QUARK MODEL AND VECTOR DOMINANCE RELATION IN HADRONIC QUASI-TWO-BODY REACTIONS*

By F. Wagner**

Lawrence Berkeley Laboratory, University of California, Berkeley

(Received August 26, 1977)

The Additive Quark Model relates Δ to vector meson production. The predictions of this model are formulated and compared to the data in a number of reactions involving vector mesons and Δ production. In all cases excellent agreement with data has been found. With the quark model we generalize the vector dominance relation proposed by Cho and Sakurai to the Δ case. Experiments agree well with the predictions. Empirically the predictions appear to be valid also in other reactions like A_1 production.

1. *Introduction*

In the additive quark model (hereafter AQM) of Białas and Zalewski [1], any hadronic reaction is described by a sum of all possible combinations of quark-quark or quark-antiquark interactions which are possible with the quark content of the external particles. In this paper we want to discuss the predictions of the AQM in the special case of reactions involving a Δ. Since the spin of the Δ is greater than $\frac{1}{2}$, the number of independent quark amplitudes is less than the number of independent helicity amplitudes for the original reaction. Therefore the helicity amplitudes have to satisfy certain constraints, which are usually called Class A relations [1]. These constraints imply that a pΔ vertex, as it occurs in the reaction

$$ap \to b\Delta^{++} \tag{1.1}$$

with arbitrary particles a and b, has the same spin structure as the reaction

$$a\pi \to bV \tag{1.2}$$

where V denotes a vector meson. If one identifies this πV vertex with real vertices $\pi\varrho$ or $\pi\omega$ depending on the internal quantum numbers, one gets relations between Δ production

---

* Work partially supported by the U. S. Atomic Energy Commission.

** Present address: Max-Planck-Institut für Physik und Astrophysik, Föhringer Ring 6, 8000 München 40, West Germany.

and V production amplitudes (so called Class B relations). These Class A and B predictions are discussed in Section 2 and compared with recent data.

The dynamical assumption that the basic quark-quark interaction conserves helicity leads to helicity conservation for the whole reaction (Class C predictions), However, it has been argued [2] that in a relativistic version of the quark model they cannot be valid. Empirically, these Class C relations have been found incompatible with experiment [3]. Nevertheless it is interesting to see whether reactions (1.1) satisfy more constraints than those given by Class A. One possible approach is that of Cho and Sakurai [4]. From the requirement that the ϱ production amplitudes in $\pi N \rightarrow \varrho N$ should extrapolate smoothly to the corresponding ones in photoproduction of π, they derived relations between the longitudinal and transverse ϱ production amplitudes. These relations we call vector meson dominance relations (VMDR). In Section 3 we generalize these ideas to the following special case of reaction (1.1)

$$\pi p \rightarrow V\Delta^{++} \tag{1.3}$$

under the assumption that the $\Delta^{++}p$ vertex satisfies the Class A constraints. In the derivation of these VMDR, only a smooth dependence of the production amplitudes on the vector meson mass is involved, but not the actual existence of a massless vector meson. Since the AQM relates the Δp vertex to a πV vertex, we can apply the ideas of Ref. [4] to the Δp vertex in reaction (1.3). Finally we compare the experimental predictions of the VMDR with the data on reactions of the type (1.3).

One important consequence of the AQM is the reduction in the number of independent amplitudes for reactions (1.1) or (1.3), which allows one to perform an amplitude analysis for these reactions even in the case of a bubble chamber experiment with limited statistics and no information about the polarization of the nucleons. Various possibilities are discussed in Section 4. The remainder of the paper is devoted to applications of the VMDR in other reactions than (1.3).

Cho et al. [4] compared the VMDR for the reaction $\pi^- p \rightarrow \varrho^0 n$ with the data before the accurate data of the CERN-Munich group [5] became available. The purpose of Section 5 is to show that the new data confirm these VMDR.

As the example of the Δ reaction (1.3) shows, VMDR are not confined to vector mesons. One can go even further and demand the same relations for the production amplitudes for axial vector meson states; for example, A_1 production in the reaction $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$. By A_1 we mean a s-wave $\pi\varrho$ state in the 3π system, but not necessarily a resonance. The experimentally observed spin structure of this reaction is sufficiently simple that the corresponding VMDR leads to helicity conservation for the A_1 along a direction between the s-channel helicity (SCH) and the t-channel helicity (TCH) direction (Section 6). If helicity conservation is true for this diffractively produced system, it should also be observed in Q or N* production. This we investigate in the case of reaction $pp \rightarrow (\pi^+ n)p$.

The notation for spins and momenta of the particles in the quasi two body reaction $ab \rightarrow cd$ will be the same throughout the paper (see Fig. 1). The four momenta of particle $a(b, c, d)$ are denoted by $p(p', k, k')$ and their masses by $m(m', M, M')$. If particle $b(c, d)$

carries spin, the helicity is denoted by $r_p(r, r')$. In many cases $c(d)$ will be a resonance decaying into two particles. The unit vector of the three momentum of the decay particle in the $c(d)$ rest frame is described by $\hat{q}(\hat{q}')$. If c is a $\omega \to 3\pi$, $\hat{q}$ denotes the normal to the 3π plane. In the decay $\Delta^{++} \to p\pi^+$ the proton carries helicity γ. The abbreviations

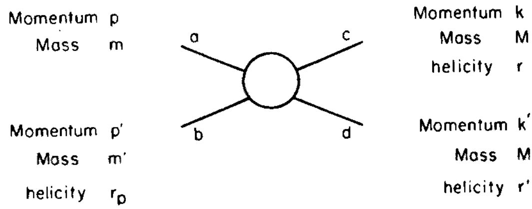

Fig. 1. Notation for quasi two body reaction

$P = p'+k'$ and $P' = k+p$ will be useful. Throughout the paper the SCH coordinate system will be used, unless stated otherwise. As usual s is the total energy squared and t the momentum transfer $t = (p-k')^2$. We will always assume that $|t/s| \ll 1$.

2. *Quark model predictions for Δ reactions*

In this section we want to sketch the derivation of Class A and B predictions of the AQM for reactions involving a pΔ vertex. The we consider the special case of associated vector meson-Δ productions. Finally we compare the Class A and B predictions with the data.

The most general two body reaction involving a Δ reads as

$$ap \to c\Delta^{++} \quad . \qquad (2.1)$$
$$\quad\quad \lfloor\!\to \pi^+ p$$

Particles a and c will be specified later on. The amplitude for the process (2.1), including the Δ decay, can be written as

$$F_{r_p\gamma} = \frac{1}{\pi} \sum_{\gamma'} H_{r_p r'} \cdot D^{3/2*}_{r'\gamma}(\hat{q}'), \qquad (2.2)$$

where $H_{r_p r'}$ denotes the amplitudes for the incoming proton with helicity r_p and the outgoing Δ with helicity r'; the D function describes the Δ decay into π^+p decay depending on the helicity γ of the decay proton and its momentum direction $\hat{q}$ in the Δ rest frame. For reasons which will be clear later on, we introduce the relative proton-Δ spin $l = 1, 2$ and its magnetic quantum number, m, is equal to the helicity flip

$$H_{r_p r'} = \sum_{l=1,2} \frac{2l+1}{4} \langle \tfrac{1}{2}\, r_p l m | \tfrac{3}{3}\, r' \rangle \cdot \tilde{H}_{lm}. \qquad (2.3)$$

For application, helicity amplitudes are not the best ones to use because of the constraints due to parity conservation. The following linear combinations of $\tilde{H}_{lm}$ are eigenamplitudes of the parity operator P times a rotation around the normal to the production plane with angle π:

$$T_i = \sum_m \varepsilon_i^R(m)\tilde{H}_{1m}, \tag{2.4}$$

$$U_i = \sum_{|m|\leqslant 1} \varepsilon_i^R(m)\tilde{H}_{2m}, \tag{2.5}$$

$$V_\pm = \sum_{|m|=2} \psi_\pm(m)\tilde{H}_{2m}, \tag{2.6}$$

where the coefficients ε and ψ are given by

$$\varepsilon_0(m) = \delta_{m,0}, \quad \varepsilon_+^{(m)} = -\frac{i}{\sqrt{2}}\delta_{|m|,1}, \quad \varepsilon_-^{(m)} = -\frac{m}{\sqrt{2}}, \tag{2.7}$$

$$\psi_+^{(m)} = -\frac{i}{\sqrt{2}}\delta_{|m|,2}, \quad \psi_-^{(m)} = -\frac{m}{2\sqrt{2}}. \tag{2.8}$$

The phases in Eq. (2.7) have been chosen such that the vector $\vec{T} = (T_-, T_+, T_0)$ transforms under rotations like an ordinary vector. Parity conservation implies that T_+, U_0, U_-, V_+ are related to natural and T_0, T_-, U_+, V_- to unnatural exchange. The cross section, including the Δ decay, for not observing any proton polarization is given by

$$W = \tfrac{1}{2}\sum_{r_p\gamma} |F_{r_p\gamma}|^2 \tag{2.9}$$

and the normalization factors in Eqs (2.2) and (2.3) are chosen such that the Δ-production cross section reads as

$$W_0 = \int d^2\hat{q}' W = \sum_i (|T_i|^2 + |U_i|^2 + |V_i|^2). \tag{2.10}$$

As explained in Ref. [6], the relative spin l is equivalent to the spin of the two interacting quarks in the p and Δ. This spin cannot be bigger than 1. Therefore the Class A prediction of AQM in this case is

$$\tilde{H}_{2m} = 0, \quad \text{or} \quad U_i = V_i = 0. \tag{2.11}$$

Prediction (2.11) is not limited to the quark model. Any interaction of basically vector type will involve only $\tilde{H}_{1m}$ as the dipole model of Ref. [7]. For pure natural exchange, the Stodolsky-Sakurai model [8] predicts only T_+ being nonzero. To see the experimental consequences from Eq. (2.11) we write the cross section (2.9) in terms of the quark model amplitudes T_i

$$W = \frac{1}{4\pi}\sum_{ik} T_i A_{ik}(\hat{q}')T_k^*, \tag{2.12}$$

where the matrix A is given by

$$A_{ik} = \tfrac{3}{2}\hat{q}_i'\hat{q}_k' + \tfrac{1}{2}\delta_{ik}. \tag{2.13}$$

The matrix A in the more complicated case of nonvanishing U amplitudes is given in the Appendix. For the remainder of the paper, we will always assume that the Δp coupling obeys the AQM. This assumption allows us to express the amplitudes in terms of measurable moments of the cross section W:

$$\mathrm{Re}\, T_i T_k^* = \int d^2\hat{q}'(5\hat{q}_i'\hat{q}_k' - \tfrac{4}{3}\,\delta_{ik})W. \tag{2.14}$$

In order to compare the AQM predictions with experiment, we now specify particles a to be a π and c to be a ϱ. By ϱ we mean a P-wave resonance state plus an S-wave contribution, as it occurs in the reaction

$$\pi^+ p \rightarrow \pi^+\pi^-\Delta^{++} \tag{2.15}$$

with the dipion mass M in the ϱ mass band. To obtain the amplitudes for reaction (2.15) we have to expand the general production amplitude T_i on the meson side into $\pi\pi S$ and P waves

$$T_i = \frac{1}{\sqrt{4\pi}} \sum_n \sqrt{3}\, P_{in}\hat{q}_n + S_i, \tag{2.16}$$

where P_{in} describes the P wave and S_i the S-wave emission of the π pair; $\hat{q}_n = (\hat{q}_-, \hat{q}_+, \hat{q}_0)$ corresponds to the unit vector of π^+ momentum in the $\pi\pi$ rest frame. P_{in} are related to the helicity amplitudes $H_{1m,r}$ of ϱ production in $\pi^+p \rightarrow \varrho^0\Delta^{++}$ in the same way as T_i are related to the $\tilde{H}_{1m}$ amplitudes for the Δ in Eq. (2.4):

$$P_{in} = \sum_{m,r} \varepsilon_i^{R*}(m)\varepsilon_n^R(r)\tilde{H}_{1m,r}. \tag{2.17}$$

The joint decay angular distribution for reaction (2.15) is obtained by

$$W(\hat{q}, \hat{q}') = \frac{1}{(4\pi)^2} \sum_{\substack{ik\\mn}} A_{ik}(\hat{q}')\,[3\hat{q}_m\hat{q}_n P_{in}P_{km}^* + S_iS_k^* + \sqrt{3}\, q_n(S_i^*P_{kn} + S_kP_{in}^*)]. \tag{2.18}$$

The matrix A_{ik} is given by (2.13). The following moments of this distribution allow the determination of the various interference terms:

$$W_{ik,mn} = \int d^2\hat{q}d^2\hat{q}'(5\hat{q}_i'\hat{q}_k' - \tfrac{4}{3}\,\delta_{ik})\,(\tfrac{5}{3}\,\hat{q}_m\hat{q}_n - \tfrac{1}{2}\,\delta_{mn})W, \tag{2.19}$$

$$W_{ik,m} = \int d^2\hat{q}d^2\hat{q}'\sqrt{3}\,\hat{q}_m(5\hat{q}_i\hat{q}_k' - \tfrac{4}{3}\,\delta_{ik}). \tag{2.20}$$

The explicit relation between those moments and the amplitudes reads as follows:

$$W_{ik,mn} = \tfrac{1}{2}\,\mathrm{Re}\,(P_{im}P_{kn}^* + P_{in}P_{km}^*) + \tfrac{1}{3}\,\delta_{m,n}\,\mathrm{Re}\, S_iS_k^*, \tag{2.21}$$

$$W_{ik,m} = \mathrm{Re}\,(S_iP_{km}^* + S_kP_{im}^*). \tag{2.22}$$

The simplicity of Eqs (2.21) and (2.22) compared to the usual complicated formalism [9] makes it very easy to extract the amplitudes from the measurable moments. The number of nonzero moments and amplitudes is restricted by parity. An even number of + indices has to occur in the amplitudes P_{mn}, S_m and the moments (2.21) and (2.22). Therefore, only one amplitude with natural exchange occurs, P_{++}, four P-wave unnatural exchange amplitudes $P_{00}, P_{0-}, P_{-0}, P_{--}$ and two S-wave amplitudes with unnatural exchange S_0 and S_-. In addition the moments have to be symmetric in both ik and mn indices. This leaves us with 20 independent moments of the type (2.21) and 10 S-P interference moments (2.22). This means 17 constraints among the moments, if AQM holds. There are two types of constraints: equality of moduli lead to linear, equality of phases lead to nonlinear constraints between the moments in Eqs (2.19), (2.20). The linear ones we can visualize in the following way: moments with natural exchange at the Δ vertex and unnatural exchange at the meson vertex must vanish by parity conservation. Therefore we get

$$W_{++,00}W_0 = W_{++,0-}W_0 = W_{++,--}W_0 = 0, \tag{2.23a}$$

$$W_{++,0}W_0 = W_{++,-}W_0 = 0. \tag{2.23b}$$

To get a common scale we have divided the moments by the total intensity W_0. The experimental moments in the ϱ region for reaction (2.15) are shown as a function of $\sqrt{-t}$

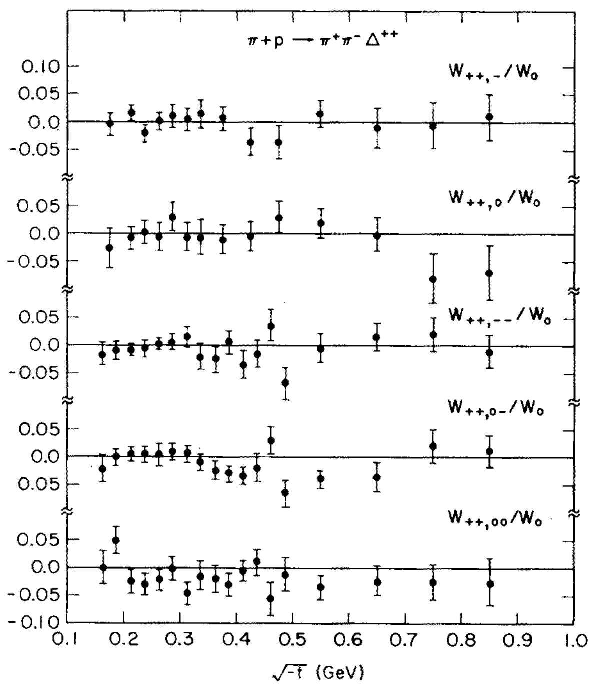

Fig. 2. Normalized moments from Eqs (2.23) for $\pi^+p \to \pi^+\pi^-\Delta^{++}$ at 7 GeV/c [10] in the ϱ region ($0.70 \leqslant M_{\pi\pi} \leqslant 0.86$ GeV) as a function of $\sqrt{-t}$ in the SCH system. Quark model Class A relations predict them to be zero

in Fig. 2 for the 7 GeV/c data of Ref. [10]. The agreement with the AQM predictions (2.23) is very good. The same investigation can be done for

$$K^+p \to K^+\pi^-\Delta^{++} \tag{2.24}$$

in the K^* region. The AQM prediction are again in good agreement with the data at 12 GeV/c [11] as Fig. 3 shows. We can replace the $\pi\pi$ state in reaction (2.15) by a 3π state in the ω region. Since there is little background [12, 13] under the ω, we are studying the reaction

$$\pi^+ p \to \omega \Delta^{++}. \tag{2.25}$$

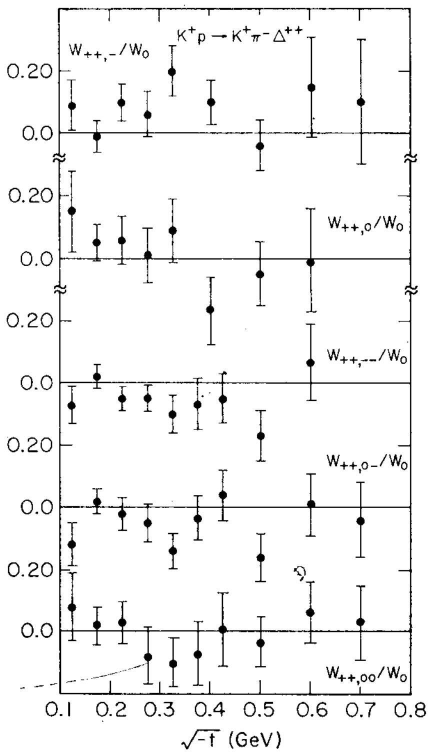

Fig. 3. Normalized moments from Eqs (2.23) for $K^+p \to K^+\pi^-\Delta^{++}$ at 12 GeV/c [11] in the K^{*0} region ($0.84 \leqslant M_{K\pi} \leqslant 0.94$ GeV) as function of $\sqrt{-t}$ in the SCH system. Quark model Class A relations predict them to be zero

Besides the constraints (2.23a), we can obtain additional constraints by projecting out natural exchange at the $\omega\pi$ vertex. The same argument as before leads to

$$W_{00,++}/W_0 = W_{0-,++}/W_0 = W_{--,++}/W_0 = 0. \tag{2.26}$$

Both the moments (2.23a) and (2.26) are shown in Fig. 4 for reaction (2.25) as a function of $\sqrt{-t}$. The data are taken from Ref. [12] at 7 GeV/c. The agreement with the AQM predictions is excellent.

These comparisons tell us that the quark model coupling for the Δp vertex are substantiated by the data for associated production of vector mesons and Δ. A meaningful test of the nonlinear constraints requires much higher statistics than available in present experiments.

Now we turn to the so called Class B predictions of the AQM. They identify the Δ in reaction (2.15) with real vector mesons. Therefore the vector meson Δ production amplitudes P_{in} should be symmetric

$$P_{in} = P_{ni}. \tag{2.27}$$

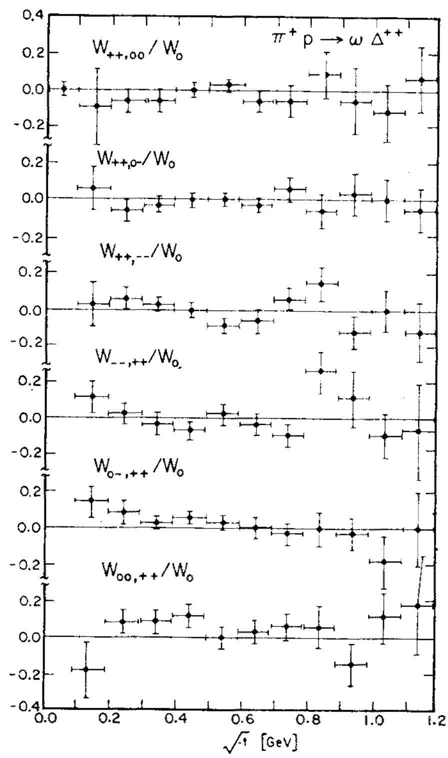

Fig. 4. Normalized moments from Eqs (2.23a) and (2.26) for $\pi^+p \to \omega\Delta^{++}$ at 7 GeV/c [12] as function of $\sqrt{-t}$ in the SCH system. Quark model Class A relations predict them to be zero

Due to the different masses of ϱ and Δ, (2.27) can hold only in one reference frame. According to Ref. [14] Eq. (2.27) should be used in the TCH frame, which has been verified experimentally at 3.9 GeV/c [3]. Due to this model dependence we want to make only a few tests of the Class B relations. Even if Eq. (2.27) holds, the single Δ and ϱ decay moments are not the same, as one can see from the different factors 5 and 5/2 in front of the quadratic terms in $\hat{q}$ and $\hat{q}'$ in Eq. (2.19). These terms give rise to ordinary $L = 2$ decay moments $\langle Y^L_M \rangle$. From the symmetry of P we get, therefore, a relation between the single Δ and ϱ decay moments

$$\langle Y^2_M \rangle_\varrho = 2\langle Y^2_m \rangle_\Delta. \tag{2.28}$$

These decay moments in the TCH system for the three reactions $\pi^+p \to \omega\Delta^{++}$, $\pi^+p \to \varrho^0\Delta^{++}$ at 7 GeV/c [10, 12], and $K^+p \to K^{+0}\Delta^{++}$ [11] at 12 GeV/c are shown in Figs 5 and 6 as

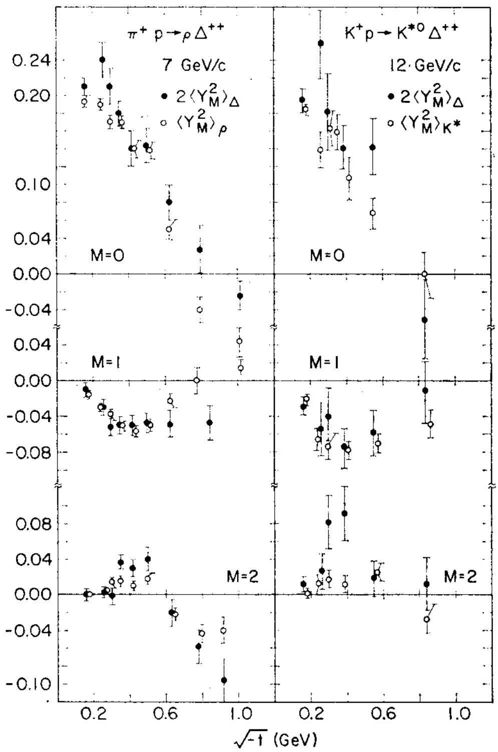

Fig. 5. Comparison of vector meson and Δ decay moments $\langle Y_M^2 \rangle$ in the TCH system as function of $\sqrt{-t}$ (ϱ-data from Ref. [10], K^*-data from Ref. [11]), Quark model Class B predicts $\langle Y_M^2 \rangle_V = 2\langle Y_M^2 \rangle_\Delta$

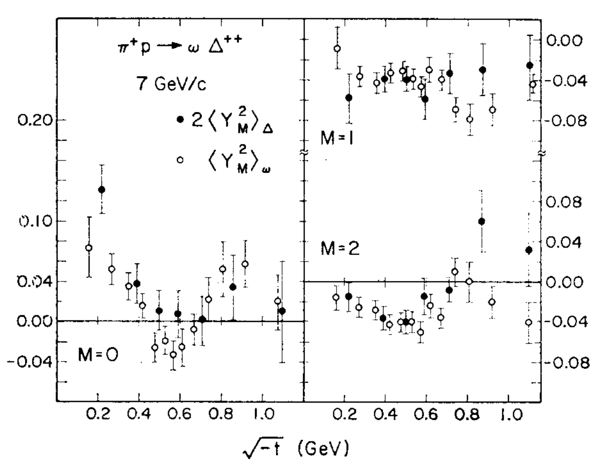

Fig. 6. Comparison of ω and Δ TCH decay moments $\langle Y_{2}^{M} \rangle$ in the reaction $\pi^+ p \to \omega\Delta^{++}$ at 7 GeV/c [12] as function $\sqrt{-t}$. Quark model Class B relations predict $\langle Y_2^M \rangle_\omega = 2\langle Y_2^M \rangle_\Delta$

a function of $\sqrt{-t}$. The data indicate that the Class **B** prediction (2.28) is in reasonable agreement with experiment. Equation (2.28) is not restricted to cases where ϱ and Δ^{++} are produced in the same reaction. Comparing the two reactions $\pi^- p \to \varrho^0 n$ and $pp \to \Delta^{++}n$, the same relation (2.28) should hold provided the production mechanism is the same. Both reactions are dominated by absorptive π exchange at low $|t|$ and natural exchange at high $|t|$ [5, 16]; therefore a common production mechanism seems to be a reasonable

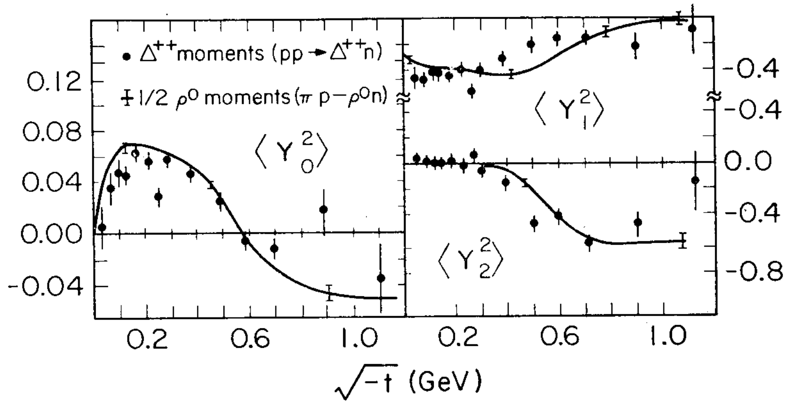

Fig. 7. Comparison of ϱ TCH decay moments in $\pi^- p \to \varrho^0 n$ and Δ^{++} decay TCH moments in $pp \to \Delta^{++}n$ at 17 GeV/c [5, 16] as function of $\sqrt{-t}$. The ϱ-moments are represented by the solid lines. Quark model Class B relations predict $\langle Y_M^2\rangle_\Delta = \frac{1}{2}\langle Y_M^2\rangle_\varrho$

assumption. Both reactions have been measured at 17 GeV/c by a CERN-MUNICH-UCLA collaboration [5, 16]. The comparison of the Δ moments in $pp \to \Delta^{++}n$ and ϱ moments in $\pi^+ p \to \varrho^0 n$ as function of $\sqrt{-t}$ (Fig. 7) indicates good agreement with the Class **B** prediction (2.28) also in this case. Recently Field [17] compared K^* production $K^+ n \to K^{*0} p$ at 6 GeV/c with $pp \to \Delta^{++}n$ data with a polarized beam and found the Class **B** relation in good agreement with the data.

3. *Vector dominance relation of Cho and Sakurai*

Cho and Sakurai [4] derived relations for the ϱ-production amplitudes for the reaction

$$\pi^- p \to \varrho^0 n \tag{3.1}$$

from the requirement that the ϱ couples like a photon to a conserved current. We will give a slightly different derivation for the simple case that the nucleons in reaction (3.1) are spinless particles of opposite parity and generalize these ideas to the more complicated case of

$$\pi p \to V\Delta^{++}, \tag{3.2}$$

where V stands for ω or ϱ. The original reaction (3.1) will be discussed in the next section. Since the AQM relates a $p\Delta$ vertex in reaction (3.2) to a $\pi\varrho$ vertex, we can apply this formalism also to the Δ. Finally we compare the prediction for reaction (3.2) with the data.

The amplitude F for reaction (3.1) with spinless nucleons including the decay $\varrho \to \pi\pi$ are given by

$$F = \frac{1}{\sqrt{4\pi}} \sum_n 3P_n \hat{q}_n, \tag{3.3}$$

where $\hat{q}$ denotes the decay π direction in the ϱ rest frame. Parity conservation requires $P_+ = 0$. The vector $\vec{P}$ is in the same way related to the helicity amplitudes as before in Eq. (2.4). Invariant amplitudes are introduced by

$$P_n = \sum_r \varepsilon_n^{*R}(r)\varepsilon_\mu(r) j^\mu = \chi_{n\mu} j^\mu. \tag{3.4}$$

The four vector $\varepsilon_\mu(r)$ is uniquely determined by $\varepsilon_\mu k^\mu = 0$ (k_μ is the ϱ momentum) and $\varepsilon_\mu = \varepsilon_m^R$ in the ϱ rest frame. By adding terms $\sim k_\mu$ to j^μ, we can always achieve the conservation law

$$k_\mu j^\mu = 0. \tag{3.5}$$

Using (3.5) the explicit relation between the amplitudes $\vec{P}$ and the "current" j^μ becomes very simple

$$\vec{P} = \vec{j} - \left(1 - \frac{M}{k_0}\right) \hat{k}(\hat{k} \cdot \vec{j}), \tag{3.6}$$

where M denotes the vector meson mass. In the rest frame Eq. (3.6) reduces to $\vec{P} = \vec{j}$. In a general reference frame it reflects the spin rotation. For massless vector mesons only transverse components of $\vec{j}$ can contribute to $\vec{P}$. Since j_μ is a four vector, it can be decomposed into scalar amplitudes A and B by:

$$j_\mu = 2(C_\mu A + K_\mu B), \tag{3.7}$$

with

$$C_\mu = p_\mu + \frac{t - m^2}{2s} P_\mu - \tfrac{1}{2} k_\mu, \tag{3.8}$$

and

$$K_\mu = \frac{M^2}{2s} P_\mu - \tfrac{1}{2} k_\mu. \tag{3.9}$$

These linear combinations of the particle momenta (see Fig. 1) are chosen such that Eq. (2.5) is satisfied up to $O(1/s)$ terms. Inserting j_μ into Eq. (3.6) and choosing the SCH-direction as z-axis, we find for P_0 and P_-

$$P_0 = M(A - B), \tag{3.10}$$

$$P_- = -2\sqrt{-t} \cdot A. \tag{3.11}$$

The crucial difference between the amplitudes A and B is that B enters with K_μ into j_μ, which is explicitly M^2 dependent. B does not give any contribution to production of a massless vector particle (γ). Presence of a significant B term would make any relation between ϱ and γ production meaningless. To impose a smooth transition between γ and ϱ production we require $B = 0$. This means that P_0 and P_- must satisfy the relation

$$P_0 = -\frac{M}{2\sqrt{-t}} P_-. \tag{3.12}$$

Due to the derivation, we call Eq. (3.12) and the analogous relations in other cases vector meson dominance relations (VMDR). Equation (3.12) has been derived in Ref. [4] by writing j_μ as a linear combination of the external momenta and imposing the conservation law (3.5) as an identity in M. This masks, however, the important omission of any M^2 dependent term $\sim K_\mu$ in j_μ. Note that Eq. (3.5) can always be satisfied by adding appropriate terms $\sim k_\mu$ to j_μ. An obvious consequence of the VMDR is the following. Choosing as quantization axis the vector $\vec{C}$ in the V rest frame, only P_0 can be nonzero. This means helicity conservation along $\vec{C}$. This direction lies between the TCH direction $\vec{p}$ and the SCH direction $\vec{P}$ forming an angle $\theta = \operatorname{arc\,tg}(2\sqrt{-t}/M)$ with $\vec{p}$. Therefore the Donohue-Högaasen angle [18] is predicted to equal θ. As we will see, this helicity conservation holds only in this simple case where we have pure unnatural exchange. It will be true in general if only one amplitude contributes.

Before we can apply the same idea to reaction (3.2), we have to find the analogue of the AQM coupling (2.4) for the invariant amplitudes for reaction (3.2). Ignoring for the moment the V-meson spin, the helicity amplitudes $H_{r_p\gamma'}$ for the Δp vertex are given in the Rarita-Schwinger representation [19]:

$$H_{r_p r'} = \bar{u}_\mu(k', r')J^\mu \bar{u}(p', r_p). \tag{3.13}$$

The AQM for the invariant amplitudes means that J_μ does not depend on any γ-matrices connecting the Δ spinor u_μ and the proton spinor u. To see this, we insert the explicit form of u_μ in terms of a direct product of spin $\frac{1}{2}$ and 1 representations

$$\bar{u}_\mu = \sum_{m,\alpha} \langle \tfrac{1}{2}\, \alpha 1 m | \tfrac{3}{2}\, \gamma' \rangle \varepsilon_\mu(m, k')\bar{u}(\alpha, k') \tag{3.14}$$

into Eq. (3.13). By taking J_μ outside the spinor product and comparing this expression with (2.3) we find

$$\tilde{H}_{1m} = \tfrac{4}{3}\sqrt{(m_p+M')^2-t}\, \varepsilon_\mu^*(m, k')J^\mu, \tag{3.15}$$

$$\tilde{H}_{2m} = 0. \tag{3.16}$$

Absorbing the kinematical factor in Eq. (3.15) into the definition of J_μ, we see that the relation of the AQM between the Δp amplitudes $H_{\gamma_p\gamma'}$ and an effective vector meson production amplitude $\tilde{H}_{1m}$ holds also for the invariant amplitudes. With the help of Eq. (3.15) we can now express the production amplitudes P_{mn} of Section 2 for reaction (3.2) in terms of invariant amplitudes

$$P_{mn} = \chi_{m\mu}^*(k')\chi_{n\nu}^*(k)J^{\mu\nu}. \tag{3.17}$$

The tensors χ are given by Eqs (3.4) and (3.6). The most general tensor $J^{\mu\nu}$ which satisfies the conservation laws $k'_\mu J^{\mu\nu} = k_\nu J^{\mu\nu} = 0$ on both vertices is obtained by

$$J^{\mu\nu} = \left(g^{\mu\nu} - \frac{2}{s}\, k'^\mu k^\nu\right) C_A + 4C'^\mu C^\nu A_U - \frac{2}{s}\, \varepsilon^{\mu\lambda_1\lambda_2\lambda_3}\varepsilon^{\nu\kappa_1\kappa_2\kappa_3}$$

$$k'_{\lambda_1} k_{\lambda_2} p'_{\lambda_3} k_{\kappa_1} k'_{\kappa_2} P_{\kappa_3} A_N + 4K'^\mu C^\nu B_1 + 4K'^\mu K^\nu B_2. \tag{3.18}$$

The vectors C_μ and K_μ are given by Eqs (3.8), (3.9) and C', K' are the analogous vectors for the Δ side:

$$C'_\mu = p'_\mu + \frac{t - m_p^2}{2s}\, P_\mu - \tfrac{1}{2}\, k'_\mu, \tag{3.19}$$

$$K'_\mu = \frac{M'^2}{2s}\, P'_\mu - \tfrac{1}{2}\, k_\mu. \tag{3.20}$$

The physical meaning of the various terms in Eq. (3.18) can be read off from the connections between the scalar amplitudes and the SCH amplitudes:

$$P_{++} = C_A - 4tA_N, \tag{3.21}$$

$$P_{--} = -C_A - 4tA_U \tag{3.22}$$

$$P_{0-} = -2M' \sqrt{-t}\,(A_U - B_1), \tag{3.23}$$

$$P_{-0} = -2M \sqrt{-t}\, A_U, \tag{3.24}$$

$$P_{00} = MM'(A_U - B_1 + B_2). \tag{3.25}$$

C_A contributes to both natural and unnatural exchange spin flip amplitudes and can be finite at $t = 0$. It is therefore dominated by Regge cut contributions. A_N appears only in P_{++} which means it describes A_2 exchange. A_U, B_1 and B_2 describe the various unnatural exchanges. The vector dominance arguments discussed before for the vector meson in reaction (3.2) require $B_2 = 0$ in order to forbid the M dependent term $\sim K_\mu$ in (3.18). This is equivalent to the VMDR for the helicity amplitudes

$$P_{0-} = -\frac{2\sqrt{-t}}{M}\, P_{00}. \tag{3.26}$$

These VMDR are a consequence of a smoothness property of $J^{\mu\nu}$ and j^μ for reaction (3.1) but do not necessarily require the existence of a massless vector meson. Therefore we can postulate the same smoothness property for the Δ as far as the M' dependence concerns. This leads to the requirement of $B_1 = 0$ and to the following VMDR for the Δp side:

$$P_{-0} = -\frac{2\sqrt{-t}}{M'}\, P_{00}. \tag{3.27}$$

The VMDR (3.26) and (3.27) do not mean helicity conservation along $\vec{C}$ or $\vec{C}'$, since C_A and A_2 will not be zero; however, (3.26) and (3.27) lead to constraints for the moments $W_{ik,mn}$ which can be tested experimentally. Before doing so we want to make a comment on the possible validity of the VMDR (3.27) in π exchange dominated reactions. The π exchange pole term gives the following expression for $J_{\mu\nu}$

$$J^{\pi}_{\mu\nu} = 4p'_{\mu}p_{\nu}\frac{G}{m_{\pi}^2 - t}. \tag{3.28}$$

Using Eqs (3.8) and (3.19) to express p and p′ in terms of our vectors C, C', K and K' we see that, due to $m'^2 \neq m_{\pi}^2$, B_2 must contain the π-pole and cannot be neglected. Therefore we expect both VMDR (3.26) and (3.27) to be valid only for the reaction

$$\pi^+ p \to \omega^0 \Delta^{++}, \tag{3.29}$$

where π-exchange is impossible. For the reaction

$$\pi^+ p \to \varrho^0 \Delta^{++} \tag{3.30}$$

only (3.26) holds and for $K^+p \to K^*\Delta^{++}$ none of the two relations will be valid, since in this case both B_1 and B_2 contain π-pole contributions.

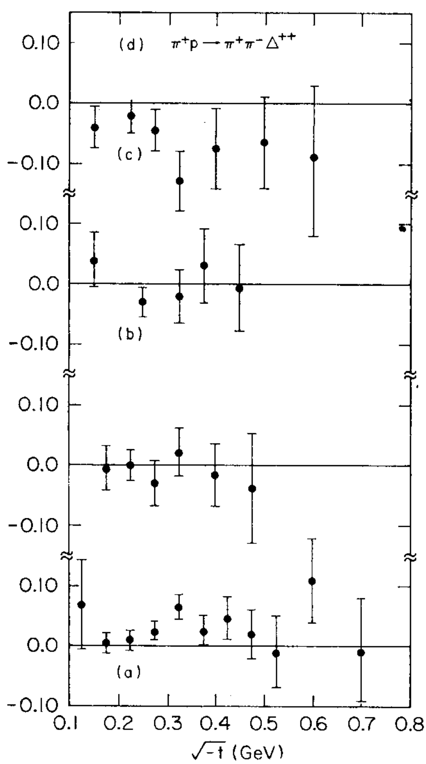

Fig. 8. Normalized moments from Eqs (3.31) and (3.32) for $\pi^+p \to \varrho^0\Delta^{++}$ at 7 GeV/c [10] as function of $\sqrt{-t}$. a), b), c) and d) correspond to Eqs (3.31a–c) and (3.32). The VMDR for ϱ predicts them to be zero

To test the VMDR experimentally we first investigate the ϱ VMDR (3.26). Inserting the relation (3.26) into the moments (2.18) leads as before in the test of Class A relations to linear and nonlinear constraints. We consider only the following three linear constraints among the normalized moments

$$[W_{0+,-+} + \alpha W_{0+,0+}]/W_0 = 0, \tag{3.31a}$$

$$[W_{00,0-} + \alpha(W_{00,00} - W_{00,++})]/W_0 = 0, \tag{3.31b}$$

$$[W_{00,--} + \alpha W_{00,0-} + W_{00,++}]/W_0 = 0, \tag{3.31c}$$

where $\alpha = 2\sqrt{-t}/M$. Note that for reaction (3.30), $W_{00,\,++}$ is not zero due to the S-wave background under the ϱ. The S–P interference moments have to satisfy the following constraint if Eq. (3.26) holds

$$[W_{00,-} + \alpha W_{00,0}]/W_0 = 0. \tag{3.32}$$

The experimental values for the four moments (3.31) and (3.32) for reaction (3.30) at 7 GeV/c [10] in the SCH system are shown as functions of $\sqrt{-t}$ in Fig. 8. This indicates that VMDR are in reasonable agreement with the data. For the ω reaction (3.29), the Eq. (3.31) holds with $W_{00,\,++} = 0$. Imposing in addition the VMDR (3.27) for the Δ side, we

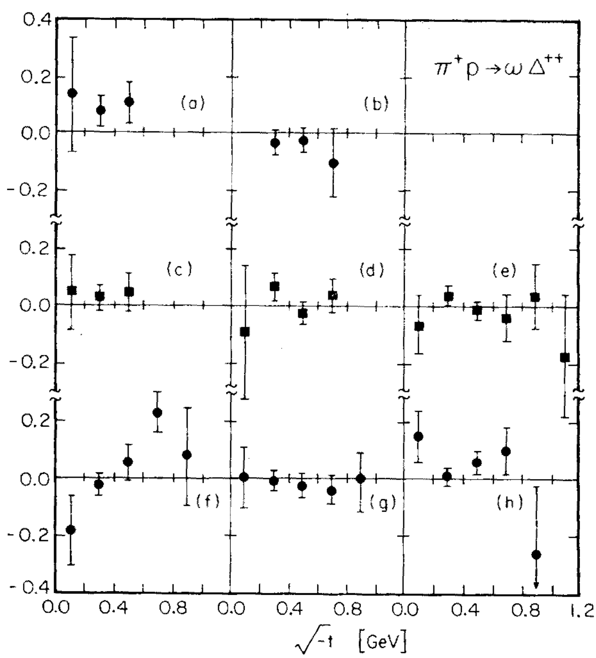

Fig. 9. Normalized moments from Eqs (3.31) and (3.33) for $\pi^+ p \to \omega\Delta^{++}$ at 7 GeV/c [12] as function of $\sqrt{-t}$. a) through h) correspond to Eqs (3.33b), (3.31c), (3.33d), (3.33e), (3.31b), (3.33c), (3.31a), and (3.33a). VMDR for ω and Δ predict them to be zero

find five more linear constraints among the moments

$$[W_{0-,00}+\alpha' W_{00,00}]/W_0 = 0, \quad (3.33a)$$

$$[W_{--,00}+\alpha' W_{0-,00}]/W_0 = 0, \quad (3.33b)$$

$$[W_{-+,0+}+\alpha' W_{0+,0+}]/W_0 = 0, \quad (3.33c)$$

$$[W_{0-,--}-\alpha' W_{00,--}+2\alpha W_{0-,0-}]/W_0 = 0, \quad (3.33d)$$

$$[W_{--,0-}-\alpha W_{--,00}+2\alpha' W_{0-,0-}]/W_0 = 0, \quad (3.33e)$$

where $\alpha' = 2\sqrt{-t}/M'$. In Fig. 9 we show the linear combinations of the moments as in Eqs (3.31) and (3.32) as functions of $\sqrt{-t}$ for $\pi^+p \to \omega\Delta^{++}$ at 7 GeV/c [10] in the SCH coordinate system. From the reasonable agreement we conclude that the VMDR holds also for the Δ in reaction (3.29). From the 12 amplitudes allowed by parity conservation AQM predicts only five nonzero. The additional VMDR reduce this number to three.

TABLE I

Number of amplitudes and constraints

Meson state present	Assumptions	P-wave amplitudes	S-wave amplitudes	Measure-ments	Linear constraints	Nonlinear constraints
P	—	12	—	20	—	—
$S+P$	—	12	4	30	—	—
P	$V_i = 0$	9	—	20	—	3
$S+P$	$V_i = 0$	9	2	30	—	9
P	$U_i = V_i = 0$	5	—	20	6	5
$S+P$	$U_i = V_i = 0$	5	2	30	5	12
P	$U_i = V_i = 0$ + VMDR(3.26) (3.27)	3	—	20	14	1
$S+P$	$U = V = 0$ + VMDR(3.26)	4	2	30	9	10

In Table I we list the number of independent amplitudes in the case of reactions (3.29) and (3.30) together with the number of measurable moments, linear and non-linear constraints, if one imposes the AQM and/or the VMDR.

4. $\pi^-p \to \varrho^0 n$ *revisited*

The VMDR have been derived originally in Ref. [4] for the reaction

$$\pi^-p \to \varrho^0 n. \quad (4.1)$$

We want to repeat the treatment here for two reasons. First, there are more amplitude analyses done for reaction (4.1) which clearly confirms the VMDR, and, second, there is an important difference between our treatment and that of Ref. [4]. Writing the helicity amplitudes for reaction (4.1) in terms of invariant amplitudes, we can omit all combinations leading to spin no flip at the pn vertex for unnatural exchange [21]. With this restriction the helicity amplitudes $H_{r_p r', r}$ in the SCH system can be decomposed as

$$H_{r_p r', r} = G_{\mathrm{nnp}} \bar{u}(r_p) \gamma_5 J^\mu u(r') \varepsilon_\mu^*(r) \tag{4.2}$$

with

$$J_\mu = 2C_\mu A_\pi + 2K_\mu B + \sigma_{\mu\nu} k^\nu C_A + \frac{i}{\sqrt{2}} \varepsilon_{\mu\lambda_1\lambda_2\lambda_3} k^{\lambda_1} p^{\lambda_2} (\gamma^{\lambda_3} A_\mathrm{N}^\mathrm{f} + (m\gamma^{\lambda_3} - p^{\lambda_3}) A_\mathrm{N}^\mathrm{nf}), \tag{4.3}$$

where A_π and B are related to π-exchange, C_A corresponds to the π-cut (the poor man's absorption model [21] sets $C_A = 1$) and $A_\mathrm{N}^\mathrm{f}(A_\mathrm{N}^\mathrm{nf})$ to the helicity flip (no flip) natural exchange contribution from A_2 exchange. We use the same vectors C, K as defined in Eq. (3.8) and (3.9) to ensure the validity of the conservation law $k_\mu J^\mu = 0$. In principle the Dirac equation for $\bar{u}$ and u can be used to express the A_N^f in terms of A_π, B and C_A as done in Ref. [2], but this will lead to a VMDR which is incompatible with experiment.

The restrictions imposed by the arguments of Cho and Sakurai [2] lead to $B = 0$. Since we use a linearly dependent set of scalar amplitudes, this has no consequence for the helicity amplitudes. Nevertheless we can make the following qualitative comparison. At $|t| < 0.1$ GeV^2 A_2 exchange can be neglected compared to π-exchange. With $B = 0$ one obtains from (4.2) and (4.3) for the $r = 0$ helicity amplitude

$$H_{r_p r', 0} = -\sqrt{-t} \frac{M}{\sqrt{M^2 - 4t}} A_\pi \delta_{r_p, -r'}. \tag{4.4}$$

The usual π-pole exchange amplitude is given by

$$H_{r_p r', 0} = -\sqrt{-t}\, \delta_{r_p, -r'} \cos \chi_{st} \frac{1}{m_\pi^2 - t} F(t), \tag{4.5}$$

where χ_{st} is the crossing angle between SCH and t-channel helicity (TCH) system with

$$\cos \chi_{st} = \frac{M + \dfrac{t - m_\pi^2}{M}}{\sqrt{\left(M - \dfrac{m_\pi^2 - t}{M}\right)^2 - 4t}} \tag{4.6}$$

and $F(t)$ a form factor. Using the form (4.4) for π-exchange is equivalent to replacing the numerator in the crossing angle (4.6) by its value at the π-pole (the additional weak t-dependence in the denominator can be absorbed into the form factor $F(t)$). This approximation has been made by *all* amplitude analyses [22] done on the CERN-Munich data [5], mainly because there is no experimental evidence for a zero at $|t| = M^2 - m_\pi^2$ in the $r = 0$ amplitude. Another consequence of using Eq. (4.3) for π exchange is that $B = 0$. This and $C_A \sim 1$ leads to a vanishing $\langle Y_2^2 \rangle$ in the TCH system, which is in good agreement with the data (see Fig. 7). All these comparisons have to remain qualitative since they depend on neglecting the A_2 exchange amplitudes, which are dominant for $|t| \sim 0.5$ GeV^2. This was not known at the time, when Cho and Sakurai derived the VMDR for reaction (4.1). This relation is obtained by requiring $B = A_N^f = 0$. Then the helicity amplitudes become linearly dependent, which gives the VMDR of Ref. [4]:

$$H_{\frac{1}{2}-\frac{1}{2},0} = -\frac{M}{\sqrt{-2t}} H_{\frac{1}{2}-\frac{1}{2},-1}. \tag{4.7}$$

However, equality of $r = 0$ and $r = 1$ amplitudes is incompatible with the known dominance of natural exchange for $|t| > 0.3$ GeV^2. Even if the original VMDR of Ref. [4] turns out to be incorrect, its weaker form $B = 0$ explains why in cos χ_{st} for the π exchange in Eq. (4.5) the factor $M^2+t-m_\pi^2$ has to be replaced by its value at the pole $t = m_\pi^2$. Finally we mention that $B = 0$ is required by the so called electric Born term model [4] (which is a special solution of the VMDR (4.7)) and by some dual models [23].

5. *Application to amplitude analyses*

As Table I shows, an amplitude analysis for a reaction like $\pi^+p \to V\Delta^{++}$ cannot be done without polarized target experiments. The least restrictive assumption for such an analysis is the neglection of the double flip amplitudes V_i (in the notation of Section 2) at the Δ vertex, which leads to a constraint fit. Usual statistics of bubble chamber experiments still require more assumptions. In this case the AQM assumption $U = V = 0$ may be used. In the analysis of reaction $\pi^+p \to \omega\Delta^{++}$ at 3.9 GeV/c [24] some nonquark model amplitudes have been claimed, which contradicts our finding. In Ref. [27] AQM has been used to determine the cross section for the S-wave under the ϱ^0 and K^* in reactions $\pi^+p \to \varrho^0\Delta^{++}$ and $K^+p \to K^{*0}\Delta^{++}$. This cross section turned out to be in reasonable agreement with the most recent $\pi\pi$ phase shift analysis [16]. Together with VMDR for the ϱ the phase between P_{00} and P_{++} has been determined [25].

For an amplitude analysis of the reaction

$$\pi^+p \to \pi^+\pi^-\pi^0\Delta^{++} \tag{5.1}$$

both in the ω region and for high 3π masses using $V_i = 0$ we refer to future publications [13, 27]. An application to inclusive reactions as

$$\pi^+p \to X\Delta^{++} \tag{5.2}$$

seems to me especially worthwhile to pursue. Extrapolating the cross section for reaction (5.2) to $t = m_\pi^2$ gives a measurement of the $\pi\pi$ total cross section. However, in general the background makes this rather unreliable. The background can be (hopefully) reduced under the assumption of the AQM, if one extrapolates the Δ moments (2.14) in the TCH system:

$$|T_{\substack{0\\+}}|^2 = \sum_X \int d^2\hat{q}'(5\hat{q}'^2_{\substack{0\\+}}-\tfrac{4}{3})\sigma(\pi^+p \to X\Delta^{++}). \tag{5.3}$$

$(m_\pi^2-t)^2\,|T_0|^2$ at the pole is related to the $\pi\pi$ cross section and $(m_\pi^2-t)^2\,|T_+|^2$ should extrapolate to zero which serves as a check of the method.

6. *Vector meson dominance relations and helicity conservation*

In the simple case of the reaction studied in Section 3, we established helicity conservation along $\vec{c}$ for the vector meson, if the VMDR is valid. Presence of other exchange mechanisms prevent this from being true as a general rule. The following reaction is experimentally known to be particularly simple

$$\pi^{\pm}p \rightarrow A_1^{\pm}p. \tag{6.1}$$

By $A_1^{\pm}$ we mean a 1^+ S-wave state between $\varrho^0\pi^{\pm}$, but not necessarily a resonance. The data show [28, 29] that this state is produced only by natural exchange and by spin coherent nucleon amplitudes. So the protons can be considered as scalar particles, and the formalism of the example in Section 3 can be applied (reversing the role of unnatural and natural exchange). If the arguments for the current j_μ can be also generalized to the axial current involved in reaction (6.1), we predict helicity conservation along direction $\vec{c}$. That means the experimental values of the Donahue-Høgaasen angle [18] should coincide with the angle θ between TCH direction $\vec{p}$ and $\vec{c}$ given by

$$\theta = \operatorname{arctg}\left[\frac{2\sqrt{-t}}{M_{3\pi}}\;\frac{m_\pi^2-t}{M_{3\pi}^2-3t-m_\pi^2}\right]. \tag{6.2}$$

The measured angles at 40 GeV/c from Ref. [28] are displayed in Fig. 10 for two intervals of the 3π mass $M_{3\pi}$. In view of the many assumptions entering the experimental analysis we consider the agreement as surprisingly good. In contrast to the reactions studied in the preceding sections, reaction (6.1) is dominated by Pomeron exchange. If this helicity conservation is not accidental, the same should be true for other Pomeron exchange reactions, for example pp → (π^+n)p. Due to the limited statistics available and

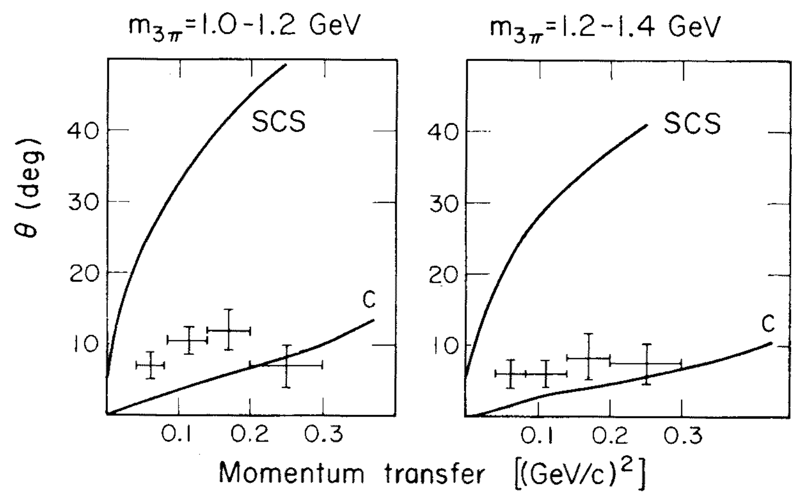

Fig. 10. Donohue-Høgaasen angle relative to the TCH system for A_1 production in $\pi^-p \rightarrow \pi^-\pi^+\pi^-p$ at 40 GeV/c [28] as function of $-t$ in two 3π mass intervals. The data represent the result of a partial wave analysis and the curve the prediction of VMDR for axial vector mesons

the increased spin complication, no amplitude analysis has yet been performed. However, helicity conservation along a certain axis should result in a flat distribution in the corresponding ψ angle. This ψ distribution for the data on $pp \to \pi^+np$ at 24 GeV/c [30] integrated over $M_{\pi^+n} \leqslant 1.7$ GeV and $|t_{pp}| \leqslant 0.3$ GeV^2 is shown in Fig. 11 for the three

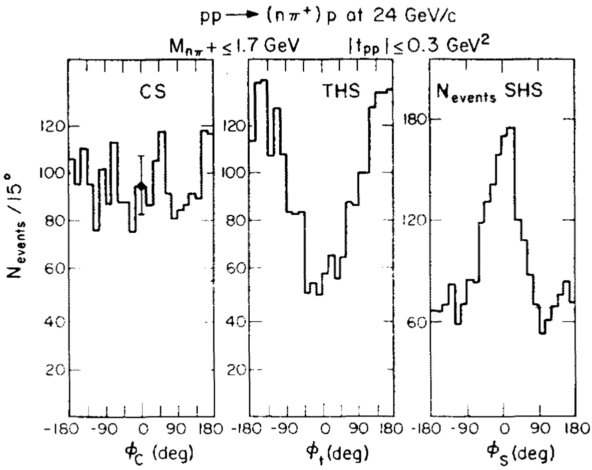

Fig. 11. Azimuthal angle ψ distribution for $pp \to (\pi^+n)p$ in a coordinate system using $\vec{c}$ as defined by Eq. (3.8) as z axis, the TCH system and the SCH system. A ψ distribution should be isotropic if the helicity is conserved

coordinate systems TCH, SCH and using $\vec{c}$ as axis. The data neither allow SCH or TCH helicity conservation, but the $\vec{c}$ direction is compatible with helicity conservation.

Finally we mention that the nonresonant 3π background in the reaction $\pi^+p \to \pi^+\pi^-\pi^0\Delta^{++}$ at 7 GeV/c [27] follows nicely the rule of helicity conservation in that coordinate system.

7. *Summary and conclusions*

We investigated the consequences of the AQM relating a $p\Delta$ vertex to a πV vertex. The restrictions imposed on the Δ couplings led to constraints for the joint decay moments in associated vector meson Δ production. These have been found in excellent agreement with the data on $\pi^+p \to \omega\Delta^{++}$, $\varrho^0\Delta^{++}$ and $K^+p \to K^{*0}\Delta^{++}$. The assumption of a quark type coupling turns out to be a very useful tool for amplitude analyses of Δ reactions.

Cho and Sakurai proposed a smoothness relation for the ϱ production amplitudes in $\pi^-p \to \varrho^0n$ from vector dominance. Since the AQM treats the Δ as a spin 1 particle, this idea can also be applied to the Δ. We found the VMDR in good agreement with the data on $\pi p \to \omega\Delta$ for both ω and Δ, and on $\pi p \to \varrho\Delta$ for the ϱ. In cases with one amplitude dominating the process, VMDR lead to helicity conservation along a direction $\vec{c}$ between the SCH and TCH direction. Surprisingly enough we found this helicity conservation to be true also in diffractive processes. While the theoretical justification seems to be rather poor, the experimental success in various cases makes us believe this distinction of the $\vec{c}$

vector may be a general rule. Apart from theoretical implication, such a rule may be helpful in future amplitude analyses of $\pi p \to \pi(p\pi\pi)$ or $pp \to p(p\pi\pi)$. The LBL analysis on $\pi^+ p \to (3\pi)^0 \Delta^{++}$ [27] demonstrates the advantage of using $\vec{c}$ as quantization axis.

We are mostly indebted to the members of Group A at LBL for the permission to use the DST's on their 7 GeV/c π^+p and 12 GeV/c K^+p experiments, and to the members of the Bonn–Hamburg–München collaboration, especially Dr W. Schrankel, for providing us with Fig. 11. We thank A. Białas, L. Van Hove, W. Ochs and M. Tabak for helpful suggestions and discussions.

APPENDIX

An observed asymmetry for the Δ may indicate the need for $\frac{1}{2}^-$ S-wave background. To include this we add to Eq. (2.2) the corresponding $j = \frac{1}{2}$ term

$$F_{r_p\gamma} = \sum_{\substack{j=\frac{1}{2},\frac{3}{2} \\ r'}} \frac{2j+1}{4\pi} \cdot H^j_{r_p r'} D^{j*}_{r' r}(\hat{q}'). \tag{A.1}$$

$H^{3/2}_{r_p r}$ is identical to that Eq. (2.3) and $H^{1/2}_{r_p r}$ given by

$$H^{\frac{1}{2}}_{r'_p r} = \delta_{r'_p r} \cdot S. \tag{A.2}$$

Ignoring the double flip amplitudes V we can order the amplitudes into a vector $X_l = (T_+, T_0, T_-, S, U_+, U_0, U_-)$. The integrated cross section is simply given by

$$W_0 = \sum_l |X_l|^2$$

and the decay angular distribution as

$$W = \frac{1}{4\pi} \sum_{l,\bar{l}} X_l A_{l\bar{l}}(\hat{q}') X^*_{\bar{l}} \tag{A.3}$$

with the symmetric matrix A:

$$A_{l\bar{l}}(q) = \left| \begin{array}{lllllll}
\frac{3}{2}\hat{q}^2_+ + \frac{1}{2} & & & & & & \\
\frac{3}{2}\hat{q}_+ q_0 & \frac{3}{2} q^2_0 + \frac{1}{2} & & & & & \\
\frac{3}{2}\hat{q}_+\hat{q}_- & \frac{3}{2}\hat{q}_0\hat{q}_- & \frac{3}{2} q^2_- + \frac{1}{2} & & & & \\
\sqrt{2}\,\hat{q}_+ & \sqrt{2}\,\hat{q}_0 & \sqrt{2}\,\hat{q}_- & 1 & & & \\
\frac{\sqrt{3}}{2}\hat{q}_+\hat{q}_- & -\frac{\sqrt{3}}{2}\hat{q}_0\hat{q}_- & -\frac{\sqrt{3}}{2}(q^2_+ - \hat{q}^2_0) & 0 & \frac{3}{2}(1-\hat{q}^2_-) & & \\
-\frac{3}{2}\hat{q}_0\hat{q}_- & 0 & \frac{3}{2}\hat{q}_0\hat{q}_+ & 0 & -\frac{\sqrt{3}}{2}\hat{q}_0\hat{q}_+ & \frac{3}{2}\hat{q}^2_0 + \frac{1}{2} & \\
-\frac{\sqrt{3}}{2}(\hat{q}^2_- - \hat{p}^2_0) & -\frac{\sqrt{3}}{2}\hat{q}_0\hat{q}_+ & \frac{\sqrt{3}}{2}\hat{q}_+\hat{q}_- & 0 & -\frac{3}{2}\hat{q}_+\hat{q}_- & \frac{\sqrt{3}}{2}\hat{q}_-\hat{q}_0 & \frac{3}{2}(1-\hat{q}^2_+)
\end{array} \right|$$

REFERENCES

[1] A. Białas, K. Zalewski, *Nucl. Phys.* **B6**, 449, 465 (1968).
[2] H. J. Lipkin, *Nucl. Phys.* **B20**, 652 (1970).
[3] G. S. Abrams, K. W. J. Barnham, J. J. Bisognano, *Phys. Rev.* **D8**, 1435 (1973).
[4] C. F. Cho, J. J. Sakurai, *Phys. Lett.* **30B**, 119 (1969); C. F. Cho, *Phys. Rev.* **D4**, 194 (1971).
[5] G. Grayer, B. Hyams, C. Jones, P. Schlein, P. Weilhammer, W. Blum, H. Dietl, W. Koch, E. Lorentz, G. Lütjens, W. Männer, J. Meissburger, W. Ochs, V. Stierlin, *Nucl. Phys.* **B75**, 189 (1974).
[6] A. Kotański, K. Zalewski, *Nucl. Phys.* **B12**, 72 (1969).
[7] G. S. Abrams, K. W. J. Barnham, *Phys. Rev.* **D7**, 1395 (1973).
[8] L. Stodolsky, J. J. Sakurai, *Phys. Rev. Lett.* **11**, 90 (1963).
[9] See, for example, H. Pilkuhn, B. E. Y. Svensson, *Nuovo Cimento* **38**, 518 (1965).
[10] S. Protopopescu, M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté, J. H. Friedman, T. A. Lasinski, G. R. Lynch, M. S. Rabin, F. T. Solmitz, *Phys. Rev.* **D7**, 1280 (1973). The actual data points are based on the DST for this experiment.
[11] M. J. Mattison, A. Barbaro-Galtieri, M. Alston-Garnjost, S. M. Flatté, J. H. Friedman, G. R. Lynch, M. S. Rabin, F. T. Solmitz, *Phys. Rev.* **D9**, 1872 (1974). The data points are based on the DST for this experiment.
[12] W. F. Buhl, G. Gidal, D. F. Grether, W. Ko, M. Alston-Garnjost, A. Barbaro-Galtieri, G. R. Lynch, F. T. Solmitz, *Phys. Lett.* **48B**, 388 (1974). Plots of these data are based on the actual DST for that experiment.
[13] D. Chew, M. Tabak, F. Wagner, Lawrence Berkeley Laboratory Report LBL-3396, 1975.
[14] A. Białas, A. Kotański, K. Zalewski, *Nucl. Phys.* **B28**, 1 (1971).
[15] V. Blobel, D. D. Carmony, H. Fesefeldt, H. Franz, B. Hellwig, P. Kobe, D. Monkemeyer, W. Schrankel, B. Schwarz, F. Selonke, J. Seyerlein, F. Wagner, B. Wessels, preprint Munich, 1974 and *Nucl. Phys.* (1974).
[16] G. Grayer, B. Hyams, C. Jones, P. Weilhammer, W. Hoogland, T. Meyer, R. Poster, P. Schlein, W. Blum, H. Dietl, W. Koch, E. Lorenz, W. Männer, J. Meissburger, V. Stierlin, contribution to the XVI International Conf. on High Energy Physics, Batavia, September 1972.
[17] R. D. Field, CALTECH preprint, CALT-466, 1974.
[18] J. T. Donohue, H. Høgaasen, *Phys. Lett.* **25B**, 554 (1967).
[19] W. Rarita, J. Schwinger, *Phys. Rev.* **60**, 61 (1941).
[20] G. Grayer, B. Hyams, C. Jones, P. Weilhammer, W. Blum, H. Dietl, W. Koch, E. Lorentz, G. Lütjens, W. Männer, J. Meissburger, W. Ochs, V. Stierlin, *Nucl. Phys.* **B50**, 29 (1972).
[21] P. K. Williams, *Phys. Rev. D*1, 1312 (1970).
[22] P. Estabrooks, A. D. Martin, *Phys. Lett.* **41B**, 350 (1972); P. Estabrooks, A. D. Martin, G. Grayer, B. Hyams, C. Jones, W. Blum, H. Dietl, W. Koch, E. Lorentz, G. Lütjens, W. Männer, V. Stierlin, Tallahassee conference on $\pi\pi$ scattering, AIP Conference proceedings 1973 No. 13, p. 37; B. Hyams, C. Jones, P. Weilhammer, W. Blum, H. Dietl, G. Grayer, W. Koch, E. Lorenz, G. Lütjens, W. Männer, J. Weissburger, W. Ochs, F. Wagner, *Nucl. Phys.* **B64**, 134 (1973).
[23] C. Michael, *Nucl. Phys.* **B63**, 431 (1973).
[24] A. C. Irving, *Nucl. Phys.* **B63**, 449 (1973).
[25] F. Wagner, Lawrence Berkeley Laboratory Report LBL-3352, 1974.
[26] F. Wagner, Proceedings of the XVII International Conf. on High Energy Physics, London 1974, p. 11–27, and LBL-3091.
[27] F. Wagner, M. Tabak, D. Chew, Lawrence Berkeley Laboratory Report LBL-3395, *Phys. Lett.* **58B**, 201 (1975); M. Tabak, Ph. D. Thesis, LBL-3397, 1975.

[28] R. Klanner, Thesis, Munich 1973, CERN-NP-73-9; Y. M. Antipov, G. Ascoli, R. Busselo, M. N. Focacci, W. Kienzle, R. Kenner, A. Lebedev, P. Lecornte, V. Roinishvili, A. Weitsch, F. A. Yotch, *Nucl. Phys.* **B63**, 153 (1973).
[29] M. Tabak, E. E. Ronat, A. H. Rosenfeld, T. A. Lasinski, R. J. Cashmore, Proceedings at the IV International Conf. on Experimental Meson Spectroscopy, Boston, 1974, and LBL-3010.
[30] Bonn–Hamburg–München collaboration, private communication of W. Schrankel.