ON THE MECHANISM OF CUMULATIVE PROCESSES

By B. N. Kalinkin, A. V. Cherbu and V. L. Shmonin

Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna*

(Received August 5, 1977)

A phenomenological model for cumulative processes is considered. This model assumes the formation of the compound-system by the initial nucleon and nuclear nucleons successively collected by it and the inclusive spectrum scale invariance of the decay products of this system. The model is almost independent of what happens when internucleon distances in nuclei are very small. Good agreement with the experimental data is obtained.

1. Introduction

The phenomenon of cumulative meson production [1] has been discovered in the experiments performed in the High Energy Laboratory of the JINR. The results have raised many interesting problems for theoretical relativistic nuclear physics.

Attempts have been made to explain the observed effects on the basis of already known mechanisms. However, they encounter great difficulties. Consideration of the Fermi motion gives no desired result. The existence of the core in the NN-interaction and the data on the distribution of $\mu^{\pm}$-pairs over the effective mass [3, 4] produced in the p, A-interaction make it difficult to validate the assumption on the existence of fluctuations of nuclear density in a small volume with probability sufficient to explain the phenomenon. Neither can the use of the "rescattering" mechanism [5] be considered satisfactory, since the production mechanism is supposed to be instantaneous. This treatment was used earlier in models of the cascade type. As is known, it does not provide an understanding of some important effects observed in the multiple production processes in nuclear matter.

Approaches have also been developed on quark-parton representations of the hadron structure. In our opinion the most consistent along this line is the model in Ref. [6]. However, it solves the problem only qualitatively. In this paper we shall consider a phenomenological model of the cumulative phenomenon taking into account the space-time factors in the production process and allowing one, in our opinion, to overcome the above difficulties.

* Address: Joint Institute for Nuclear Research, Head Post Office, P.O. Box 79, Moscow, USSR.

2. *The space-time description of the cumulative meson production*

The physical phenomenon is realized in time and space intervals different from zero. Let us consider the cumulation process from this point of view. Suppose that in the first act of the inelastic interaction there arises a compound system which breaks-up during a short but finite period of time. Then we assume the possible inelastic collisions of this system with the nuclear nucleons resulting in the growth of its mass and, consequently, in the increase of the maximal energy of the emitted π-meson. Thus, the cumulation may be considered as a scheme describing the "gathering" (collection) of nucleons by an initial particle into a compound system with increasing mass.

Since for the production of a cumulative meson the compound system should have a mass near the maximal value (the inelasticity coefficient K near 1), then the partial cross--section of its production should be

$$\sigma \ll \sigma_{\mathrm{NN}}^{\mathrm{in}}. \tag{1}$$

By some estimations $\sigma(K \to 1) \approx (0.1 \div 0.2)\sigma_{\mathrm{NN}}^{\mathrm{ni}}$, in the interval of $5 \div 6\ \mathrm{GeV} \leqslant E_{\mathrm{p}} \leqslant 10 \div 12\ \mathrm{GeV}$ [7]. There are also indications that in this interval σ may depend on energy. For instance, the dependence close to

$$\sigma(K \to 1) \sim 1/\gamma_{\mathrm{cms}}^2 \tag{2}$$

is used.

The mean time $\bar{\tau}_0$ of the cumulative π-meson emission should be, on the one hand, less than the mean time of the decay of the thermodynamic equilibrium system $\bar{\tau}$ and, on the other hand, larger than the collision time $\bar{\tau}_{\mathrm{coll.}}$:

$$\bar{\tau}_{\mathrm{coll.}} \approx \frac{\langle r_{\mathrm{N}} \rangle}{c\gamma_{\mathrm{cms}}} < \bar{\tau}_0 < \bar{\tau} \approx \frac{r_0 \langle n_s \rangle^{1/3}}{c}, \tag{3}$$

where $\langle r_{\mathrm{N}} \rangle$ is the mean nucleon radius, r_0 is the radius of nuclear forces and $\langle n_s \rangle$ is the mean multiplicity when $K \approx 1$.

Assume that

$$\bar{\tau}_0 = \tau_0/\sqrt{S}\,. \tag{4}$$

Equation (4) takes into account the possible decrease of the mean time of the π-meson emission with increasing mass of the compound-system. The number of states emitting the cumulative π-mesons is naturally given by the usual exponential dependence (this dependence is due to the decay of the whole system and the dissipation of energy to other degrees of freedom)

$$\eta_{\mathrm{cms}} \simeq \exp\left[-t/\bar{\tau}_0\right]. \tag{5}$$

Passing to the laboratory system and expressing t in terms of the coordinate and velocity, we obtain

$$\eta(z) \simeq \exp\left[-\frac{z - z_{\mathrm{in}}}{\bar{\tau}_0 V_{\mathrm{cms}} \gamma_{\mathrm{cms}}}\right], \tag{6}$$

where z_{in} are the coordinates of the system production or of the act of its increasing mass. Since in the successive acts the product $\bar{\tau}_0 V\gamma$ is changing, we should write

$$\eta(z) \simeq \exp\left[-\frac{z-z_{in}}{(\bar{\tau}_0 V\gamma)_n}\right] = \exp\left[-a_n(z-z_{in})\right] \tag{7}$$

(n is the number of the collected nucleus nucleons). Therefore, one should use S_n instead of S in Eq. (4). Then, from the kinematics, we obtain

$$a_n = \left[\frac{\tau_0 c}{\sqrt{S_{n'}}} \frac{(E_p^2-m_p^2)^{1/2}}{\sqrt{S_{n'}}}\right]^{-1} = \frac{1}{\tau_0 c} \frac{(n'^2+1)m_p^2+2n'm_pE_p}{(E_p^2-m_p^2)^{1/2}};$$

$$n' = n-1. \tag{8}$$

It immediately follows from (8) that with $E_p \gg m_p$:

$$(a_n)_{E_p \gg m_p} \approx n' \frac{2m}{\tau_0 c} = (n-1)\frac{2m}{\tau_0 c} = \text{const}\,(E_p). \tag{9}$$

The parameters σ and τ_0 are to be refined by comparing the model with experiment.

Further we assume that the system formed at any stage of cumulation decays so that there occurs (approximately) the scale invariance of the inclusive spectrum of produced particles (distribution over the variable $x = p_\parallel / p_\parallel^{max}$ is invariant with respect to the mass of the intermediate system). This principle as the basic one has essentially been used even in the first investigations of the cumulative effect [8]. In paper [2] for the invariant cross--section in the elementary act, it was assumed that

$$\varrho(x, p_\perp) \approx \frac{E_\pi}{\sigma_{NN} p_\pi^2} \frac{d^2\sigma}{dp_\pi d\Omega} = F(x)\exp(-ap_\perp^2) \tag{10}$$

and for $F(x)$ the expression was obtained describing well the data on pion production in p-p-collisions at different energies. Since most experiments were performed for $\Theta_\pi = 180°$ we assume that $\exp(-ap_\perp^2) \approx 1$.

Now we pass to the basic relations describing the process within the above model. The scheme of the process is presented in Fig. 1 for a certain value of the impact parameter "b" of an incident proton.

The probability of producing the compound system in the "gathering" of n nucleons with the given distribution of points z_n in which its mass increases, is

$$W^{(n)}(b; z_1, z_2, \ldots, z_n) = w_1(z_1)w_2(z_2)\ldots w_n(z_n), \tag{11}$$

where $w_1(z_1), \ldots, w_n(z_n)$ are the probabilities of acts at points $z_1, \ldots, z_n$ respectively. The coordinates of events are distributed in the interval $[z_{min}, z_{max}]$ under the condition that

$$z_{min} = -\sqrt{R^2-b^2} \leqslant z_1 \leqslant z_2 \ldots \leqslant z_{n-1} \leqslant z_n \leqslant z_{max} = \sqrt{R^2-b^2}. \tag{12}$$

Here R is the radius of the nucleus with mass number A ($R = r_0 A^{1/3}$, $r_0 = 1.2$ fm). All the possibilities of the distribution of acts over the points z_i and the condition (12) result in the following integral

$$W^{(n)}(b) = \int_{z_{\min}}^{z_{\max}} w_1(z_1)dz_1 \int_{z_1}^{z_{\max}} w_2(z_2)dz_2 \dots \int_{z_{n-1}}^{z_{\max}} w_n(z_n)dz_n. \tag{13}$$

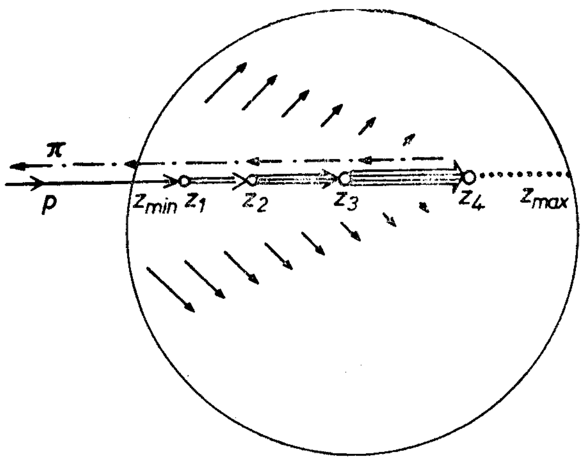

Fig. 1. The scheme of the "gathering" process

Then the invariant cross-section of the cumulative pion production on the nucleus can be written as follows:

$$\mathscr{R}_A = 2\pi \sum_n F(x_n) \int_0^R bdbW^{(n)}(b) = \sum_n F(x_n)W^{(n)}, \tag{14}$$

$$W^{(n)} = 2\pi \int_0^R bdbW^{(n)}(b). \tag{15}$$

Expression (14) is analogous to that in Ref. [1]. The only difference concerns the coefficient $W^{(n)}$. In paper [1] it is denoted by P_n and is of the combinatorial nature due to the use of the fluctuation model.

In (14) $x_n = p_{\|}^{\pi}/(p_{\|}^{\pi})_{(n)}^{\max}$; $(p_{\|}^{\pi})_{(n)}^{\max}$ is the maximal value of the momentum determined by the kinematics of the cumulation of an n-th order:

$$(p_{\|}^{\pi})_{(n)}^{\max} \simeq (E_\pi)_{(n)}^{\max} \simeq n\frac{m_p}{2}\frac{\left(1-\frac{m_p}{E_p}\right)\left(1+\frac{m_p}{2nE_p}\right)}{1+\frac{n^2+1}{2n}\frac{m_p}{E_p}}. \tag{16}$$

m_p is the proton mass and E_p is its total energy in the lab. system.

For the calculation of (13) and (14) we shall take the simple, frequently used in nuclear optics approximations. First, assume the nucleus to be a sphere uniformly filled in with matter with density

$$\varrho = \begin{cases} A/V_A = \dfrac{3}{4\pi r_0^3}; & r < R, \\ 0 \qquad ; & r > R. \end{cases} \tag{17}$$

Second, the trajectories of the initial particle and resulting from it compound systems are considered to be rectilinear.

Third, in analogy with optics, we take

$$w_n'(z) = \sigma\varrho e^{-\sigma\varrho(z-z_{n-1})} \tag{18}$$

as the basis of the expression $w_n(z)$. However, (18) does not take into account the possibility that the system decays (and, consequently, leaves the considered channel) between the acts of increasing mass. To this end we multiply (18) by the time factor (7) when $n \geqslant 2$. Then, as $w_n(z)$, we have

$$w_1(z_1) = \sigma\varrho \exp\left[-\sigma\varrho(z-z_0)\right], \qquad z_0 = z_{\min},$$
$$w_{n\neq 1}(z_n) = \sigma\varrho \exp\left[-\sigma\varrho(z_n-z_{n-1})\right] \exp\left[-a_n(z_n-z_{n-1})\right]. \tag{19}$$

Direct integration in (15) with $w_n(z_n)$ from (19) results in rather cumbersome expressions $W^{(n)}$. However, at least for $1 \leqslant n \leqslant 4$ (these cases are particularly important in practice), $W^{(n)}$ can be written in a compact way:

$$W^{(n)} = \frac{\pi}{2} \frac{(\sigma\varrho)^n}{\prod\limits_{\kappa=1,\dots,n} (\sigma\varrho+a_\kappa)} \left\{2R^2 + \sum_{k=1}^{n} C_k^{(n)} \frac{\gamma[2; 2(\sigma\varrho+a_k)R]}{(\sigma\varrho+a_k)^2}\right\}, \tag{20}$$

where $\gamma[2; x]$ is the incomplete γ-function, $a_1 \equiv 0$ and

$$C_{k\neq 1}^{(n)} = \prod_{\substack{\lambda=1,\dots,n \\ \lambda\neq k}} (\sigma\varrho+a_\lambda)\Big[\prod_{\substack{r=1,\dots,n \\ r\neq k}} (a_k-a_r)\Big]^{-1},$$

$$C_{k=1}^{(n)} = -\Big[1+(-1)^n \sum_{k=2}^{n} C_k^{(n)}\Big]. \tag{21}$$

It is interesting to consider the behaviour of the $W^{(n)}$ functions as functions of A in the two limiting cases, small and large times i.e. when $a_n > 1$ and $a_n \ll 1$ (see (3), (7) and (8)). Note, then, that since $\sigma \ll \sigma_{\rm NN}^{\rm in}$ for at least light and intermediate nuclei

$$\omega = 2\sigma\varrho R < 1. \tag{22}$$

Then for small times ($a_n > 1$) we have from (20) and (21)

$$W^{(n)} \approx \frac{\pi}{2} \frac{(\sigma\varrho)^n}{\prod\limits_{\kappa=1,\dots,n} (\sigma\varrho+a_\kappa)} \left\{2R^2 - \frac{\gamma[2; 2\sigma\varrho R]}{(\sigma\varrho)^2}\right\} \approx \frac{4\pi R^3(\sigma\varrho)^{n+1}}{3 \prod\limits_{\kappa=1,\dots,n} (\sigma\varrho+a_\kappa)} \sim A. \tag{23}$$

(To derive (23) we have used the explicit expression for $\gamma[2;\omega]$ and its expansion in the small parameter ω.)

For large times ($a_n \ll 1$) it is more convenient to perform integration in (13) assuming at once that $\eta \approx 1$ (see (8)). Then $W^{(n)}$ are expressed through the linear combinations of the gamma-functions $\gamma[n_0, \omega]$ with indices $2 \leqslant n_0 \leqslant n+1$. Expanding them again in a series of ω, we obtain

$$W^{(n)} \sim (\sigma\varrho)^n R^2 R^n \sim A^{\frac{2}{3}+\frac{1}{3}n}. \tag{24}$$

The "volume" dependence (23) can be explained as follows: at small times (σ is also small) the process of "gathering" proceeds in a relatively small nuclear region of the order of magnitude of $\overline{\Delta z} \approx 1/a_2 \approx \text{const}\,(R)$. At large times $W^{(1)} \sim A$, since σ is still small. The reason for the further increase of the power of A in (24) with increasing n by a factor of 1/3 is related to the fact that the contribution of each order of cumulativity is proportional to the effective path in the nucleus $\sim R \sim A^{1/3}$.

The A-dependences of the type (23) and (24) were obtained earlier in [1] by a different method.

3. *The comparison of the model of "gathering" with experiment*

The calculations have been performed using the expressions (14)–(16), (20) and (21). The best agreement with experiment [9] is achieved if

$$\sigma = 0.25 \cdot \sigma_{\rm NN}^{\rm in}, \quad (\bar{\tau}_0)_{n'=1} \approx 4 \cdot 10^{-24}\ \text{sec}; \quad (P_{\rm p} = 8.4\ \text{GeV}/c). \tag{25}$$

We should like to note two important consequences following from (25).

First, the value of $\bar{\tau}_0$ is indeed inside the interval pointed out in (3):

$$\bar{\tau}_{\rm coll.} \simeq 1.5 \cdot 10^{-24}\ \text{sec} < (\bar{\tau}_0)_{n'=1} (= 4 \cdot 10^{-24}\ \text{sec}) < \bar{\tau} \approx 10^{-23}\ \text{sec}. \tag{26}$$

The quantity σ does not contradict the estimates in [7].

Second, $\overline{\Delta z}$ is the mean length of the nuclear "active" zone which to an order of magnitude is

$$\overline{\Delta z} \approx \frac{1}{a_{n'=1}(\bar{\tau}_0)} \approx 2.5\ \text{fm} \gg r_{\rm core} \tag{27}$$

that is, it exceeds almost by an order of magnitude the assumed value of the radius of the core $r_{\rm core}$ in the nucleon-nucleon interaction and is essentially larger than the internucleon distance in nuclei. This allows one to overcome the difficulties in considering small internucleon distances (within the given model they give a small contribution). The results of calculation and the experimental data are given in Fig. 2 for the invariant cross-section $\mathscr{R}_A(T_\pi)$ where $T_\pi > (T_\pi)^{\rm max}_{\rm N,N}$ and $A = C^{12}$, Pb^{208} ($P_{\rm p} = 8.4$ GeV/c). The agreement is quite good. It indicates the fact that A^N-dependence of the yield of cumulative pions [1] is reproduced correctly by the model (see Fig. 3).

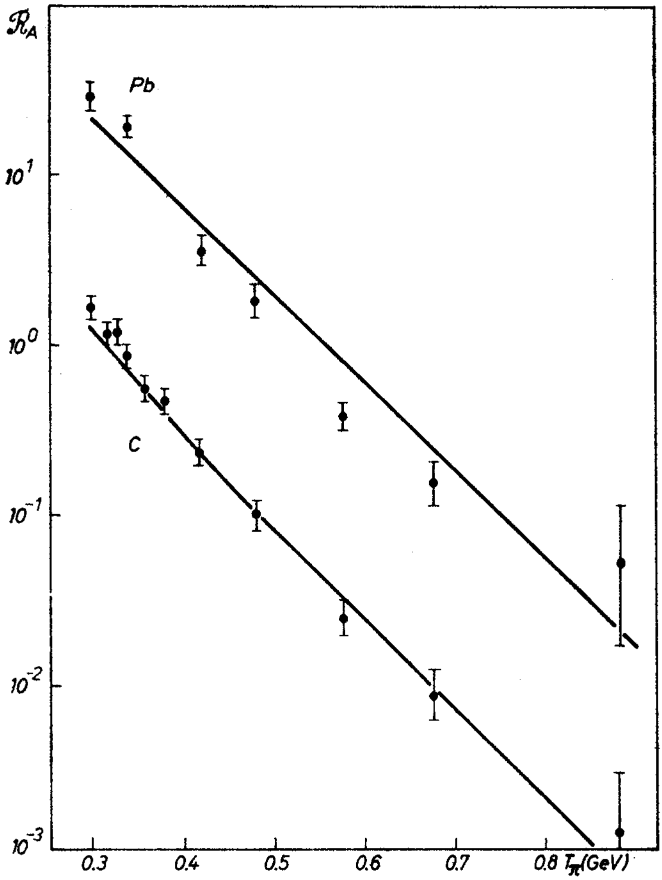

Fig. 2. The function $\mathcal{R}_A(T_\pi)$ for P_p = 8.4 GeV/c

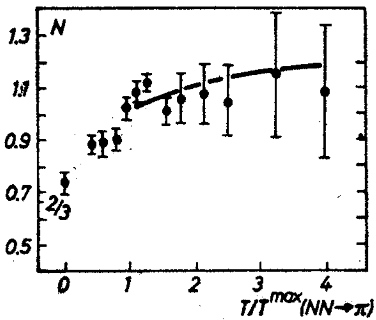

Fig. 3. The dependence $\mathcal{R}_A \sim A^N$ in different orders of cumulation

Further, it was found out [10] that the function $1/A\, \mathscr{R}_A(P_p)$ at $\kappa = T_\pi/(T_\pi)^{max}_{NN} = $ const for $P_p \geqslant 5$ GeV/c is practically constant. Such a behaviour follows from the model provided that $\sigma(P_p)$ is constant or changes very slightly in the region 5 GeV/$c \leqslant P_p \leqslant 8.4$ GeV/c. Indeed, for the rather large momenta of the initial particle $x_n \approx$ const (P_p) (see (16)) and $a_{n'} =$ const (see (9)). Therefore $[F(x_n) \cdot W^{(n)}]_{\kappa=\text{const}} \approx$ const (P_p).

If one assumes that $\sigma(P_p)$ when $P_p < 5$ GeV/c is of the form (2), then with decreasing P_p the function $1/A\, \mathscr{R}_A(P_p)$ (by $\kappa =$ const) increases what is observed experimentally. However, it is difficult to separate such a dependence since at small P_p it may be contributed to a mechanism based on the Fermi-motion.

We have discussed above the yield of π-mesons from the nucleus. Being produced, for instance, at point z_4 a π-meson should pass, on the average a considerable distance $z_4 - z_{min}$. There arises a question: Why are π-mesons emitted from the nucleus almost nonabsorbed? There exists a possibility of neglecting this effect for energetic π-mesons. Since the mean free path of the initial proton $\Lambda \approx 1/\varrho\sigma^{tot}_{NN} \approx r_0$, it interacts almost with every nucleon along the trajectory of its motion. The nucleons acquiring the transverse momenta $\langle p_\perp \rangle \approx 0.4$ GeV/$c \neq 0$ should be displaced in the direction away from the proton trajectory (arrows in Fig. 1) releasing the channel in the nucleus.

We can roughly estimate the degree of the displacement S of nucleons. The time required for the process of "gathering" from point z_1 to point z_4 and for the relativistic meson to pass this distance $(z_4 - z_1)$ is of the order of

$$t \approx 2(\overline{\Delta z})/c. \tag{28}$$

Hence, the displacement is

$$S \approx t \cdot \bar{v}_{N,\perp} \approx \frac{2(\overline{\Delta z}) \cdot \langle p_\perp \rangle}{m_p c} \approx 2.5 \text{ fm} > r_0. \tag{29}$$

Consequently, the density of matter in the channel decreases sharply, and the processes of "absorption" become unimportant. There were attempts to use this "trailing" effect for multiple production developing in the forward hemisphere [11] at rather large energies. However, just in this particular case, it does not "work": S is negligible due to a small path difference of particles of the "leading" and "pionization" groups.

Note that π-mesons at very low energy cross the nucleus under different conditions. For them the channel may disappear, be washed out due to the presence of the heat and Fermi-motion of the excited nucleus. Therefore, the A-dependence of the yield of these π-mesons will not be "volume" and should tend to $A^{2/3}$.

4. Conclusions

The model allows one to overcome the known difficulties (see § 1) and describes well the main regularities of cumulative meson production. It realized the aforesaid ideas:

a) The approximate scale invariance of the inclusive spectrum of particles produced in the collective interaction of hadrons [8], and

b) the necessary consideration of the space-time factors in the production processes [12].

The analysis of the data on its basis establishes the scale of the space-time interval inside of which the particles are produced in the hadron-hadron collisions (see (25), (27)).

The model is phenomenological (it contains only two parameters). However, it shows the quantities (e. g. $\sigma(P_p)$, $\bar{\tau}_0$) which should be explained on the basis of more detailed considerations of the hadronic structure. On the other hand, the phenomenological parameters in the quark-parton representation may, in principle, result in important relations between the dynamic factors of the structure models. It is most probable that the approach formulated above may appear to be a useful analogue for constructing the models of the hadron-hadron interaction. This idea was claimed earlier in Ref. [13].

The authors are grateful to A. M. Baldin, B. N. Valuev, A. V. Efremov, V. I. Ogievetsky and M. I. Shirokov for discussions and useful remarks.

REFERENCES

[1] A. M. Baldin, preprint JINR E2-9138, Dubna 1975; Santa Fe Conf. on High Energy Physics and Nuclear Structure, p. 373, 1975.
[2] A. M. Baldin et al., *Yad. Fiz.* **18**, 79 (1973).
[3] J. H. Christenson et al., *Phys. Rev.* **D8**, 2016 (1973).
[4] N. N. Nikolaev, B. I. Zakharov, *Yad. Fiz.* **21**, 434 (1975).
[5] L. A. Kondratyuk, V. B. Kopelyovich, *Zh. Eksp. Teor. Fiz. Pisma* **21**, 88 (1975).
[6] A. V. Efremov, preprint JINR E2-9529, Dubna 1976.
[7] E. L. Feinberg, *Usp. Fiz. Nauk* **104**, 539 (1971).
[8] A. M. Baldin, preprint JINR P7-5769, Dubna 1971.
[9] A. M. Baldin et al., preprint JINR E1-8054, Dubna 1974.
[10] A. M. Baldin et al., preprint JINR I-8249, Dubna 1974.
[11] V. S. Barashenkov et al., *Yad. Fiz.* **13**, 743 (1971).
[12] B. N. Kalinkin, V. L. Shmonin, preprints JINR P2-7869, P2-7870, P2-7871, Dubna 1974; *Yad. Fiz.* **21**, 628 (1975); IV International Seminar on High Energy Physics Problems, D1, 2-9224, Dubna 1975; preprint JINR E2-9137, Dubna 1975.
[13] A. M. Baldin, International Conf. on Problems of Heavy Ions Physics, Dubna 1971; preprint JINR D7-5769, 607, 1971.