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HOW THE TRANSVERSE MOMENTA OF CLUSTERS AFFECT THE PREDICTED SHORT RANGE TWO-PARTICLE CORRELATIONS*

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It is shown that taking into account the transverse momenta of clusters, one obtains for the short range correlations formulae, which are about as simple as those well-known for the $p_T = 0$ case, but agree much better with experiment.

Particles produced in high energy hadron-hadron collisions exhibit short range positive (attractive) correlations. For recent data see Refs [1] and [2]. It has been noticed by a number of people (Refs [3–9]) that these correlations can be easily explained if the final particles are decay products of clusters, which have spherically symmetrical decay distributions. Simple analytic formulae for the short range part of the correlation function have been derived assuming that the decaying clusters have zero transverse momentum. More general cases have been studied by Monte Carlo methods.

Here we show that for correlations among pions the simple analytic formulae can also be derived, when the condition of zero transverse momentum for the cluster is relaxed. Using for the mass and transverse momentum distribution of the clusters the estimates from Ref. [10], we find for the short range part of the correlation function

$$C(\eta_1, \eta_2) = \bar{K} \exp\left[-\frac{(\eta_1-\eta_2)^2}{4\sigma^2}\right], \quad \sigma \approx 0.6, \tag{1}$$

where η_1 and η_2 are the pseudorapidities $\left(\eta = \ln \operatorname{ctg}\frac{\theta}{2}\right)$ of the two particles, and $\bar{K}$ (see derivation) is a constant. Previous analytic formulae were giving $\sigma \approx 0.9$ (note that we are using pseudorapidities, so that the finite mass corrections given e. g. in Ref. [11] are

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not applicable). The recent experimental estimates are $\sigma = 0.65$ from Ref. [1] and $\sigma = 0.60$ from Ref. [2].

The derivation of formula (1), which is a simple extension of the derivation given for the $p_T = 0$ case in Ref. [5], is as follows. Let us consider a cluster with mass M, transverse momentum $p_T = M \sinh Y$, and longitudinal momentum $p_L = 0$. The assumption $p_L = 0$ is not very restrictive, because with respect to Lorentz transformations along the beam axis the resulting formula (6) is almost invariant (it would be rigorously invariant, if the pseudorapidities were replaced by rapidities). Consequently, it is not necessary to specify in which Lorentz frame $p_L = 0$. The single particle density in η for the decay products is

$$\varrho(\eta) = \int f(E_0) P dE \frac{d \cos \theta}{d\eta} d\varphi, \tag{2}$$

where the subscript "0" refers to quantities evaluated in the rest frame of the cluster.

Using the Lorentz transformation

$$E_0 = E\left(\cosh Y - \frac{P \sinh Y}{E \cosh \eta} \cos \varphi\right) \tag{3}$$

and making the ultrarelativistic approximation $P \approx E$ (see discussion at the end of the paper, this approximation is not applicable for particles heavier than pions), we find

$$\varrho(\eta) = \frac{n}{2 \cosh^2 \eta} \frac{\cosh Y}{[1+\sinh^2 Y \operatorname{tgh}^2 \eta]^{3/2}}, \tag{4}$$

where n is the number of particles considered (e. g. of positive pions) among the decay products of the cluster. For $Y = 0$ this density reduces to the familiar $\cosh^{-2}\eta$ distribution. As well-known, for $Y = 0$ formula (4) may be closely approximated by a Gaussian. We found that in the whole region $0 \leqslant Y \leqslant 2$ a good approximation is

$$\varrho(\eta) = \frac{n}{\sigma(Y)} \sqrt{\frac{\pi}{2}}\, e^{-\frac{\eta^2}{2\sigma^2(Y)}}, \tag{4'}$$

where

$$\frac{1}{2\sigma^2(Y)} = 1.161 \cosh^2 Y - 0.277. \tag{5}$$

The choice of the "best" value of $\sigma(Y)$ deserves a comment. We assumed that (4′) should agree with (4) as to the value of η where $\varrho(\eta) = \frac{1}{2}\varrho(0)$. For $Y = 0$ this yields $\sigma(0) = 0.75$, which is smaller than the value $\sigma(0) = 0.91$ derived by cosmic ray physicists (cf. e. g. Ref. [12]) from the condition $\sigma^2(0) = \int\limits_0^\infty x^2 \cosh^{-2} x dx$. We think that our choice corresponds more closely to the kind of fits made in Refs [1] and [2].

The short range part of the correlation function corresponding to the density (4′) is (see Ref. [5] for a discussion)

$$C(\eta_1, \eta_2) = \left\langle N \frac{n(n-1)}{2\pi\sigma^2(Y)} e^{-\frac{(\eta_1+\eta_2-2H)^2}{4\sigma^2(Y)}} \right\rangle e^{-\frac{(\eta_1-\eta_2)^2}{4\sigma^2(Y)}}. \tag{6}$$

Here N, n, H (capital η) denote respectively the number of clusters, the multiplicity in the decay of one cluster and the pseudorapidity of the cluster. The brackets $\langle\ \rangle$ denote averaging over N, n and H. The result of averaging over H depends on the experimental situation. In the two most interesting cases, however, when either H has a flat distribution in a range large compared with $\sigma(Y)$ [1], or one of the pseudorapidities η_i is fixed and no other constraints H are imposed [2], averaging over H introduces an additional factor $\sigma(Y)$. Assuming that N and n are not correlated to Y we have therefore

$$C(\eta_1, \eta_2) = \frac{K}{\sigma(Y)} e^{-\frac{(\eta_1-\eta_2)^2}{4\sigma^2(Y)}}, \tag{7}$$

where K does not depend on Y. Averaging (7) over the distribution [10]

$$\frac{1}{N}\frac{dN}{dp_T^2} = \beta e^{-\beta p_T^2}, \qquad \beta = 1.1\ \text{GeV}^{-2}, \tag{8}$$

we find finally

$$C(\eta_1, \eta_2) = K'\left[\frac{e^{-\gamma}}{\gamma} + \frac{\sqrt{\pi}}{2}\frac{\text{erfc}\sqrt{\gamma}}{\gamma^3}\right] \approx \overline{K} e^{-\frac{(\eta_1-\eta_2)^2}{4\sigma^2}}. \tag{9}$$

Here

$$\gamma = \frac{\beta M^2}{2\sigma^2(0)1.161} + \frac{(\eta_1-\eta_2)^2}{4\sigma^2(0)}, \tag{10}$$

$\overline{K}$ and K' are constants, and in the Gaussian approximation $\sigma = 0.605$ so that formula (1) is obtained.

Finally, let us discuss the ultrarelativistic approximation $P \approx E$ i. e. $m \approx 0$. The correction of order m^2 to the single particle distribution is

$$\varrho_1(\eta) = \frac{m^2}{2}\int_m^\infty P_0 f(E_0) E_0^{-2} dE_0 \left[\frac{\cosh Y}{[1+\sinh^2 Y \,\text{tgh}^2\,\eta]^{3/2}} - 1\right]\frac{1}{\cosh^2\eta}. \tag{11}$$

For $Y = 0$ this correction vanishes. Otherwise it can be split into two terms by opening the square bracket. The first term just renormalizes the leading term (4). The second gives a negative, approximately Gaussian distribution, thus it makes the total distribution broader. The normalization of this terms depends on the form of the function $f(E_0)$. Putting $f(E_0) \sim e^{-\beta E_0}$, with β fixed to get $\langle E_0\rangle = 375$ MeV as in Ref. [10], we found that for pions, at $\eta = Y = 0$ the negative term is about 11 per cent of the renormalized main term. Thus, for pions it is a reasonable approximation to use $P \approx E$. For heavier particles our derivation is not applicable.

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