LETTERS TO THE EDITOR

ON SU(4) MASS FORMULAE

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The generalization of the SU(3) Schwinger mass formula to the SU(4) is given.

1. In the nonperturbative approach to the broken SU(4) symmetry (the "asymptotic" SU(4)) the following constraints on the masses and mixing parameters of the 16-plet of mesons have been obtained [1]

$$\lambda_1^2 m_\eta^2 + \delta_1^2 m_{\eta_g}^2 + \kappa_1^2 m_{\eta_c}^2 = \tfrac{1}{3}(4m_K^2 - m_\pi^2), \tag{1a}$$

$$\lambda_1\lambda_2 m_\eta^2 + \delta_1\delta_2 m_{\eta_g}^2 + \kappa_1\kappa_2 m_{\eta_c}^2 = -\frac{\sqrt{2}}{3}(m_K^2 - m_\pi^2), \tag{1b}$$

$$\lambda_2^2 m_\eta^2 + \delta_2^2 m_{\eta_g}^2 + \kappa_2^2 m_{\eta_c}^2 = \tfrac{3}{2}m_D^2 + \tfrac{1}{9}m_K^2 - \tfrac{4}{9}m_\pi^2, \tag{1c}$$

$$m_F^2 - m_K^2 = m_D^2 - m_\pi^2. \tag{1d}$$

The physical noncharmed isosinglets η, η_g and η_c are mixed states of two members of 15-plet (η_8 and η_{15}) and a singlet η_0. The mixing parameters λ_i, δ_i and κ_i ($i = 1, 2, 3$) are defined by

$$\begin{pmatrix} |\eta\rangle \\ |\eta_g\rangle \\ |\eta_c\rangle \end{pmatrix} = \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \delta_1 & \delta_2 & \delta_3 \\ \kappa_1 & \kappa_2 & \kappa_3 \end{pmatrix} \begin{pmatrix} |\eta_8\rangle \\ |\eta_{15}\rangle \\ |\eta_0\rangle \end{pmatrix}. \tag{2}$$

The mass formulae (1) follow immediately by the infinite momentum method from the

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"exotic" commutation relations

$$[\dot{T}_\alpha, T_\beta] = 0, \tag{3}$$

where T_α is the SU(4) generator, $\dot{T}_\alpha$ is its time derivative $\left(\dot{T}_\alpha \equiv \frac{d}{dt} T_\alpha\right)$ and (α, β) stands for "exotic" combinations of indices, i. e. such combinations for which the commutators $[\dot{T}_\alpha, T_\beta]$ do not belong to a 15-plet.

2. In the present paper we give the additional constraints

$$\lambda_1^2 m_\eta^4 + \delta_1^2 m_{\eta_g}^4 + \kappa_1^2 m_{\eta_c}^4 = m_\pi^4 + \tfrac{8}{3}\, m_K^2(m_K^2 - m_\pi^2), \tag{4a}$$

$$\lambda_1\lambda_2 m_\eta^4 + \delta_1\delta_2 m_{\eta_g}^4 + \kappa_1\kappa_2 m_{\eta_c}^4 = -\frac{2\sqrt{2}}{3}\, m_K^2(m_K^2 - m_\pi^2), \tag{4b}$$

$$\lambda_2^2 m_\eta^4 + \delta_2^2 m_{\eta_g}^4 + \kappa_2^2 m_{\eta_c}^4 = m_\pi^4 + \tfrac{1}{3}\, m_K^2(m_K^2 - m_\pi^2) + 3m_D^2(m_D^2 - m_\pi^2), \tag{4c}$$

which follow from the "exotic" commutation relations

$$[\dot{T}_\alpha, \dot{T}_\beta] = 0. \tag{5}$$

Such relations can be obtained, for example, in the free quark model.

From Eqs (1a) – (1c) and (4a) – (4c) one obtains three mass formulae. For example, the mass formula containing masses of η and η_g mesons is

$$\begin{aligned}
&\{8(m_K^2 - m_\pi^2)^2 - (3m_\eta^2 - 4m_K^2 + m_\pi^2)(4m_K^2 - m_\pi^2 - 3m_{\eta_g}^2)\} \\
&\{(m_\eta^2 - 3m_D^2 + 2m_\pi^2)(m_{\eta_g}^2 - 3m_D^2 + 2m_\pi^2) - 3(m_D^2 - m_\pi^2)^2 \\
&+ (m_\eta^2 - \tfrac{1}{3}\, m_K^2 - \tfrac{2}{3}\, m_\pi^2)(m_{\eta_g}^2 - \tfrac{1}{3}\, m_K^2 - \tfrac{2}{3}\, m_\pi^2) \\
&+ \tfrac{5}{9}\,(m_K^2 - m_\pi^2)^2\} = 4(m_K^2 - m_\pi^2)^2(m_\eta^2 + m_{\eta_g}^2 - 2m_K^2)^2.
\end{aligned} \tag{6}$$

Replacing in Eq. (6) the pair (η, η_g) by (η, η_c) and (η_g, η_c) one obtains the other two formulae. These formulae can be regarded as a generalization of the SU(3) Schwinger mass formula[1] to the SU(4).

To verify the obtained mass formulae in the pseudoscalar case we calculate the masses of the η and η_c mesons in two cases

(a) if η_g = E(1420), then $m_\eta = 0.543$ GeV and $m_{\eta_c} = 2.800$ GeV;
(b) if η_g = X(960), then $m_\eta = 0.495$ GeV and $m_{\eta_c} = 2.675$ GeV.

In both the cases there exists the solution with mixing parameters corresponding to almost pure charmonium structure of the η_c state. So, the 16-th pseudoscalar meson η_c may be identified with the x(2.83). We see also that in the E case the prediction is closer to the experimental value not only for the η(0.549) mass but also for the x(2.83) mass.

[1] In the SU(3) the Schwinger mass formula follows from the "exotic" commutation relations (3) and (5) [2].

In the vector meson case we solve the equations with respect to the masses of the φ and φ_c mesons. We obtain

$$m_{\varphi} = 1.012 \text{ GeV and } m_{\varphi_c} = 2.754 \text{ GeV}.$$

There exists the solution in which the φ_c meson is the pure orthocharmonium state. The identification of the $\varphi_c(2.754)$ state with the $\Psi(3.1)$ meson seems to be rather unsatisfactory. The $\varphi_c(2.754)$–$\Psi(3.1)$ problem is discussed in our paper [3] where exactly the same predictions on masses of 0^- and 1^- mesons have been obtained in the broken $SU(8)_w$ symmetry.

We would like to note that in the present approach there exists a mass formula containing an additional singlet, i. e. for mixing of the 15-plet with two singlets. Predictions of that formula and predictions of the Eq. (6) for mesons with other spin-parities will be given elsewhere.

REFERENCES

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