SOME RADIATIVE DECAYS OF VECTOR MESONS AND BARYONS IN THE MIT BAG MODEL

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The radiative decays $\varrho \to \pi\gamma$, $\omega \to \pi\gamma$, $K^* \to K\gamma$, and $\Delta^+ \to p\gamma$ are discussed in the MIT bag model. It is shown that except for the case of $\omega \to \pi\gamma$ the calculated results are generally in rough agreement with experiment. The connection with magnetic moment calculations in the bag model is stressed.

Particular attention [1–10] has been devoted in the literature to the MIT bag model. One of the main virtues of this model lies in providing a consistent relativistic framework in which the quark wave functions may be explicitly calculated at least in the cavity approximation. The model has been successfully applied to the calculations of the low lying hadron mass spectrum, static parameters of light hadrons, and various types of hadron decays. On the other hand, there are also other cases where the model does less well although it is still in rough agreement with experiment. For example, the magnetic moment of the proton as calculated in the bag model [3] reproduces the experimental value to about 30%. In addition there are also cases where the bag model seems to be in strong disagreement with experiment. An example of the latter is the treatment of the vector meson decays discussed in Ref. [5]. Hence, in order to investigate to what extent other vector meson decays as calculated in the bag model compare with experiment we proceed to discuss in this paper some of the most important radiative decays not considered in Ref. [5], namely $K^* \to K\gamma$, $\varrho \to \pi\gamma$ and $\Delta^+ \to p\gamma$. Furthermore, to compare our treatment with that presented in Ref. [5] we shall also consider the decay $\omega \to \pi\gamma$.

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We shall see that except for the case of $\omega \to \pi\gamma$ all of the remaining decays are in rough agreement with experiment and relate this to the fact that magnetic moments as calculated in the bag model are also in approximate agreement with experiment.

We shall start by writing down the quark and antiquark wave functions in the bag model. These are given by

$$\psi(\vec{r}) = \frac{N(x)}{\sqrt{4\pi}} \begin{pmatrix} i\alpha u \\ -\beta\vec{\sigma}\cdot\hat{\vec{r}}u \end{pmatrix} \tag{1}$$

in the case of quarks, and

$$\eta(\vec{r}) = \frac{N(x)}{\sqrt{4\pi}} \begin{pmatrix} -i\beta\vec{\sigma}\cdot\hat{\vec{r}}u \\ \alpha u \end{pmatrix} \tag{2}$$

in the case of antiquarks. In the above equation we have defined

$$\alpha = \left(\frac{\omega+m}{\omega}\right)^{1/2} j_0\left(\frac{xr}{R}\right) \tag{3}$$

and

$$\beta = \left(\frac{\omega-m}{\omega}\right)^{1/2} j_1\left(\frac{xr}{R}\right). \tag{4}$$

Furthermore, $N(x)$ is a normalization factor which is given by

$$N^{-2}(x) = R^3 j_0^2(x) \frac{2\omega\left(\omega-\frac{1}{R}\right)+\frac{m}{R}}{\omega(\omega-m)}, \tag{5}$$

with ω defined by

$$\omega = \frac{1}{R}[x^2+(mR)^2]^{1/2}, \tag{6}$$

where R is the radius of the spherical cavity, m is the quark or antiquark mass, and $x = x(m, R)$ is the smallest positive root of the equation

$$\tan x = \frac{x}{1-mR-[x^2+(mR)^2]^{1/2}}. \tag{7}$$

In order to compute the radiative decay $A \to B\gamma$ we then proceed to calculate the matrix element of the electromagnetic current γ_μ^{em} between the states A and B. We thus obtain

$$\langle B|j_\mu^{\text{em}}|A\rangle\varepsilon^\mu = \sum_i m_{BA}^i(\vec{k}), \tag{8}$$

where

$$m_{BA}^i(\vec{k}) = \int d^3x e^{i\vec{k}\vec{r}} \frac{N_B^i N_A^i}{4\pi} \psi_B^{i\dagger}(\vec{r})\gamma_0\gamma_\mu\psi_A^i(\vec{r})\varepsilon^\mu \tag{9}$$

in the case of quarks and

$$m^{i}_{BA}(\vec{k}) = \int d^3x e^{i\vec{k}\vec{r}} \frac{N^i_B N^i_A}{4\pi} \varphi^{i\dagger}_A(\vec{r})\gamma_0\gamma_\mu\varphi^i_B\varepsilon^\mu \tag{10}$$

in the case of antiquarks. In the preceding the sum in Eq. (8) extends over all the quarks and antiquarks contained in the states and the vector $\vec{k}$ in Eqs (8) to (10) denotes the photon momentum.

The procedure now is to make a Taylor expansion in $\vec{k}$ in Eqs (9) and (10). We then see that the 0th order contribution vanishes and as first nonvanishing contribution we thus obtain

$$m^i_{BA}(\vec{k}) = i(\vec{k}\times\vec{\varepsilon})\cdot\vec{\mu}^i_{BA} \tag{11}$$

with

$$\vec{\mu}^i_{BA} = \frac{N^i_B N^i_A}{4\pi}\int_{\text{Bag}} d^3x(\alpha^i_B\beta^i_A+\alpha^i_A\beta^i_B)\,\frac{r}{3}\,\mu^{i\dagger}_B\vec{\sigma}u^i_A. \tag{12}$$

For comparison we also note that the expression for the magnetic moment $\vec{\mu}_A$ of a baryon A in the bag model is given by [3]

$$\vec{\mu}_A = \sum_i \vec{\mu}^i_A \tag{13}$$

with

$$\mu^i_A = \int_{\text{Bag}} d^3x\,\tfrac{1}{2}\,\vec{r}x\psi^{i\dagger}_A(\vec{r})\vec{\alpha}\psi^i_A(\vec{r}), \tag{14}$$

where the sum in Eq. (13) extends over all the quarks contained in particle A.

Upon substitution of Eq. (1) in Eq. (14) we then obtain

$$\mu^i_A = \tfrac{2}{3}\,\frac{(N^i_A)^2}{4\pi}\int_{\text{Bag}} d^3x r\alpha^i_A\beta^i_A u^{i\dagger}_A\vec{\sigma}u^i_A. \tag{15}$$

We thus note the strong similarity between transition matrix elements for radiative decays and magnetic moments in the bag model. In fact

$$\vec{\mu}^i_{AA} = \vec{\mu}^i_A. \tag{16}$$

Furthermore, we also note the similarity of Eqs (11) and (16) to the analogous expressions which relate the transition matrix elements for radiative decays to the magnetic moment of the proton in the nonrelativistic quark model [12–14].

In order to compute the radiative decays $A \to B\gamma$ in the bag model it thus suffices to calculate explicitly the expression in Eq. (12) and then proceed in a fashion completely analogous to the nonrelativistic quark model.

We next write down the expression in Eq. (12) for the cases under consideration. In the case of $\Delta^+ \to p\gamma$, $\varrho \to \pi\gamma$ and $\omega \to \pi\gamma$ (i. e. for $A \to B\gamma$ with A and B nonstrange particles) we then obtain

$$\vec{\mu}^{i}_{AB} = \frac{1}{6}\frac{R_2^{3/2}}{R_1^{1/2}}\frac{x}{x-1}\frac{1}{\sin^2 x}\left\{\frac{R_1}{xR_2}\left[\frac{\sin x\left(\frac{R_2}{R_1}-1\right)}{x\left(\frac{R_2}{R_1}-1\right)} - \frac{\sin x\left(\frac{R_2}{R_1}+1\right)}{x\left(\frac{R_2}{R_1}+1\right)}\right]\right.$$

$$\left. + \frac{\cos x\left(\frac{R_2}{R_1}+1\right)}{x\left(\frac{R_2}{R_1}+1\right)} - \frac{\sin x\left(\frac{R_2}{R_1}+1\right)}{x^2\left(\frac{R_2}{R_1}+1\right)^2}\right\} u^{i\dagger}_B \vec{\sigma} u^i_A. \tag{17}$$

In the above equation R_1 and R_2 denote the bag radii corresponding to particles A and B respectively and the integration in Eq. (12) has been carried out over the bag with smallest radius. For the case of $A \to B\gamma$ (with A and B strange) an analogous calculation may also be performed, leading to obvious modifications of Eq. (17). The results for the vector meson decays are summarized in Table I.

TABLE I

Radiative decays of vector mesons. The decay widths computed in the nonrelativistic quark model are given in column 2 while the decay widths calculated in the bag model and the experimental values are presented in columns 3 and 4 respectively. All values are given in KeV

Radiative decays	$\Gamma^{\rm NR}_{\rm V\to p\gamma}$	$\Gamma^{\rm Bag}_{\rm V\to p\gamma}$	$\Gamma^{\rm exp}_{\rm V\to p\gamma}$
$\omega \to \pi\gamma$	1170	161	880
$\varrho \to \pi\gamma$	130	18	35 ± 10
$K^* \to K\gamma$	281	46	75 ± 35

We note from the preceding Table that the decay widths computed in the bag model are smaller than those calculated in the nonrelativistic quark model.

Before concluding we wish to present also our results for the case $\Delta^+ \to p\gamma$. We analyse this decay by itself since bag model calculations are most reliable in case of baryons while, for example, in the case of mesons the π meson mass is already badly reproduced in the bag model. Making use of Eq. (14) we thus obtain for the z component $\sum_i \mu^{i(z)}_{\Delta^+ p}$ of $\sum_i \vec{\mu}^{i}_{\Delta^+ p}$ the result

$$e\sum_i \mu^{i(z)}_{\Delta^+ p} = (0.65)\,\frac{2\sqrt{2}}{3}\,\mu_p, \tag{18}$$

where e denotes the electron charge and μ_p the experimental value for the magnetic moment of the proton.

The result in Eq. (18) is to be compared with the expression for $\sum_i \mu_{\Delta+p}^{iNR(z)}$ obtained in the nonrelativistic quark model, namely

$$e\sum_i \mu_{\Delta^+p}^{iNR(z)} = \frac{2\sqrt{2}}{3}\mu_p \tag{19}$$

as well as with the experimental value given by

$$e\sum_i \mu_{\Delta^+p}^{i\,\exp(z)} = (1.25\pm0.02)\frac{2\sqrt{2}}{3}\mu_p. \tag{20}$$

From Eqs. (18) and (20) we thus arrive at

$$\frac{\sum_i \mu_{\Delta^+p}^{i(z)}}{\sum_i \mu_{\Delta^+p}^{i\,\exp(z)}} = 0.52. \tag{21}$$

For purposes of comparison note also that the ratio in Eq. (21) is not much different from the ratio between the magnetic moment of the proton μ_p^{Bag} calculated in the bag model and its experimental value μ_p, namely

$$\frac{\mu_p^{Bag}}{\mu_p} = 0.68. \tag{22}$$

The discrepancy between the calculated value for the radiative decay $\Delta^+ \to p\gamma$ and its experimental value is thus related to the fact that the value calculated for the magnetic moment of the proton in the bag model is only in approximate agreement with experiment. The ratio in Eq. (21) is somewhat smaller than that in Eq. (22) due to the fact that Δ^+ and p have different bag radii.

In conclusion, we have computed the decay widths for the radiative decays $\varrho \to \pi\gamma$, $\omega \to \pi\gamma$, $K^* \to K\gamma$, and $\Delta^+ \to p\gamma$ in the bag model and found that except for the case of $\omega \to \pi\gamma$ the remaining decays are in rough agreement with experiment. This was then related to the fact that magnetic moments as calculated in the bag model reproduce the experimental results to about 30%. In contrast to the previously mentioned decays it is also apparent from our calculations that there is a manifestly large discrepancy between the value computed for the decay $\omega \to \pi\gamma$ in the bag model and its presently quoted experimental result.

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