HIGH ENERGY NUCLEUS-NUCLEUS COLLISIONS

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The data on the interactions of cosmic nuclei with emulsion are presented. The average energy of incoming nuclei is about 19 GeV/nucleon. The cross-sections for nucleus-nucleus collisions are determined and compared with Glauber theory. The average and the dispersion of multiplicity distribution observed in nucleus-nucleus collisions are presented. They are discussed in the framework of the incoherent production model in collision of two nuclei.

1. *Introduction*

In this paper we present data on the interactions of the heavy cosmic nuclei with the emulsion nuclei at the average primary energy of about 19 GeV/nucleon. The investigation of the nucleus-nucleus collisions is now very attractive. One expects that these interactions provide much more information about the elementary nucleon-nucleon process than in the case of hadron-nucleus collisions. In the collision of two nuclei the elementary interaction is more amplified than in the case of hadron interacting with nucleus. The data we show, although with low statistics and not monoenergetic primary nuclei beam, indicate how certain characteristics of hadron-nucleus interactions extend to nucleus-nucleus interactions.

The paper is organized as follows. In Section 2 we describe the experimental procedure and data. The values of the interaction mean free paths are determined in Section 3. In the same Section we compare the experimental values of the cross-sections for nucleus-nucleus collisions with the Glauber theory and with the geometrical cross-sections. In Section 4 the average multiplicities and dispersions are presented as well as their dependence on the mass and energy of the incoming nucleus. We discuss the consistency of the average multiplicities and dispersions with the values calculated on the basis of the model of incoherent particle production in nucleus-nucleus collisions [1]. The conclusions are given in Section 5.

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2. *Experimental material*

We present the data on the interactions of heavy cosmic nuclei with the emulsion nuclei. These interactions were found in the ICEF emulsion stack. The description of the emulsion stack and of the balloon flight is given in Ref. [2]. In the part of stack which was in Cracow laboratory a systematic scanning of the tracks of all heavy nuclei entering the stack and having the projected length of more than 3 mm per plate was performed. All these tracks were traced until they interacted or left the stack. The tracks of the secondary alpha particles were scanned also. A total of 557 interactions of nuclei with Z greater than 2 with emulsion nuclei were found. 140 interactions of alpha particles which were the fragments of the primary nuclei were also recorded. From the known energy spectrum of heavy nuclei in the cosmic radiation ($n(E) \sim E^{-2.6}$) [3] and the energy of the magnetic cut-off (7 GeV/nucleon), we get the average energy of incoming nuclei 19 GeV/nucleon. According to the evaporation theory [4] we assume the same average energy for α-particles.

In each event found we measured the number of relativistic particles N_s ($\beta \geqslant 0.7, Z = 1$), the number of heavily ionizing particles N_h ($\beta < 0.7, Z \geqslant 1$) and the number of fragments of primary nucleus with $Z \geqslant 2$, N_f. The charges of the primary nuclei (Z) as well as these of the heavy fragments (Z_f) were determined by the ionization

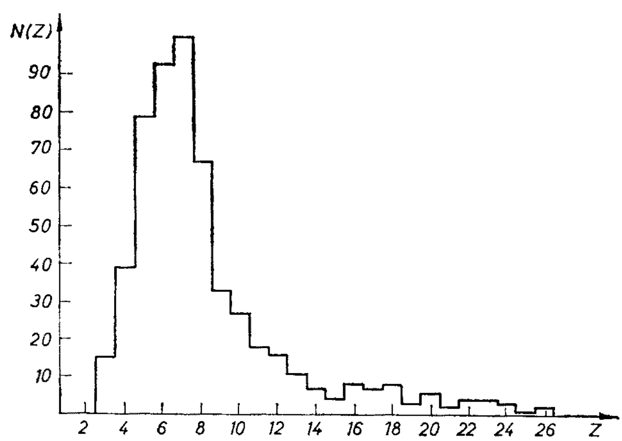

Fig. 1. Charge distribution of primary cosmic nuclei

measurements[1]. For α-particles and lithium nuclei the charges were estimated from the measurements of the gap density on the nucleus track. For heavier nuclei the method of counting knock-on electrons was applied (see Ref. [4], p. 587). The details of the charges measurements are given in Ref. [5]. In Fig. 1 we present the charge distribution of the primary nuclei.

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[1] These measurements were done in Łódź laboratory [5].

3. Cross-sections for the nucleus-nucleus interactions

The interaction mean free paths of heavy nuclei in emulsion were estimated by the maximum likelihood procedure of Bartlett [6]. This method can be used when the information about the flux of the incident particles is not available. It is just our case, since we have only data about these nuclei which interacted in emulsion stack and no information about the nuclei which left the stack without interaction. The mean free path λ is obtained by maximizing (over λ) the following function (called the maximum likelihood function):

$$L(\lambda; l_1 \dots l_n) = \prod_{i/1}^{n} \frac{e^{-l_i/\lambda}}{1-e^{-L_i/\lambda}} \cdot \frac{1}{\lambda}, \tag{1}$$

where n is the number of interactions, l_i is the path length of the i-th heavy primary from the point of entry to the collision point in the emulsion. L_i is the potential length of the path of this nucleus, i. e. the total length available in the stack for the i-th heavy primary in case of no interaction. The values l_i are measured experimentally and L_i are determined on the basis of the known geometry of the emulsion stack in which the scanning was performed. The maximizing procedure was done individually for five groups of incoming nuclei: $Z = 2$, $Z = 3$–5, $Z = 6$–9, $Z = 10$–19 and Z equal or greater than 20 [2]. The results are shown in Fig. 2. In this figure we also present the other available data for the same and lower energies of incident nuclei obtained by the direct method [7–9]. It can be seen that the agreement between all experimental results is very good. One also notices that there is no energy dependence of the mean free paths in the observed energy range (average energies 4.1 GeV/nucleon and 19 GeV/nucleon).

In order to reduce the experimental errors we averaged all results for a given charge group. The cross-sections are the inverses of the product of the average mean free path by the target density. The values of cross-sections for nucleus-emulsion collisions are listed in Table I and shown in Fig. 3 (points with error bars). These are the absorption cross-sections. In addition to the inelastic events (with particle production) they include also the quasielastic interactions, i. e. the interactions in which the nuclei were excited but no particle was produced. In Fig. 3 we also present the predictions of the Glauber theory [11]. The predicted numbers are calculated from the Glauber model for light nuclei (masses less than 8) and from its optical limit for heavier nuclei [12]. The calculated values do not include the quasielastic interactions so they underestimate the experimental ones. We averaged these values over the emulsion components and over the beam nuclei in order to get a direct correspondence with the experimental conditions.

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[2] In the case of α-particles apart from the Bartlett method we used also the direct method of determination of λ. It was possible because now we had full information about all the incoming α-particles. In this case λ is just the aritmethic average $\lambda = \sum_{i/1}^{N} l_i/n$, where N is the number of incident α-particles, n is the number of interactions and l_i is the path length of the i-th α-particle ($i = 1 \dots N$). Both methods give similar values of λ: Bartlett method: $\lambda = 22.8$ cm, direct method: $\lambda = 23.6^{+2.4}_{-2.2}$ cm.

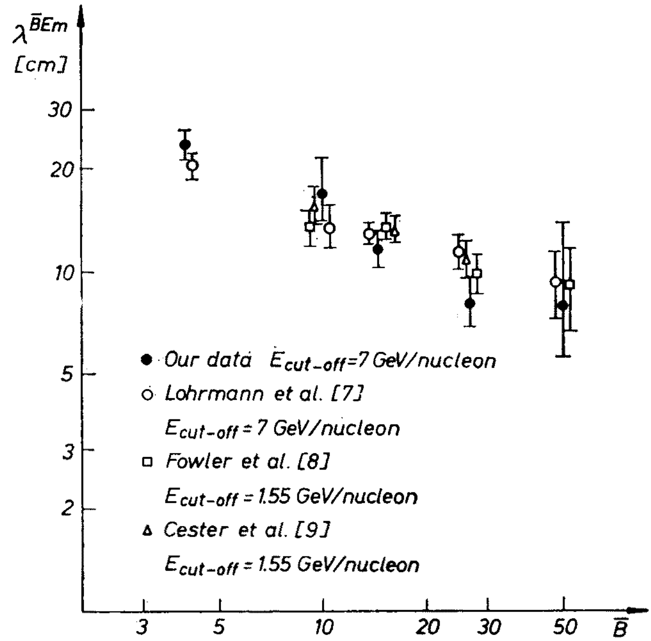

Fig. 2. Interaction mean free paths in function of $\bar{B}$

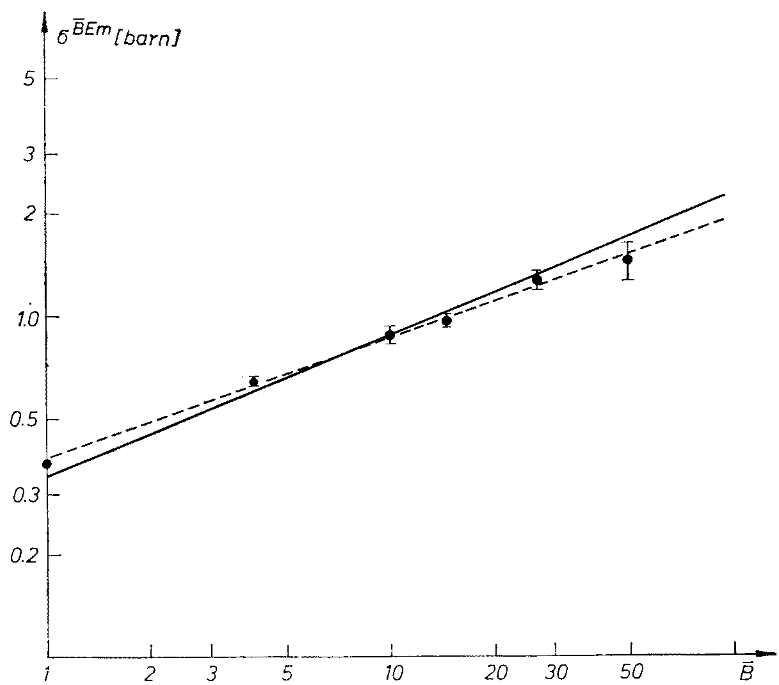

Fig. 3. Experimental values of absorption cross-sections in $\bar{B}$-emulsion collisions (for proton-emulsion data is taken from Denisov et al. [13]). ——— inelastic cross-sections calculated from Glauber model, – – – geometrical cross-sections

TABLE I

The values of the absorption cross-sections in emulsion combined from all data available. The cross-sections are given for five groups of incoming nuclei

$\bar{B}$	$\sigma^{\bar{B}Em}$ [barn]
1	0.371
4	0.639±0.017
10.0	0.875±0.053
14.4	0.965±0.021
26.6	1.280±0.067
49.0	1.445±0.184

From Fig. 3 it is seen that experimental cross-sections agree with these calculated from the Glauber theory. If we include in Glauber calculations the quasielastic cross--section, we will get small discrepancies especially for the heaviest nuclei. The Glauber cross-sections are larger than the experimental values. The reason of these discrepancies might be that the optical limit of the Glauber theory does not work well for the nuclei we are dealing with. To get a more reliable comparison we have to use the values calculated from the Glauber multiple scattering series, which calculation is not easy to do.

In Fig. 3 we also plot the geometrical cross-sections calculated from the formula:

$$\sigma_{BA}^{\mathrm{GEOM}} = \pi R_0^2 (A^{1/3}+B^{1/3})^2, \tag{2}$$

where $R_0 = 1.1$ fm and A, B are the masses of colliding nuclei. It may be seen that the geometrical cross-sections agree with the experimental numbers and do not differ much from the Glauber cross-sections. So the nuclei interact as black discs with radii R_A growing as $A^{1/3}$: $R_A = R_0 A^{1/3}$.

4. *Particle production in the nucleus-nucleus collisions*

In this Section we discuss the average multiplicities of particles produced and dispersions of multiplicity distributions in nucleus-nucleus collisions. As we mentioned in Section 2 for each interaction we measured the number of relativistic particles N_s, the number of heavily ionizing particles N_h and the number of the fast fragments of primary nucleus N_f. In nucleus-nucleus collision we may distinguish the three different processes in which particles are produced: production process, target fragmentation process and fragmentation of the incident nucleus. Unfortunately we cannot select experimentally particles produced in each of these processes. We are interested only in production process and we describe this process by the number of produced mesons. We may estimate this number with a good accuracy on the basis of experimentally measured quantities. We define N_m — the number of produced mesons by:

$$N_m = N_s - p, \tag{3}$$

where p is the number of protons into which the incoming nucleus fragments and is equal:

$$p = Z - \sum_{i/1}^{N_t} Z_t^i. \tag{4}$$

N_m defined in (3) does not contain the slow mesons (with β less than 0.7) which are included in the N_h tracks. On the other hand in N_m we have a contamination of recoil protons from the target nucleus which are relativistic ($\beta > 0.7$). If we do not use any model for the interactions between nuclei we cannot define the exact number of produced mesons. But taking into account the large multiplicities we are dealing with, the N_m defined experimentally in (3) is a good approximation for the meson multiplicity[3].

The presented sample of the cosmic nuclei and α-particles interactions with emulsion contains 607 events in which particles are created and also 90 quasielastic events ($N_m = 0$). In the following we consider only the events with N_m greater than zero. In order to avoid

TABLE II

Average multiplicities and dispersions in nucleus-emulsion collisions

Z	$\langle E \rangle \simeq 19$ GeV/nucleon				$\langle E \rangle = 4.1$ GeV/nucleon			
	No of events	$\bar{B}$	$\bar{N}_m$	D	No of events	$\bar{B}$	$\bar{N}_m$	D
2	114	4.0	5.84±0.57	6.05±0.40				
3—5	120	10.0	13.07±1.30	14.20±0.92	165	9.8	5.64±0.37	4.73±0.26
6—9	250	14.5	15.86±1.13	17.77±0.80	381	14.0	7.27±0.45	8.75±0.32
10—19	103	26.5	23.66±2.78	28.26±1.97	157	25.6	9.78±1.22	15.27±0.86
⩾20	21	47.5	31.43±7.62	34.94±5.39	38	49.3	10.84±3.14	19.38±2.22
⩾10	124	30.1	24.98±2.66	29.64±1.88	195	30.2	9.99±1.16	16.16±0.82

statistical fluctuations we divide the whole sample into five charge groups in the same way as in preceding Section. In Table II we give for each group the values of the average masses of incident nuclei ($\bar{B}$), average meson multiplicities ($\bar{N}_m$) and dispersions (D), for our data with $\langle E \rangle = 19$ GeV/nucleon. The same set of data for lower energy ($\langle E \rangle = 4.1$ GeV/nucleon) is also included in Table II. This data is taken from Refs [8–10]. These are the interactions of the cosmic nuclei found in the emulsion stack radiated by the cosmic rays in the Northern Italy. The only difference with our data is the lower average energy. In Fig. 4 we show the dependence of the average meson multiplicity on the mass of incident nuclei for these two energies.

[3] To check this we calculated corrections to our definition (including the slow mesons and rejecting the fast protons) on the basis of successive collision model [1,14–16] and using the data for nucleon-nucleon interactions [17]. Identical procedure was done for hadron-nucleus collisions [18]. Calculated corrections underestimate the values of N_m of about 7.5% and are less than our experimental errors.

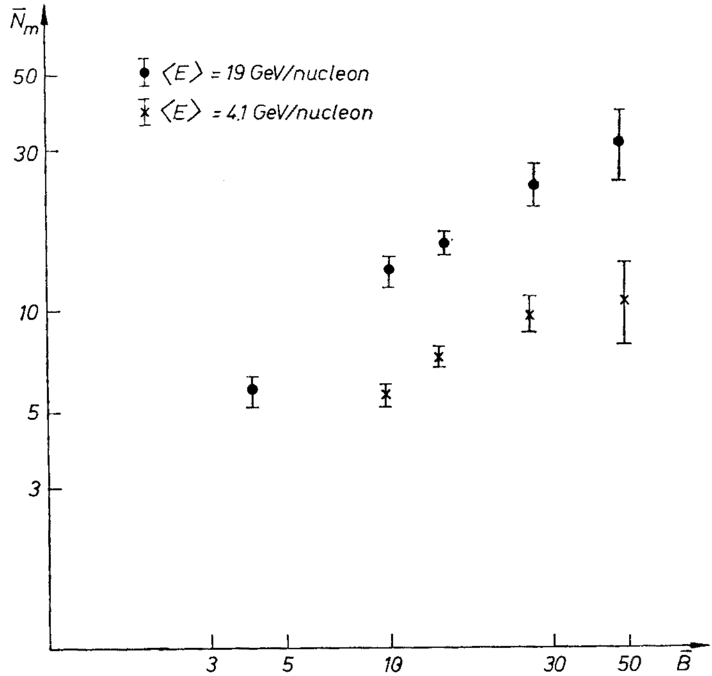

Fig. 4. Average number of produced mesons plotted versus $\bar{B}$

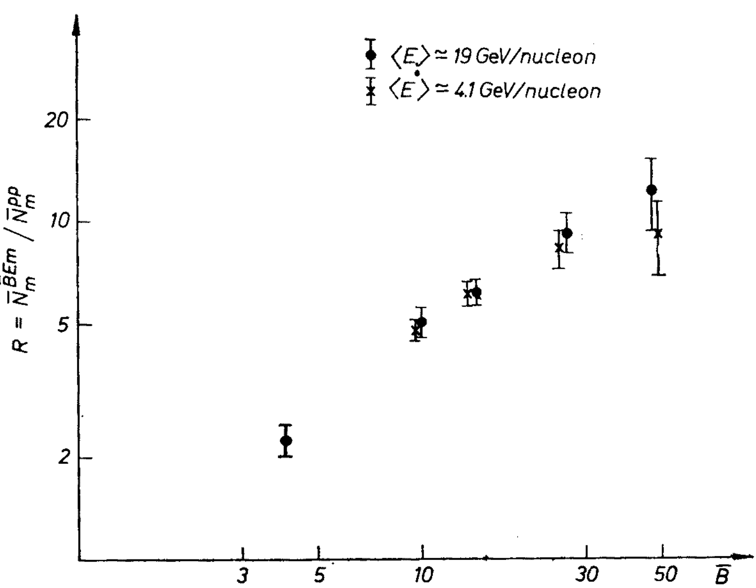

Fig. 5. Dependence of the R ratio on the masses of the primary nuclei

A very suitable parameter which describes the multiplication of the elementary nucleon-nucleon process in the collisions of composite objects is the ratio R:

$$R^{\bar{B}Em} = \frac{\overline{N}_{\rm m}^{\bar{B}Em}}{\overline{N}_{\rm m}^{\rm pp}} . \tag{5}$$

In Fig. 5 we present R as a function of $\overline{B}$ for 19 and 4.1 GeV/nucleon. One observes the interesting fact that in the limit of experimental errors R is independent of the primary energy for all incoming nuclei. This effect is well known for hadron-nucleus interactions [18–20].

Another parameter which gives us more information about the multiplicity distribution is its dispersion. There exist many data concerning the dependence of the dispersion

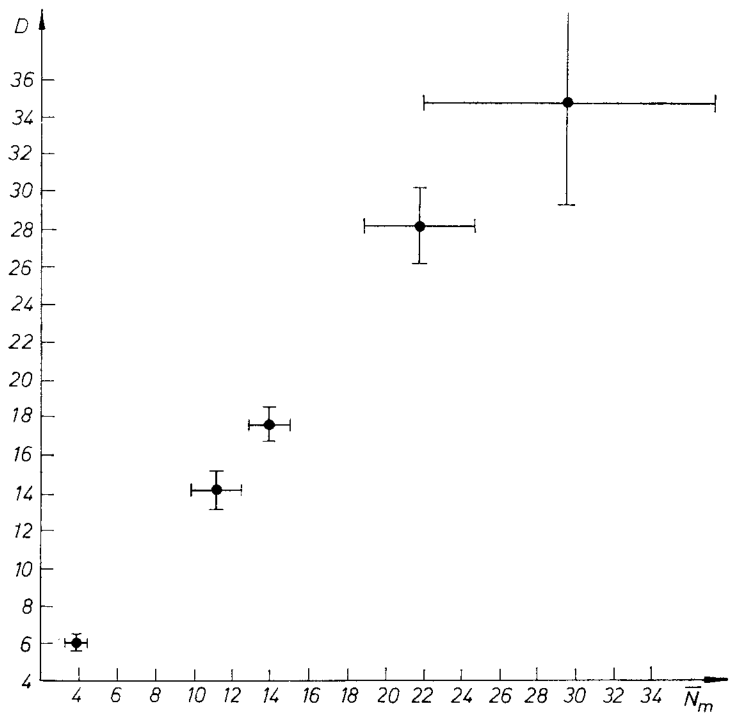

Fig. 6. D vs $\overline{N}_m$ for $\bar{B}$-emulsion collisions at average primary energy 19 GeV/nucleon

on the average multiplicity in hadron-nucleon [23] and hadron-nucleus collisions [20–22]. These data show the universality of the $D/\overline{N}$ ratio which does not depend neither on the energy of the incoming hadron nor on the target nucleus. From the data on the nucleus--nucleus interactions one sees that this universality is much wider. It includes also the independence of the $D/\overline{N}$ ratio of the mass of incoming nucleus. In Fig. 6 this universality is well seen for our data. The points lie on a straight line passing through the origin. For the lower energy data $D/\overline{N}$ ratio depends on $\overline{B}$. May be this energy is too low for this universality to set in.

Now we are going to compare the experimental values of the average multiplicity and the dispersion with the predictions of the incoherent production model [1]. The main assumption of this model is that the contributions of individual nucleons add incoherently. The average multiplicity is proportional to the number of "wounded nucleons" i. e. the nucleons which underwent at least one inelastic collision. On the basis of our data we want to test whether multiplicity is really proportional to the number of wounded nucleons $\bar{w}_{BA}$, each of them giving the same contribution to the final multiplicity:

$$\overline{N_{\rm m}^{BA}} = \bar{w}_{BA} \cdot (\tfrac{1}{2}\,\bar{n}_{\rm H}), \tag{6}$$

where $\bar{n}_{\rm H}$ is the average multiplicity in nucleon-nucleon collision and $\bar{w}_{BA}$ is the average number of wounded nucleons given by:

$$\bar{w}_{BA} = (A \cdot \sigma_B + B \cdot \sigma_A)/\sigma_{BA}.$$

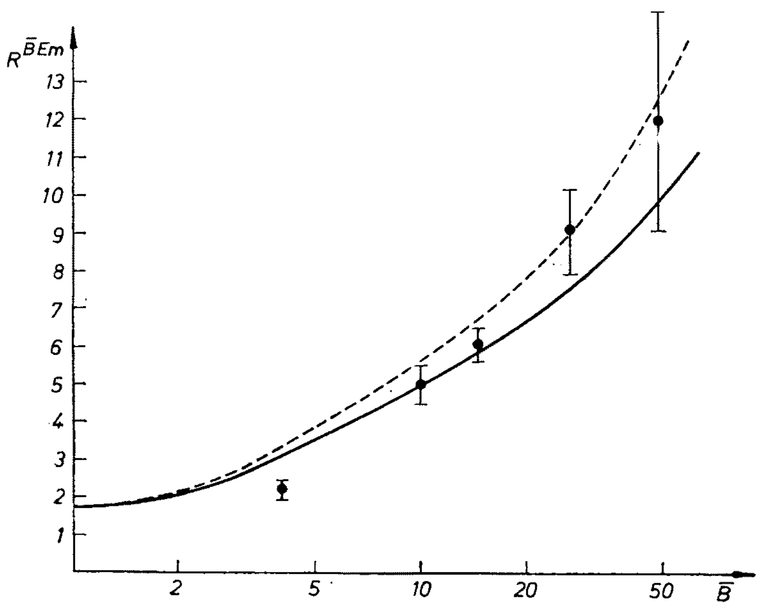

Fig. 7. $R^{\bar{B}Em}$ vs $\bar{B}$. Predictions of the incoherent production model: ——— average multiplicity proportional to the average number of wounded nucleons; – – – average multiplicity proportional to the average number of collisions

One may also expect the direct extrapolation of the formula which worked for the nucleon-nucleus collisions to hold, namely

$$\overline{N_{\rm m}^{BA}} = \tfrac{1}{2}\,(\bar{\nu}_{BA} + 1) \cdot \bar{n}_{\rm H}, \tag{7}$$

where $\bar{n}_{\rm H}$ is as before and $\bar{\nu}_{BA}$ is the average number of nucleon-nucleon collisions and is equal:

$$\bar{\nu}_{BA} = B \cdot A \cdot \sigma_{\rm pp}/\sigma_{BA}.$$

σ_{BA}, $\sigma_{\rm pp}$, σ_A are the inelastic cross-sections for nucleus-nucleus, nucleon-nucleon and nucleon-nucleus collisions respectively. For hadron-nucleus collisions equations (6) and

(7) are exactly the same, since $\bar{w}_{hA} = \bar{\nu}_{hA}+1$, but for nucleus-nucleus interactions there is no simple relation between $\bar{w}_{BA}$ and $\bar{\nu}_{BA}$.

In the work [1] the average multiplicities and dispersions were calculated for various colliding nuclei at a given energy. In order to compare these predictions with our data we performed averaging over the emulsion components, beam nuclei and energy spectrum. In Fig. 7 we plot the calculated values of R as a function of $\bar{B}$ for these two parametrizations ((6) and (7)) and compare them with experimental numbers. The predictions of

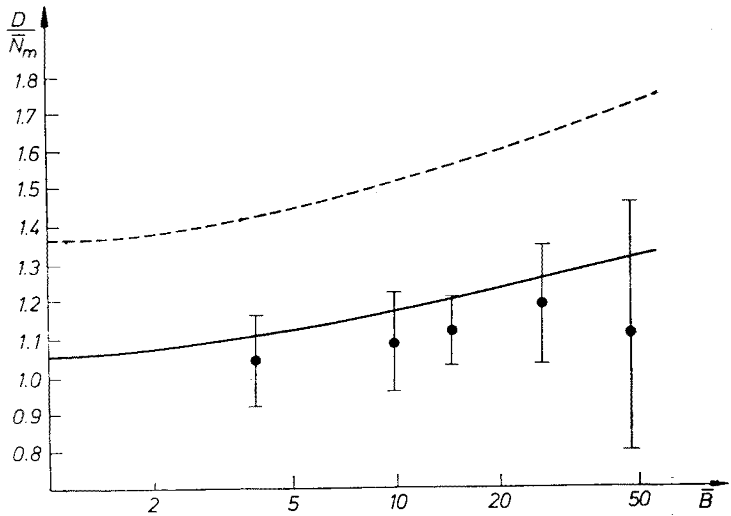

Fig. 8. Dependence of the $D/\bar{N}_m$ ratio on $\bar{B}$. ——— multiplicity proportional to w_{BA}; – – – multiplicity proportional to ν_{BA}

these two hypotheses are similar in the investigated $\bar{B}$ range, and so the experimental data do not favour any of them.

In contrast to the average multiplicities the predictions for dispersions are more model dependent. In Fig. 8 we compare our data on $D/\bar{N}$ ratio with those predicted by the model[4]. It is clearly seen that the multiplicity in nucleus-nucleus collisions is proportional to the average number of wounded nucleons rather than to the average number of nucleon-nucleon collisions.

5. *Conclusions*

The data on the cross-sections for nucleus-nucleus interaction and particle production in these collisions are presented. Our values of absorption cross-sections combined with other available data show a good agreement with those calculated from the Glauber

[4] We did not do the same comparison for the data with lower average energy. These data exhibit strong $\bar{B}$ dependence of $D/\bar{N}$ (see values listed in Table II). Both models predict weak dependence of $D/\bar{N}$ on $\bar{B}$, therefore the lower energy data do not agree with either of these predictions.

model and its optical limit. Small discrepances observed for the heaviest nuclei, may indicate that the optical approximation used in the calculations does not work well in this range of nuclear masses. The experimental cross-sections are also consistent with the values given by the simple geometrical formula, thus we may treat the colliding nuclei as black discs with radii growing with mass number A as $A^{1/3}$.

The average multiplicities of produced mesons as well as the dispersions of the multiplicity distributions in nucleus-nucleus collisions support the regularities observed in hadron-nucleus interactions. The ratio of the average number of mesons produced in the nucleus-nucleus collision to that produced in nucleon-nucleon interaction is independent of the primary energy (for energies 4.1 and 19 GeV/nucleon). Universality of $D/\overline{N}$ ratio well known for hadron-nucleus collisions is observed in nucleus-nucleus interactions also. This ratio, for our data, does not depend on the mass of the incoming nucleus.

The average multiplicities and dispersions are compared with the predictions of the model of incoherent particle production in nucleus-nucleus collisions. We observe a good agreement with this model and our data support the hypothesis that the relevant parameter which describes the particle production in collision of two nuclei is the number of wounded nucleons w_{BA}, not the number of collisions ν_{BA} of the nucleons in the target and beam nuclei.

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