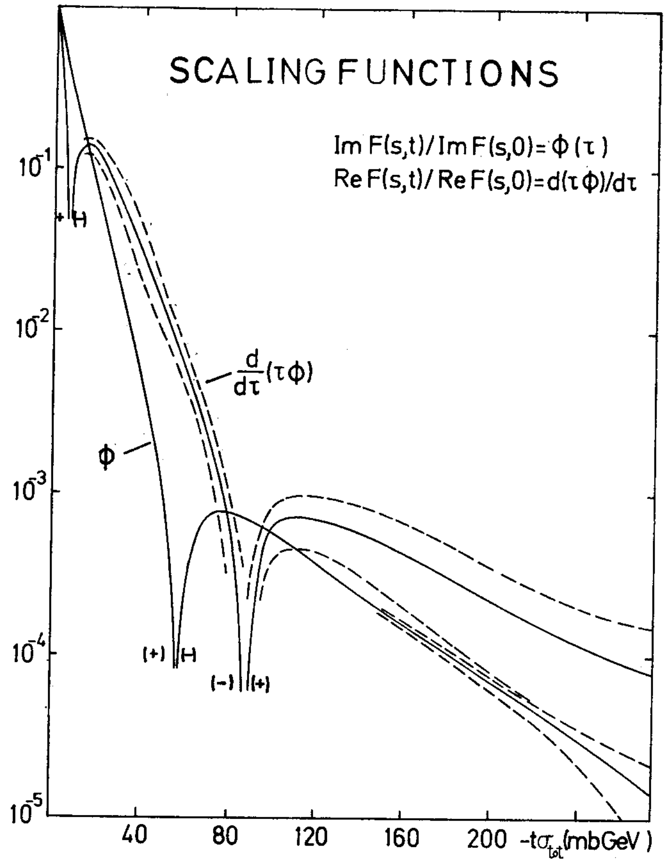

Fig. 4

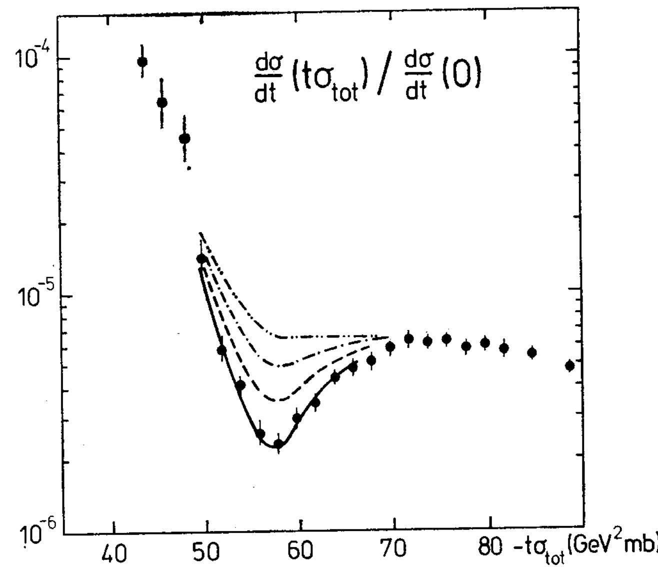

Fig. 5

Fig. 4. The scaling function ϕ and the derivative $d(\tau\phi)/d\tau$ as found by solving numerically the GS differential equation (4) (solid lines). The dashed lines represent the error corridors

Fig. 5. Predictions for $d\sigma/dt$ around the dip at high energies as calculated from the solution shown in Fig. 4. Solid line: fit to the data [4] with $\varrho = 0.078$, dashed line: $\varrho = 0.10$, dash dot line: $\varrho = 0.12$, dash dot dot line $\varrho = 0.14$

5. *Conclusions*

We discussed in this paper the role of the GS real part in the elastic differential cross-section, in particular in the dip region and at large $|t|$. In the dip region, from the shape of the cross-section around the dip we predict accurately the forward ratio $\varrho = \mathrm{Re}F/\mathrm{Im}F$. In the large $|t|$ region we obtain a flattening as well as a not very appreciable energy variation of the elastic differential cross-section due to the real part correction. The numerical solution of the GS differential equation relating $d\sigma/dt$ to $\mathrm{Im}F(s, t)$ and $\mathrm{Re}F(s, t)$ provides a smooth, fairly reliable amplitude. The plots of the normalized $\mathrm{Im}F(s, t)$ and $\mathrm{Re}F(s, t)$ allow predictions of the energy dependence of $d\sigma/dt$ and from them we predict the vanishing of the dip at higher energies. All these results are either in agreement, qualitative at least, with experiment or will be easily tested in the future.

Regarding the behaviour at the dip we would like to stress again that both the t dependence (Fig. 1) as well as the s dependence (Fig. 2) confirm the expectations of GS and demonstrate that $\mathrm{ImF}(s, t)$ has a zero (and not a minimum) at the dip. Concerning the large $|t|$ behaviour the arguments for a flattening and for hindred energy variation of $d\sigma/dt$ are of course more general than the parametrization, an exponential in $\sqrt{|t|}$, used in deriving them. Similar arguments can be derived from an exponential in $|t|$, but in that case several local exponentials have then to be considered in the $3 \lesssim |t| \lesssim 10\,\mathrm{GeV}^2$ region. An exponential in $\sqrt{|t|}$ provides in our case a simpler parametrization and is indeed a very natural large $|t|$ behaviour for a large class of models [11].

However, the deviation of the two last points (compare Fig. 3) from our example cannot be accounted for by the real part and may indicate that either the imaginary part of the amplitude changes its behaviour and GS may still be correct or that GS breaking factors become important.

The behaviour of $\mathrm{Im}F(s, t)$ at large $|t|$ has a very interesting consequence for the impact parameter picture. The small slope and the absence of additional dips result in the deviation of the eikonal from a simple Gaussian. In fact different impact parameter analyses have shown that the eikonal (as well as the overlap function) flattens off at small b [5,12]. This flattening is observed not only in pp scattering at high energy, but as well in other reactions and at low energies [13]. It is certainly a relevant stable feature of diffraction to be included in any phenomenologically acceptable model for the eikonal. Models failing to do that and blaming the eikonal or a related expansion for the disagreement with data are clearly wrong.

Even at the largest presently measured $|t|$ values the idea of GS may still prove to be useful. Clearly as one moves into larger and larger $|t|$ values one expects GS to break down and a regime of hard scattering and dominantly real amplitudes eventually to take over. However, to $|t| \simeq 10\,\mathrm{GeV}^2$ at $\sqrt{s} \simeq 53$ corresponds a centre of mass angle of $\theta \simeq 6.8°$. Scattering in this region cannot yet be considered as genuine large angle scattering and it is probably still governed by diffraction.

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