ON WEAK FORM FACTORS IN NUCLEAR MUON CAPTURE

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The roles of the various weak hadronic form factors in the ground to ground state transition rate and the recoil polarization in muon capture by ^{12}C and ^{16}O are analysed using best available nuclear wave functions and compared with the conventional Fujii-Primakoff approximation. It is concluded that the recoil nuclear polarization of $^{12}B(1^+)$ can safely be used to examine the induced pseudoscalar and tensor form factors. A plausible value for the induced tensor form factor is obtained by comparing with the experimental value for the $^{12}B(1^+)$ average polarization and is in agreement with the recent β-decay experimental results.

1. Introduction

The study of muon capture by nuclei, despite its role as a probe to the nucleus, serves as a powerful mechanism to understand the hadronic part of the interaction Hamiltonian. However, the information about the individual roles of the various hadronic form factors are masked by the complexity of the nuclear physics. It is the purpose of this paper to examine the relative contributions of weak hadronic form factors in muon capture by ^{12}C and ^{16}O. We have chosen the ground to ground state partial transition rates and the average polarization of recoil nuclei, namely $^{12}B(1^+)$ and $^{16}N(2^-)$ respectively, since these two observables involve the weak hadronic form factors in different (different from one another) combinations and so can provide an independent check on our conclusions.

In muon capture, five hadronic form factors contribute since the momentum transfer of the process is large ($\sim$ 100 MeV/c) when compared to β-decay. Using the conservation of the hadronic vector current and muon-electron universality, the value of the vector form factor $g_V(0)$ is 0.986 G, where G is the four-fermion coupling. The CVC theory predicts 3.7 $g_V(0)$ for the weak-magnetism term $g_M(0)$ and 0 for the induced scalar form factor g_S. While the Partial Conservation of Axial part of the hadronic Current (PCAC) predicts nothing about the induced tensor form factor g_T, it gives, along with the Adler–Weisberger [1] sum rule, $-1.25\, g_V(0)$ for the axial form factor $g_A(0)$. In the limit of only about 7%

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breaking of the Chiral SU(2) symmetry [2], the induced pseudo-scalar form factor g_P is 7.7 $g_A(0)$, according to the Goldberger–Treiman relation. Even though recent experimental results suggest the existence of g_T, as the theoretical implications are still in a fluid state, we assume the absence of such second class currents. Even if they exist, g_T can be swamped in g_P giving rise to an *effective coupling* $\eta = g_P + g_T$, in muon capture. However, in Section 4 we discuss the induced tensor coupling in details, in the light of the recent experimental measurements.

In Section 2 we give the conventional Fujii-Primakoff approach and compare with our complete calculations. We conclude that while the qualitative agreement between the two approaches is good, the quantitative agreement is inadequate. We give reasons for the inadequacy in Section 4.

2. *Fujii–Primakoff approach*

The partial capture rate and the recoil nuclear polarization in Fujii–Primakoff approximation (hereafter referred to as FPA) [3] are given by

$$\Gamma_{\mathrm{FPA}} \propto (2J+1)G_A^2 + J(G_P^2 - 2G_P G_A) \ldots \tag{1}$$

and

$$\vec{P}_N^{\mathrm{FPA}} \propto (J+1)\{(2J+1)G_A^2 - 2JG_A G_P\}\vec{P}_\mu \ldots \tag{2}$$

where J is the final nuclear spin, P_μ is the polarization vector of muon in $1S$-atomic orbit and

$$\begin{aligned} G_A &= g_A - (g_V + g_M)\alpha, \\ G_P &= (g_P - g_A - g_V - g_M)\alpha, \end{aligned} \tag{3}$$

with $\alpha = \nu/2M$, ν being the neutrino momentum and M the nucleon mass. Substituting (3) in (1) and (2), FPA and P_N^{FPA} can be expressed in terms of g_V, g_M, g_A, g_P, the weak hadronic form factors, and their relative contributions can be evaluated. In analysing the relative contributions of the weak form factors in P_N^{FPA}, we have kept the denominator constant, as this is the capture rate, whose value can be taken from experimental data. The results for the ground to ground state partial transitions in ^{12}C and ^{16}O are given in the second row of Tables I and III respectively. The notation followed is: g_V^2 term is denoted by VV, g_A^2 by AA, $g_A g_V$ by AV etc. From the results, it is obvious that the dominant contribution is from the Gamow–Teller matrix element, the AA term. The interference terms AM and AP contribute almost on equal strength with opposite sign. Thus, as far as the partial capture rate is concerned, the effect of the induced pseudo-scalar interaction (though AP term, for example) is annuled by the weak-magnetism interaction (though AM term). The results for the average polarization of the recoil nucleus $^{12}B(1^+)$ and $^{16}N(2^-)$ are given in the second row of Tables II and IV respectively. Here also, the dominant interaction is the Gamow–Teller type (AA term). As in the case of the capture rate, the interference terms AM and AP contribute almost on equal strength with opposite sign. In both cases, the other terms like VM or VP contribute very little. Thus, in FPA, Gamow–Teller interaction is dominant in capture rate and recoil nuclear polarization and the effect of pseudo-scalar interaction is cancelled (almost) by weak-magnetism interaction. A similar analysis has been carried out for ^{12}C by Mukhopadhyay [15] whose results agree fairly

TABLE I

Percentage role of hadronic form factors in the partial capture rate in $\mu^- + {}^{12}C(0^+) \rightarrow {}^{12}B(1^+) + \nu_\mu$. First row represents complete calculation and second row represents FPA results

	g_A	g_V	g_M	g_P
g_A	88.6 95.2	1.04 4.8	21.4 17.9	−18.1 −24.4
g_V		0.03 0.09	0.5 0.7	
g_M			3.2 1.3	
g_P				4.2 4.4

with ours (Table I). The slight deviations are due to the choice of nuclear models. In Ref. [15] they have used Cohen and Kurath wave functions where as we have used the modified general 1p shell wave functions given by Hirooka et al. [7].

3. Complete calculations

The complete calculation consists in writing the full Hamiltonian for muon capture by proton (it should be noted that in FPA the nucleon-velocity-dependent terms are neglected and only *S*-wave neutrino is considered while we retain all the relativistic terms and higher partial waves for neutrino are considered) and applying *Low Density Approach* for nuclear case. The exact details of calculation are given in Ref. [4]. We use Kurath [5] wave functions for ^{12}C and ^{12}B after the modification due to Hirooka et al. [7] and Migdal [6] wave functions for ^{16}O and ^{16}N. Both these wave functions have been used by Hirooka et al. [7] and Rho [8] respectively and are found to give correct β-decay μ^--capture rates. As the complete details of calculations are complicated, we give the final results only. The partial transition rate is given by

$$\Gamma^C \propto g_A^2 \left\{ b_1 + \alpha(\alpha+2)b_2 - \frac{2}{M}(1+\alpha)b_3 \right\}$$

$$+ g_V^2 \left\{ \alpha^2 b_1 - \alpha^2 b_2 - \frac{2\alpha}{M} b_4 \right\}$$

$$+ g_M^2 \{ \alpha^2 b_1 - \alpha^2 b_2 \} + g_P^2 \alpha^2 b_2$$

$$+2g_M g_V\left\{\alpha^2 b_1-\frac{\alpha^2 b_4}{M}\right\}+2g_A g_V\left\{-\alpha b_1+\alpha b_2+\frac{b_4}{M}\right\}$$

$$+2g_A g_M\{\alpha b_2-\alpha b_1\}+2g_A g_P\left\{-\alpha^2 b_2-\alpha b_2+\frac{\alpha b_3}{M}\right\}, \quad (4)$$

and the average polarization is given by

$$\Gamma^C\vec{P}_N^C \propto \Bigg[g_A^2\left\{-a_1+2\alpha a_2-\frac{2a_3}{M}\right\}$$
$$+g_M^2\{-\alpha^2 a_1-2\alpha^2 a_2\}$$
$$+g_V^2\left\{-\alpha^2 a_1-2\alpha^2 a_2-\frac{2\alpha}{M}a_4\right\}$$
$$+g_V g_M\left\{2\alpha^2 a_1-2(\alpha^2-\alpha)a_2-\frac{2\alpha}{M}a_4\right\}$$
$$+g_V g_A\left\{2\alpha a_1-2(\alpha^2-\alpha)a_2+\frac{2\alpha}{M}a_3+\frac{2a_4}{M}\right.$$
$$\left.-\frac{2\alpha a_5}{M}-\frac{2a_5}{M}\right\}$$
$$+g_A g_M\left\{2\alpha a_1-2(\alpha^2-\alpha)a_2+\frac{2\alpha}{M}a_3\right\}$$
$$+g_A g_P\left\{-2\alpha a_2+g_V g_P 2\alpha^2 a_2+\frac{2\alpha}{M}a_5\right\}\Bigg]\vec{P}_\mu,$$

where $a_1, \ldots, a_5$ and $b_1, \ldots, b_4$ are the nuclear matrix elements given in Ref. [4]. For details of the evaluation of these nuclear matrix elements, see Refs [7, 8].

TABLE II

Percentage role of hadronic form factors in the recoil nuclear polarization in $\mu^-+{}^{12}C(0^+) \rightarrow {}^{12}B(1^+)+\nu_\mu$. First and second row have the same meaning as in Table I

	g_A	g_V	g_M	g_P
g_A	120.0 103.5	−0.5 5.4	1.75 19.9	−32.0 −25.3
g_V		−0.2 0.5	0.9 0.4	0.22 −0.99
g_M			0.05 0.70	
g_P			4.1 3.7	

TABLE III

Percentage role of hadronic form factors in the partial capture rate in $\mu^- + {}^{16}O(0^+) \rightarrow {}^{16}N(2^-) + \nu_\mu$. First and second rows have the same meaning as in Table I

	g_A	g_V	g_M	g_P
g_A	105.6 104.0	0.02 4.82	18 17.8	−31 −34.8
g_V		−0.1 0.1	0.4 0.7	
g_M			1.3 1.3	
g_P				5.3 5.9

The above nuclear matrix elements are evaluated using Kurath's [7] wave function for A = 12 system and Migdal's [6] wave functions for A = 16 system and then the individual contributions from the several hadronic form factors are calculated. The results for the partial capture rate in ^{12}C and ^{16}O are given in the first row of Tables I and III which those for the average polarisation of $^{12}B(1^+)$ and $^{16}N(2^-)$ are given in the first row of Tables II and IV. It is seen that here also, the dominant contributions come from the Gamow–Teller matrix element. However, the roles of the interference terms are interesting and is discussed in the next section.

4. *Numerical results and discussion*

From Tables I to IV, we notice that the FPA and complete calculations are qualitatively in agreement. The following quantitative discrepancies make the analysis interesting.

(*i*) In Table II (for $^{12}B(1^+)$ polarization), we find that the *AM interference term* gives negligible contribution to the average polarization of $^{12}B(1^+)$ when compared to the *AP interference term* in the case of complete calculation but not so in FPA. This is because the *AM term* involves the nuclear matrix element $\langle J || \sum_{n=1}^{A} j_l(vr_n)\}(y_l(\hat{r}_n) \times \nabla_1)_L \times \sigma_1\}_J || 0 \rangle$, for the l^{th} partial wave neutrino which is neglected in FPA. This matrix element suppresses the Gamow–Teller matrix element of *AM term* and so the later term becomes small. So, FPA is inadequate to analyse g_P as the $g_A g_M$ term cancels $g_A g_P$ but not so in complete calculation.

(*ii*) In Table IV (for $^{16}N(2^-)$ polarization), the *AM interference* term is comparable to that of *AP term* but in opposite sign. This is because of the fact that the AM term involves the nuclear matrix element referred to in (*i*) and because of the overlap of 1p and 1d radial wave functions in ^{16}O this nuclear matrix element itself is small. This is not the case with ^{12}C as 1p − 1p radial wave function overlap is maximum. As the AM and AP terms contribute equally with opposite sign for A = 16 system, even in the case of complete

TABLE IV

Percentage role of hadronic form factors in the recoil nuclear polarization in $\mu^- + {}^{16}O(0^+) \rightarrow {}^{16}N(2^-) + \nu_\mu$. First and second rows have the same meaning as in Table I

	g_A	g_V	g_M	g_P
g_A	103.3 103.4	0.123 4.95	18 18	−30 −31
g_V		0.03 0.03	0.6 0.2	0 −1.2
g_M			0.42 0.42	
g_P			4.2 4.6	

calculation, the average polarization of ${}^{16}N(2^-)$ is not a better candidate for form factor analysis.

(*iii*) In all the Tables I to IV, the *AV interference term* is large in FPA but small in our approach. This is because the AV interference term contains nuclear matrix elements b_1, b_2 and a_1, a_2 respectively for partial capture and polarizations and they retard the Gamow–Teller matrix element, in complete calculation.

We wish to draw the conclusion from (*i*) and (*ii*) that while qualitative agreement via the dominance of the Gamow–Teller matrix element with FPA is good, the quantitative disagreement via AM and AP interference terms is remarkable. For drawing conclusions about the induced pseudoscalar/tensor coupling, it is more reliable to consider the average polarization of ${}^{12}B(1^+)$. Oziewicz [10] has pointed out that one can look into angular correlation measurements rather than partial capture rate, for drawing conclusions about form factors. As the angular correlation measurements are not accurate enough, we resort to polarization measurements.

In the light of the above conclusion, we wish to draw a plausible value for the induced tensor coupling. The study of induced tensor coupling is of interest not only to nuclear physicists but also to particle physicists. Senju et al. [11] speculate that the existence of g_T indicates a possible internal structure for quarks, thus evincing interest in the *extended quark model*. Such a g_T is useful in discussing the *anomalies* in neutrino reactions. So it is timely to comment upon the induced tensor form factor in muon capture. Since our conclusion is that the average polarization of ${}^{12}B(1^+)$ is best suited, we compare our model calculations with the experimental measurement [12]. Possoz et al. [12] obtain a value for $g_P = (11 \pm 5) g_A$. In fact, what one actually obtains by such comparison is not simply g_P but the combination $(g_P + g_T)$, in muon capture. Assuming for the moment that induced pseudoscalar coupling is given by Goldberger–Treiman relation with possible Coulomb corrections [13], we predict $g_T \simeq (3.5 \pm 5) \times g_A$ or approximately of similar strength to the weak magnetism term. The above prediction is corroborated by the recent β-decay measurements [14].

5. *Conclusion*

We wish to conclude that both in FPA and in the complete calculations, the dominant contribution to partial capture rate and recoil polarization is from Gamow–Teller matrix element. However, the complete calculations show that for drawing conclusions about hadronic form factors, $^{12}B(1^+)$ recoil polarization is more reliable than $^{12}N(2^-)$ recoil polarization. Thus, by comparing with the experiment of Possoz et al. [12] we find g_T to be as high as g_M which agrees with β-decay experimental results [14].

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