GREEN'S FUNCTIONS IN NONRENORMALIZABLE MASSLESS $\varphi^6_{3+\varepsilon}$ THEORY. PART I. REGULARIZED THEORY

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Symanzik's method of renormalization in ultraviolet nonrenormalizable quantum field theories is applied to the construction of Green's functions of the massless $\varphi^6_{3+\varepsilon}$ theory. The renormalization of a regularized version of the theory is presented.

Introduction

The aim of these papers[1] is to investigate Green's functions (GF) of the nonrenormalizable quantum field theory of massless scalar field selfinteracting by a sixth-power coupling in $3+\varepsilon$, $\varepsilon > 0$, dimensional space-time. We strictly follow the approach recently proposed by Symanzik [1, 2] for nonrenormalizable theories. This very promising approach was originally worked out by Symanzik for the familiar quartic interaction but for the non-realistic case of space-time with dimension greater than four. We are interested in the sixth-power coupling because it is nonrenormalizable also in the physical case of four dimensions, opening thereby the possibility of experimental verification of the nonrenormalizable models of QFT.

Following Symanzik, we begin with the Lagrangian

$$\mathscr{L} = -\tfrac{1}{2}\varphi\Box\varphi - \frac{g_B}{6!}\varphi^6 - \frac{m^2_{BO}}{2}\varphi^2 - \frac{\lambda_{BO}}{4!}\varphi^4. \tag{I.0.1}$$

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The exhibited counterterms of square and quartic type are characteristic of three-dimensional space-time where the sixth-power coupling is renormalizable. The m_{BO} is the bare mass of the zero-physical-mass theory and the λ_{BO} is the bare parameter responsible for the absence of the physical quartic interaction (induced quartic interaction in less than four dimensions would lead to extreme difficulties in the infrared region). The counterterms are understood as repeated insertions in Feynman graphs contributing to the GF.

Symanzik's approach to the removal of singularities in GF of the nonrenormalizable theory is based on investigating the cutoff dependence of GF corresponding to a regularized version of (I.0.1). The simplest cutoff which can be used for this purpose is that of Pauli--Villars [6] which on the Lagrangian level means the modification of (I.0.1) to

$$\mathscr{L}_{\Lambda} = -\tfrac{1}{2}\,\varphi\Box(1+\Lambda^{-2}\Box)\varphi - \frac{g_B}{6!}\,\varphi^6 - \frac{m_{BO}^2}{2}\,\varphi^2 - \frac{\lambda_{BO}}{4!}\,\varphi^4, \tag{I.0.2}$$

which approaches (I.0.1) when $\Lambda \to \infty$. It can be proved that for renormalized $2n$-point vertex functions (one particle irreducible amputated parts of GF) $\Gamma_{\Lambda}^{2n}(g, \mu, \varepsilon)$ corresponding to the Lagrangian (I.0.2) the following large-Λ expansion holds:

$$\Gamma_{\Lambda}^{2n}(g, \mu, \varepsilon) = h_0^{2n}(g, \mu, \varepsilon) + \sum_{j=1}^{\infty} \sum_{k=0}^{\infty} \Lambda^{-j+\varepsilon k} h_{jk}^{2n}(g, \mu, \varepsilon), \tag{I.0.3}$$

where μ is a scale of mass and g is a dimensionless coupling constant. $h_0^{2n}(g, \mu, \varepsilon)$ corresponds to the unregularized Lagrangian (I. 0.1) without the counterterms. To $h_{jk}^{2n}(g, \mu, \varepsilon)$ only graphs with number of loops $L \geqslant k$ contribute. The derivation of the expansion (I.0.3) is identical to the derivation of the analogous expansion for the regularized $\varphi_{4+\varepsilon}^4$ theory which is presented in Appendix A of Symanzik's paper [1]. Therefore, we shall not quote here this derivation and we shall concentrate on studying the $\Lambda \to \infty$ limit of the vertex functions.

In Sections 1, 2, 3 of the present article we shall construct the renormalized VF of the regularized theory. In Section 4 here, we shall analyse differentiation with respect to Λ.

We shall proceed to the heart of the matter in Section 1 of Part II. Namely, in this section we shall study the $\Lambda \to \infty$ limit of the Γ_{Λ} functions and we shall formulate a set of conditions under which one can expect this limit to exist.

These conditions will be given in the form of existence conditions for nonsingular functions of ε given as one dimensional improper integrals over functions which are computable in the perturbation theory. However, these integrals, which we shall call Symanzik's constants for the $\varphi_{3+\varepsilon}^6$ theory, are divergent when evaluated by the (finite order) perturbation theory. Actually, the number of such constants is infinite. Helas, the methods for computing Symanzik's constants are not invented yet.

The physical discussion is presented in the summary of the original Symanzik paper [1] about the $\varphi_{4+\varepsilon}^4$ model and our results for the $\varphi_{3+\varepsilon}^6$ model are not enlightening in this respect (especially as the problem of unitarity is concerned). To compare with the presentations there we have chosen to be more explicit in some parts of the reasonings (to our knowledge Symanzik's approach to nonrenormalizable theories is not very well known). We shall also currently mention technical differences between the $\varphi^4_{4+\varepsilon}$ and $\varphi^6_{3+\varepsilon}$ models.

The $\varphi^6_{3+\varepsilon}$ model can be studied at the physically interesting $\varepsilon \to 1$ limit. One can hope that in the future, when one will know how to compute Symanzik's constants, this example of nonrenormalizable interaction will turn out to be a little less "academic" than the $\varphi^4_{4+\varepsilon}$ at the limits $\varepsilon \to 1$ or $\varepsilon \to 2$.

1. *Standard subtractions*

In the first two Sections we shall analyze in how many dimensions the subtraction scheme, which was used by Symanzik in regularized $\varphi^4_{4+\varepsilon}$ theory (and which we are calling the standard subtractions), is expected to work for the $\varphi^6_{3+\varepsilon}$ theory.

Let us first indicate that (UV, overall) superficial degree of divergence of the graphical contributions to VF corresponding to (I.0.1)

$$\mathrm{d}(\Gamma^{2n}_{\Lambda=\infty}(g, \mu, \varepsilon)) = 3-n+\varepsilon L \tag{I.1.1}$$

can in fact grow arbitrarily large for $\varepsilon > 0$ (L is the number of loops in the graph). This statement of nonrenormalizability for $\varepsilon > 0$ follows from power counting (the propagators are $i(p^2+i0)^{-1}$) from the "topological" relation

$$6V-2n = 2I \tag{I.1.2}$$

and from

$$L = I-V+1 \tag{I.1.3}$$

(V denotes the number of six-legged vertices and I the number of internal lines in the graph). Let us note that

$$\mathrm{d}_c(n) = \mathrm{d}(\Gamma^{2n}_{\Lambda=\infty}(g, \mu, \varepsilon = 0)) = 3-n \tag{I.1.4}$$

can be an even or an odd number whereas for the $\varphi^4_{4+\varepsilon}$ theory the analogous number is always even. Let us also anticipate here that if we would subtract from the integrand of an integral contributing to these VF the first $\mathrm{d}_c(n)+1$ terms of its Taylor expansion with respect to the external momenta (Zimmermann's subtractions [9] performed according to (I.1.4)) then this could produce finite (unregularized) VF merely for $\varepsilon < 1/L$ (cf. general formula (I.1.6) below). This situation improves in the regularized theory based on the Lagrangian (I.0.2).

In fact, the Feynman rules now read: substitute for the vertex $-ig\mu^{-2\varepsilon}$, for the propagators substitute

$$i[(p^2+i0)(1-\Lambda^{-2}(p^2+i0))]^{-1}.$$

With these rules we get

$$\mathrm{d}_t(n, \varepsilon, L) = \mathrm{d}(\Gamma^{2n}_{\Lambda}(g, \mu, \varepsilon)) = 6-2n+(\varepsilon-3)L \tag{I.1.5}$$

which could become arbitrarily large only for $\varepsilon > 3$.

The standard subtractions are Zimmermann's subtractions in the regularized theory (I.0.2), performed not according to true degrees d_t but to the conventional degree d_c (I.1.4), corresponding to the $\Lambda = \infty$ and $\varepsilon = 0$ case. As far as the quadratically divergent part of $\Gamma^2_\Lambda(g, \mu, \varepsilon)$ is concerned, it is stipulated that not the term $p^2 \frac{\partial}{\partial p^2} \Gamma^2_\Lambda(p, -p; g, \mu, \varepsilon)|_{p=0}$ but the term $(p^2 - \Lambda^{-2}p^4) \frac{\partial}{\partial p^2} \Gamma^2_\Lambda(p, -p; g, \mu, \varepsilon)|_{p=0}$ has to be subtracted. In such a way one avoids renormalization of Λ (the kinetic energy part of the renormalized Lagrangian will be of the same form as in (I.0.2)).

The condition for standard subtractions to produce finite VF reads

$$d_c(n)+1 > d_t(n, \varepsilon, L) \tag{I.1.6}$$

and below we shall find the region of ε in which this holds. In order to convince oneself about the validity of this statement let us take as an example the case when d_t is a number (irrational, in general) from the interval [0, 1) i.e. that just one subtraction will make the contribution ot the VF UV finite. The (I.1.6) means then that $d_c = 0$ because $d_c = d_c(n)$ are the integer numbers and the number of required subtractions equals one accoding to the d_t as well as according to the d_c. The reasoning proceeds analogously if $d_t \in [1, 2)$ and so on.

Let us check now for which values of ε the condition (I.1.6) holds. This condition reduces to

$$n-2 > (\varepsilon-3)L \tag{I.1.7}$$

from (I.1.4) and (I.1.5)). The most restrictive case is that of $n = 1$ (we restrict ourselves to $n \geqslant 1$ as we will not analyze the vacuum functions in this paper). Hence, the condition (I.1.6) surely holds for $\varepsilon < 3-1/L_{\min}$, where $L_{\min} = 4$. However, this restriction will not be good enough for our purposes as the analysis for VF with insertions [10] will show.

2. *Standard subtractions for vertex functions with insertions*

Although in the regularized Lagrangian (I.0.2) the $\varphi\Box^2\varphi$ term belongs to the kinetic energy, the VF with insertions (with zero momentum transfer to the graph) of this operator $((\Gamma^{2n}_{\Lambda, \varphi\Box^2\varphi}(g, \mu, \varepsilon))$ will appear when one will differentiate VF Γ^{2n}_Λ with respect to the cutoff Λ. More precisely, the ΛDVO (cutoff differential vertex operations) for the Γ^{2n}_Λ function will lead to VF with insertions of operators

$$\begin{array}{cccc} O_1 = \varphi^2, & O_2 = \varphi^4, & O_3 = \varphi\Box\varphi, & O_4 = \varphi^6, \\ 1+\varepsilon & 2+2\varepsilon & 3+\varepsilon & 3+3\varepsilon \end{array}$$

$$\begin{array}{cc} O_5 = \varphi^2\Box\varphi^2, & O_6 = \varphi^8, \\ 4+2\varepsilon & 4+4\varepsilon \end{array}$$

$$\begin{array}{ccc} O_7 = \varphi\Box^2\varphi, & O_8 = \varphi^3\Box\varphi^3, & O_9 = \varphi^{10} \\ 5+\varepsilon & 5+3\varepsilon & 5+5\varepsilon \end{array} \tag{I.2.1}$$

(in the second rows we have listed their canonical dimensions). All these operators are needed also as counterterms in the construction of the renormalized $\Gamma^{2n}_{\Lambda,\varphi\Box^2\varphi}(g, \mu, \varepsilon)$ according to the standard subtractions (the normal product in the sense of Zimmermann and Lowenstein [11]). Moreover, the VF with arbitrary numbers of insertions of these operators will be also needed for carrying out the process of reintegration with respect to Λ (see Part II). Let us notice that at $\varepsilon = 0$ the O_5 and O_6 have smaller dimensions than the O_7, O_8 and O_9. This is in contradistinction to the $\varphi^4_{4+\varepsilon}$ theory, where in an analogous group of operators only those with equal dimensions (at $\varepsilon = 0$) appear.

All the operators (I.2.1) are written in the form

$$O_i = \varphi^{s_i}\Box^{r_i}\varphi^{s_i}. \tag{I.2.2}$$

For their dimensions it will be convenient to write

$$\dim O_i = a_i + \varepsilon b_i, \tag{I.2.3}$$

where the maximal a_i are the $a_7 = a_8 = a_9 = 5$ $(b_i \leqslant a_i)$.

The insertions of the operators O_i taken into account, the "typological" relation (I.1.2) becomes

$$6V + 2\sum_{i=1}^{\alpha} s_i - 2n = 2I' \tag{I.2.4}$$

and (I.1.3) generalizes to

$$L' = I' - V - \alpha + 1, \tag{I.2.5}$$

where $\alpha = 1, 2, \ldots$ denotes the number of these additional vertices, $I' = I + \sum_{i=1}^{\alpha} s_i$ denotes the number of internal lines in the graph and

$$L' = L + \sum_{i=1}^{\alpha} s_i - \alpha \tag{I.2.6}$$

is the number of loops $(L' \geqslant L)$. We find that the superficial degree of divergence for contributions to $\Gamma^{2n}_{\Lambda,i,j,\ldots,k}(g, \mu, \varepsilon)$ (the subscripts $i, j, \ldots$ denote the insertions of the $O_i, O_j, \ldots, i = 1, \ldots, 9, j = 1, \ldots, 9, \ldots$) is

$$\mathrm{d}_t(n, \varepsilon, L', \alpha) = (\varepsilon - 3)L' - 2n + 2\sum_{i=1}^{\alpha} s_i + 2\sum_{i=1}^{\alpha} r_i + 6(1-\alpha) \tag{I.2.7}$$

and the conventional degree

$$\mathrm{d}_c(n, \alpha) = \mathrm{d}(\Gamma^{2n}_{\infty,i,j,\ldots}(g, \mu, \varepsilon = 0)) = -n + \sum_{i=1}^{\alpha} s_i + 2\sum_{i=1}^{\alpha} r_i + 3(1-\alpha). \tag{I.2.8}$$

The condition for the standard subtractions to give also finite VF with α insertions reads

$$\mathrm{d}_c(n, \alpha) + 1 > \mathrm{d}_t(n, \varepsilon, L', \alpha), \tag{I.2.9}$$

what constitutes the obvious generalization of (I.1.6). From (I.2.6) and (I.2.7) we get that the (I.2.9) takes place for

$$\varepsilon < 2+2\frac{\alpha+V}{L'} - \frac{1}{L'} . \tag{I.2.10}$$

Thus, for $\varepsilon < 2$ (I.2.9) is certainly satisfied because $\alpha+V \geqslant 1$ for any graph.

Let us only mention here that in the Λ DVO for VF with insertions will appear VF with insertions of operators different from O_i listed in (I.2.1) but still having the form (I.2.2). From the general formula (I.2.10) it is clear that for $\varepsilon < 2$ the standard subtraction scheme is sufficient also in this case.

The fact that only for

$$0 < \varepsilon < 2 \tag{I.2.11}$$

the (minimal) standard subtractions produce the VF which are free of ε-singularities will not affect the process of reintegration of Λ DVO which leads to large Λ-expansion and, consequently, allows one to study the $\Lambda \to \infty$ limit for the VF. Namely, it will be sufficient to know that in a certain region of $\varepsilon > 1$ the VF are singularity free (see analytic continuations from $\varepsilon > 1$ in Part II). Let us recall that standard subtractions in the regularized $\varphi^4_{4+\varepsilon}$ theory allow one to define the GF of its nonrenormalizable counterpart for $\varepsilon \in (0, 3)$ [1].

3. *Renormalization conditions*

We will renormalize the VF on the mass shell, i.e. on zero external momenta. Before quoting the renormalization conditions for conventionally divergent VF let us indicate that IR degrees of divergence $d_i(\Gamma^{2n}_\Lambda(g, \mu, \varepsilon))$ ($d_i(\Gamma^{2n}_{\Lambda,i,j,\dots}(g, \mu, \varepsilon))$) for the VF with $d_c(n) \geqslant 0$ ($d_c(n, \alpha) \geqslant 0$), see (I.1.4) ((I.2.8)), are greater than zero for $\varepsilon > 0$. In fact, the IR power counting gives

$$d_i(\Gamma^{2n}_\Lambda(g, \mu, \varepsilon)) = 3-n+\varepsilon L \tag{I.3.1}$$

and

$$d_i(\Gamma^{2n}_{\Lambda,i,j,\dots}(g, \mu, \varepsilon)) = 3(1-\alpha)-n+\sum_{i=1}^{\alpha} s_i+2\sum_{i=1}^{\alpha} r_i+\varepsilon L'. \tag{I.3.2}$$

For writing the renormalization conditions it is advantageous to control the dimensions (in units of mass) of VF. These are

$$\dim \Gamma^{2n}_\Lambda(g, \mu, \varepsilon) = 3-n-\varepsilon(n-1), \tag{I.3.3}$$

$$\dim \Gamma^{2n}_{\Lambda,i,j,\dots}(g, \mu, \varepsilon) = 3-3\alpha-n-\varepsilon(n+\alpha-1)+\dim O_i+\dim O_j+ \dots . \tag{I.3.4}$$

We renormalize by setting zero for divergent VF with odd number of derivatives and

$$\Gamma^2_\Lambda(0, 0; g, \mu, \varepsilon) = 0, \qquad \frac{\partial}{\partial p^2}\Gamma^2_\Lambda(p, -p; g, \mu, \varepsilon)|_{p=0} = i,$$

$$\Gamma^4_\Lambda(0, 0, 0, 0; g, \mu, \varepsilon) = 0, \qquad \Gamma^6_\Lambda(0, 0, 0, 0, 0, 0; g, \mu, \varepsilon) = -ig\mu^{-2\varepsilon}. \tag{I.3.5}$$

For the divergent VF with single insertions of the operators listed in (I.2.1) we impose

$$\Gamma^2_{\Lambda,i}(0, 0; g, \mu, \varepsilon) = \delta_{i1}, \quad \forall\, i,$$

$$\frac{\partial}{\partial p^2}\Gamma^2_{\Lambda,i}(p, -p; g, \mu, \varepsilon)|_{p=0} = \delta_{i3}, \quad i > 2,$$

$$\left(\frac{\partial}{\partial p^2}\right)^2 \Gamma^2_{\Lambda,i}(p, -p; g, \mu, \varepsilon)|_{p=0} = 2\delta_{i7}, \quad i > 6,$$

$$\Gamma^4_{\Lambda,i}(0, 0, 0, 0; g, \mu, \varepsilon) = \delta_{i2}, \quad i > 1,$$

$$\frac{\partial}{\partial p^2}\Gamma^4_{\Lambda,i}(p, -p, 0, 0; g, \mu, \varepsilon)|_{p=0} = \delta_{i5}, \quad i > 4,$$

$$\Gamma^6_{\Lambda,i}(0, 0, 0, 0, 0, 0; g, \mu, \varepsilon) = \delta_{i4}, \quad i > 2,$$

$$\frac{\partial}{\partial p^2}\Gamma^6_{\Lambda,i}(p, -p, 0, 0, 0, 0; g, \mu, \varepsilon)|_{p=0} = \delta_{i8}, \quad i > 6,$$

$$\Gamma^8_{\Lambda,i}(0, 0, 0, 0, 0, 0, 0, 0; g, \mu, \varepsilon) = \delta_{i6}, \quad i > 4,$$

$$\Gamma^{10}_{\Lambda,i}(0, 0, 0, 0, 0, 0, 0, 0, 0, 0; g, \mu, \varepsilon) = \delta_{i9}, \quad i > 6. \tag{I.3.6}$$

For the sake of completeness one should also quote the remaining renormalization conditions for VF with double, triple and even for a VF with more insertions of the operators from (I.2.1). However, the VF on zero external momenta with more than one insertion must vanish because then no trivial one-particle-irreducible graphs exist. Therefore, such renormalization conditions will be useless for our purposes as e.g. for the determination of coefficients in Λ DVO for VF with insertions. Let us end this section with the assertion that standard subtractions with these renormalization conditions specify the VF of the regularized theory for any value of $\varepsilon \in (0,2)$. From here on we shall speak only about ε in this region.

4. *Differentiation with respect to the cutoff*

For the analysis in this section it is advantageous to note that in the Feynman rules as well as in the renormalization conditions the parameters μ and g appear only in the combination $g\mu^{-2\varepsilon}$ and, consequently, the renormalized VF can depen don these parameters also only in this particular combination.

The cutoff Λ can be of course treated as a parameter in the renormalized Lagrangian corresponding to (I.0.2) and then the parametric derivative identity (the Λ DVO) [11] satisfied by perturbatively renormalized VF can be shown to read

$$\Lambda\frac{\partial}{\partial\Lambda}\Gamma^{2n}_{\Lambda}(g, \mu, \varepsilon) = \sum_{i=1}^{9} c_i(g, \Lambda, \mu, \varepsilon)\Gamma^{2n}_{\Lambda,i}(g, \mu, \varepsilon). \tag{I.4.1}$$

The formula (I.4.1) follows most naturally from the functional approach and its derivation is strictly analogous to the one given in the original Symanzik paper [2]. The fact that the summation in (I.4.1) runs over all operators which were listed in (I.2.1) is explained in the Appendix to Part II.

It is easy to check that

$$\dim c_i(g, \Lambda, \mu, \varepsilon) = 3+\varepsilon-\dim O_i. \tag{I.4.2}$$

Because in c_i only integer powers of $g\mu^{-2\varepsilon}$ can appear we must have

$$c_i(g, \Lambda, \mu, \varepsilon) = \Lambda^{3+\varepsilon-\dim O_i} c_i(g\mu^{-2\varepsilon}\Lambda^{2\varepsilon}, \varepsilon) \tag{I.4.3}$$

where $c_i(z, \varepsilon)$ are integer power series in the variable z.

Let us notice that at $\varepsilon = 0$ the c_2, c_5 and c_6 are proportional to odd powers of Λ whereas in coefficients of Λ DVO for the analogously regularized $\varphi^4_{4+\varepsilon}$ only even powers of the cutoff (at $\varepsilon = 0$) are encountered).

From the renormalization conditions we get

$$c_1 = c_2 = c_3 = c_4 = 0 \tag{I.4.4a}$$

and

$$c_5(z, \varepsilon) = \Lambda^{2+\varepsilon} \frac{\partial}{\partial \Lambda} \frac{\partial}{\partial p^2} \Gamma^4_\Lambda(p, -p, 0, 0; g, \mu, \varepsilon)|_{p=0}, \tag{I.4.4b}$$

$$c_6(z, \varepsilon) = \Lambda^{2+2\varepsilon} \frac{\partial}{\partial \Lambda} \Gamma^8_\Lambda(0, 0, 0, 0, 0, 0, 0, 0; g, \mu, \varepsilon), \tag{I.4.4c}$$

$$\begin{aligned} c_7(z, \varepsilon) = {} & \Lambda^3 \frac{\partial}{\partial \Lambda} \left(\frac{\partial}{\partial p^2}\right)^2 \Gamma^2_\Lambda(p, -p; g, \mu, \varepsilon)|_{p=0} \\ & -c_5 \left(\frac{\partial}{\partial p^2}\right)^2 \Gamma^2_{\Lambda,5}|_{p=0} - c_6 \left(\frac{\partial}{\partial p^2}\right)^2 \Gamma^2_{\Lambda,6}|_{p=0}, \end{aligned} \tag{I.4.4d}$$

$$\begin{aligned} c_8(z, \varepsilon) = {} & \Lambda^{3+2\varepsilon} \frac{\partial}{\partial \Lambda} \frac{\partial}{\partial p^2} \Gamma^6_\Lambda(p, -p, 0, 0, 0, 0; g, \mu, \varepsilon)|_{p=0} \\ & -c_5 \frac{\partial}{\partial p^2} \Gamma^6_{\Lambda,5}|_{p=0} - c_6 \frac{\partial}{\partial p^2} \Gamma^6_{\Lambda,6}|_{p=0}, \end{aligned} \tag{I.4.4e}$$

$$\begin{aligned} c_9(z, \varepsilon) = {} & \Lambda^{3+4\varepsilon} \frac{\partial}{\partial \Lambda} \Gamma^{10}_\Lambda(0, 0, 0, 0, 0, 0, 0, 0, 0, 0; g, \mu, \varepsilon) \\ & -c_5 \Gamma^{10}_{\Lambda,5}|_{p=0} - c_6 \Gamma^{10}_{\Lambda,6}|_{p=0}. \end{aligned} \tag{I.4.4f}$$

Hence, the $c_i(z, \varepsilon)$ have perturbative expansions

$$c_i(z, \varepsilon) = 2i\delta_{i7} + \sum_{k=k_{\min i}}^{\infty} c_{ik}(\varepsilon) z^k \tag{I.4.5}$$

with $k_{\min 5} = k_{\min 6} = k_{\min 7} = k_{\min 8} = 2$ and $k_{\min 9} = 3$. The coefficients $c_{ik}(\varepsilon)$ must be non-singular for $\varepsilon \in (0,2)$ because of (I.4.1) which relates (UV) perturbatively renormalized VF. One can easily check that the $c_{ik}(\varepsilon)$ with $i \neq 7, 9, 8$ are also free of IR singularities there. Namely, the Λ-differentiations in (I.4.4) are always adding two to the IR degree (I.3.1), removing thereby IR singularities which are expected for $\varepsilon \leqslant 1/L$ and $\varepsilon \leqslant 2/L$ in the graphical contributions to the VF in (I.4.4b), (I.4.4c) and (I.4.4d), (I.4.4e), (I.4.4f), correspondingly. However, in the formulae for c_7, c_8, c_9 the terms without the Λ-differentiations are IR-singular for $\varepsilon \leqslant 1/L$. These IR-singularities are coming from the terms proportional to c_5, c_6 and they disappear for $\varepsilon > 1/L$. It might well be that they are absent also for $\varepsilon \leqslant 1/L$ — we postpone this problem to a more careful study. In the present papers we are concerned with UV structure of the theory. Therefore we will isolate ε-poles in c_7, c_8, c_9 corresponding to IR-singularities and we will consider only the remaining part of c_7, c_8, c_9. In other words, we proceed as if

$$c_5 \left(\frac{\partial}{\partial p^2}\right)^2 \Gamma^2_{\Lambda,5}|_{p=0} + c_6 \left(\frac{\partial}{\partial p^2}\right)^2 \Gamma^2_{\Lambda,6}|_{p=0},$$

$$c_5 \frac{\partial}{\partial p^2} \Gamma^6_{\Lambda,5}|_{p=0} + c_6 \frac{\partial}{\partial p^2} \Gamma^6_{\Lambda,6}|_{p=0},$$

$$c_5 \Gamma^{10}_{\Lambda,5}|_{p=0} + c_6 \Gamma^{10}_{\Lambda,6}|_{p=0},$$

were equal to zero. Thus, apart from IR-poles, $c_{ik}(\varepsilon)$ are analytic functions of (ε).

The analyticity of $c_{ik}(\varepsilon)$ does not imply that the whole perturbative series (I.4.5) is analytic. However, following Symanzik, we assume that also the $c_i(z, \varepsilon)$ are analytic functions of ε in the interval (0,2).

In Part II of the present investigation we shall analyze the problem of the removal of the cutoff.

The list of references is given at the end of Part II.