PHOTONS AS A DETECTOR OF GRAVITATIONAL WAVES

By A. Kułak

Department of Solid State Physics, Academy of Mining and Metallurgy, Cracow*

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Physical and technical problems associated with the application of the photon-graviton resonance for detection of gravitational waves are discussed. An estimation of experimental possibilities in the meter range is given.

1. Introduction

Experiments concerning general relativity are based on measurement of small physical effects. The cause of big difficulties with the detection of gravitational waves is the small cross-section of the graviton-matter interaction. The method of detection suggested by J. Weber is based on measurement of mechanical vibrations excited in a quadrupole resonator by a gravitational wave. In such a method only a small part of the gravitational wave energy changes into mechanical energy, which can be measured by physical methods. The general theory of relativity predicts interaction of gravitational waves also with zero-rest mass particles. The photon-stationary gravitational field interaction is one of the most often measured effects of general theory of relativity. Particularly accurate measurements are obtained in the experiment applying the Mössbauer effect. The question is — "Is it possible to apply photons for detection of gravitational waves?" The application of photons for detection was suggested in 1971 by Braginsky and Mensky [1]. Since that time several works have been published [2, 3]. In this article a possible solution of experimental difficulties is suggested and also a research possibility is discussed.

2. Photon-gravitational resonance

To replace masses by photons in a gravitational detector one has to solve two problems. The first one is to find a configuration of the electromagnetic field which interacts strongly with a gravitational wave. The second one is to find an experimental method of measuring changes in the electromagnetic field through which we can measure interaction between the photon field and a gravitational wave.

* Address: Zakład Fizyki Ciała Stałego, Akademia Górniczo-Hutnicza, Al. Mickiewicza 30, 30-059 Kraków, Poland.

To solve the first problem we should find a resonance configuration in which the energy exchange between both fields is maximal. To solve the second problem one has to find a method of measurement in which limitations of accuracy would have only quantum character.

To see the possibility of resonant momentum exchange between wave field and a system of particles let us consider the motion of a quadrupole rotating in the plane perpendicular to the wave propagation direction. In the particular case when the quadrupole rotation period is one an a half times longer than the wave period, both quadrupole masses are in a force field having the same direction of action. This means that the angular momentum of the quadrupole will constantly decrease as a result of interaction with the wave field. This interaction has a resonance character. An exchange of momentum between the quadrupole and wave can last as long as the periods are adequately synchronized.

The position of a maximum of the accelerated quadrupole differs from the position of the moderated quadrupole by 90°.

We can expect a similar mechanism of momentum exchange when we replace the masses by photons. The photon path geometry suitable for this purpose is assured by a toroidal waveguide. Inside the waveguide the electromagnetic field can move only lengthwise along axis. We place the waveguide plane perpendicularly to the direction of propagation of a gravitational wave. If the wave propagates in the x^1 direction and is polarized in such a way that the axes x^2 and x^3 are polarization axes, the electromagnetic field moves in space-time given by the metric

$$ds^2 = c^2dt^2 - {dx^1}^2 - [1+h(x^1, t)]{dx^2}^2 - [1-h(x^1, t)]{dx^3}^2, \tag{2.1}$$

where $h(x^1, t)$ is the wave amplitude.

Let us consider two photons with the same momentum, which move inside the waveguide along two null geodesics placed near by each other. Since the motion of photons in the waveguide is not disturbed as long as they are not totally absorbed, their momentum is changed only by the gravitational field. When photons are placed in two points such that the time coordinate difference $\delta x^0 = 0$, the difference of their space positions $\delta x^\alpha \neq 0$ causes a momentum difference δp_i. This difference can be found with the help of a parallel displacement of one photon into a position of the second along the vector δx^α. The difference δp^i will constantly change during the motion of photons. In the interval of time $dx^0 = cdt$, δp^i will change by

$$d\delta p^i = -\frac{1}{p^0} R^i_{jkl} p^j \delta x^k p^l dx^0. \tag{2.2}$$

Expressing the space interval as $\delta x^\alpha = n^\alpha \delta x$, where n^α is a spacelike unit vector, using the notation $p^\alpha = n^\alpha p^0$, assuming that the gravitational field is weak and using the symmetry properties of the curvature tensor one finds that

$$\frac{d\delta p^0}{p^0} = -cR_{0\alpha 0\beta} n^\alpha n^\beta \delta x dt. \tag{2.3}$$

This equation expresses simultaneously the relation between the difference in frequencies of both photons.

On the waveguide circumference it is convenient to introduce the angular distance between photons $\delta\gamma$ so that $\delta x = r\delta\gamma$, where r is the waveguide radius. Let us assume that a waveguide is placed at $x^1 = 0$, and consider two groups of photons circulating the waveguide with angular velocity ω_0. The coordinates of both groups in the x^2, x^3 system connected with the wave polarization axes will be γ_1 and γ_2. Expressing the components of the curvature tensor by the wave amplitude $h(0, t)$ according to the equation

$$R_{0\alpha 0\beta} n^\alpha n^\beta = \frac{1}{4c^2} \ddot{h}(t) \cos 2\gamma \tag{2.4}$$

and integrating over $\delta\gamma$ we have from (2.3)

$$\frac{d\omega_F}{\omega_F} = \tfrac{1}{8} \frac{r}{c} \ddot{h}(t) \cos(\gamma_2 + \gamma_1) \sin(\gamma_2 - \gamma_1) dt, \tag{2.5}$$

where ω_F is the initial photon frequency. For two groups of photons separated by 1/4 of the circumference i.e. for

$$\gamma_1 = \omega_0 t, \qquad \gamma_2 = \omega_0 t + \pi/2,$$

the frequency displacement reaches the value

$$\frac{d\omega_F}{dt} = -\tfrac{1}{8} \frac{r\omega_F}{c} \ddot{h}(t) \sin 2\omega_0 t. \tag{2.6}$$

When the gravitational wave frequency $h(t)$ is close to $2\omega_0$, the right-hand side of the equation (2.6) does not oscillate and the relative change of the photon frequencies increases with time. After the interaction time t the photon frequency will be

$$\omega = \omega_F + \omega_\Delta \sin 2\omega_0 t, \tag{2.7}$$

where

$$\omega_\Delta = \int_0^t \left[\frac{d\omega_F}{dt}\right] dt$$

is the integral of equation (2.6). The interaction of photons with the wave field results in the change of their energy, which moves inside the waveguide with a group velocity near the velocity of light. In such a way r is connected with the angular velocity of motion of wave packets inside the waveguide

$$r = v_g/\omega_0. \tag{2.8}$$

In the case of interaction with monochromatic radiation $h(t) = h_0 \sin \omega_g t$ and the problem leads to the equation

$$\frac{d\omega_F}{dt} = \tfrac{1}{8} \frac{v_g}{c} \frac{\omega_F}{\omega_0} \omega_g^2 h_0 \sin \omega_r t \cdot \sin \omega_g t, \tag{2.9}$$

where $\omega_r = 2\omega_0$. As a result we obtain

$$\omega_\Delta = \frac{1}{4} \frac{v_g}{c} \omega_F \omega_r h_0 t I(\omega), \tag{2.10}$$

where $I(\omega) = \sin \Delta\omega t / \Delta\omega t$ and $\Delta\omega = \omega_r - \omega_g$. We can interpret the function $I(\omega)$ as a resonance curve of an interaction for the photon-gravitational resonance. It confirms our qualitative considerations, a maximum occurs for $\Delta\omega = 0$, that is for photons circulating the waveguide with frequency equal to a half of the gravitational wave frequency. A bandwidth of interaction curve is inversely proportional to the time of interaction of photons with the wave field.

The frequencies ω_F and ω_r are not independent in the case of resonance. It follows from boundary conditions for the field which fills in the circular waveguide that

$$N\lambda_{Fg} = 2\pi r, \tag{2.11}$$

where $\lambda_{Fg} = v_p / \nu_F$, v_p is a phase velocity of the electromagnetic wave inside the waveguide, and N is a natural number. From (2.8) and (2.11) follows that

$$\omega_F = \frac{N}{2} \frac{v_p}{v_g} \omega_r. \tag{2.12}$$

3. Detector construction

As we pointed out above, the resonance interaction results in the modulation of the electromagnetic field frequency in the waveguide. Along the waveguide circumference two groups of photons with increased and decreased frequencies are created and these groups are separated by 90° of its circumference (analogy to the accelerated and moderated mass quadrupole).

The connection between frequency and phase is

$$\frac{d\psi}{dt} = \omega. \tag{3.1}$$

Thus, taking into account (2.7), the phase change of the electromagnetic wave is

$$\psi = \omega_F t + \psi_\Delta \cos \omega_r t + \psi_0, \tag{3.2}$$

where $\psi_\Delta = \omega_\Delta / \omega_r$ and ψ_0 is initially selected as a free phase. The maximum of phase deviation occurs for the time difference $\Delta t = T_r/2 = 1/2\nu_r$. If interaction with a monochromatic gravitational wave takes place, the magnitude of phase oscillation is, according to (2.10),

$$\psi_\Delta = \frac{1}{4} \frac{v_g}{c} h_0 \omega_F t I(\omega). \tag{3.3}$$

Because of the expected magnitude of h_0, the modulation phase amplitude is very small and there is a problem in measuring it. Experimentally a phase oscillation of the order of 10^{-10}rd or less is expected. The measurement of such small displacements is possible only with the help of interference methods. In a resonance, according to (2.7) and (3.2), the amplitude of phase deviation has a maximum for the time difference $T_r/2$. This occurs at points of the waveguide separated by 1/4 of its circumference.

Interference measurements of the electromagnetic field phase deviations which occur between the waveguide points separated by 1/4 of its circumference are the basis of our detection method.

Let us consider a wave running along a nondispersive waveguide; the wave is described by the equation

$$\varphi(x, t) = \varphi_0 \cos\{\omega_F t - k_F x + \psi_F \cos(\omega_r t - k_r x)\}, \tag{3.4}$$

where φ_0 is, for example, the amplitude of the electric field and x is space coordinate along the waveguide axis. In non dispersive media the phase, as well as the phase modulation, are propagated with equal velocities $\omega_F/k_F = \omega_r/k_r = v_p = v_g = v$. Electromagnetic fields leaded out from two points with coordinates $x_1 = x_0$ and $x_2 = x_0 + \lambda_g/2$ of the waveguide will interfere as follows

$$\varphi_1(x_1, t) = \varphi_0 \cos\{\omega_F t - k_F x_0 + \psi_\Delta \cos(\omega_r t - k_r x_0)\},$$

$$\varphi_2(x_2, t) = \varphi_0 \cos\left\{\omega_F t - k_F x_0 - \frac{N}{2}\pi + \psi_\Delta \cos(\omega_r t - k_r x_0 - \pi)\right\}. \tag{3.5}$$

Using a summing interferometer ($\varphi_\Delta = \varphi_1 + \varphi_2$) in the case when $N = 4n+2$, or differential interferometer ($\varphi_\Delta = \varphi_1 - \varphi_2$) when $N = 4n$ (where n is natural number) we can obtain the interference amplitude as

$$\varphi_\Delta(t) = 2\varphi_0 \psi_\Delta \cos(\omega_r t - k_r x_0) \sin(\omega_F t - k_F x_0). \tag{3.6}$$

It is convenient to use in this experiment photons from the microwave range, other ranges are not suitable for this purpose. An interaction of photons with the gravitational wave

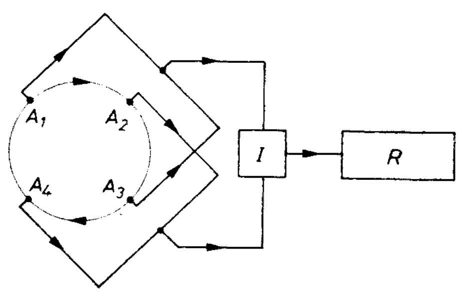

Fig. 1. Realization of phase detection process. A_1, A_2, A_3, A_4 — controlled microwave switches, I — interferometer system, R — microwave receiver

and also a separation of them can be made with the help of waveguides and microwave switches, which are easy to make for this frequency range. A full utilization of electromagnetic field inside the waveguide is assured in the detection system given in Fig. 1.

When microwave switches A_1, A_2, A_3 and A_4 operate simultaneously, photons from the circular waveguide will go to the external waveguides and will interfere in the interferometric system. The whole interference process will take 1/4 of circulation period of photons. Function (3.2) modulated in phase is changed into the expression (3.6) modulated in amplitude. After the Fourier transformation of the signal (3.6) we will obtain the detected signal spectrum

$$\varphi_\Delta(\omega) = \frac{i}{2}\,\varphi_0\psi_\Delta\{[\delta(\omega+\omega_R^+)-\delta(\omega-\omega_R^-)]+[\delta(\omega+\omega_R^-)-\delta(\omega-\omega_R^-)]\}. \tag{3.7}$$

The signal detected in the interferometer output occurs in two frequency ranges $\omega_R^+ = \omega_F+\omega_r$ and $\omega_R^- = \omega_F-\omega_r$. The receiving circuit should work in one of these ranges. It makes possible accurate measurements of the power of the detected signal. The measurement of power of this signal allows one to determine accurately the gravitational radiation flux; to this end it is necessary to know additionally the electric field amplitude and the interaction time. Such a measurement can be made with the help of conventional methods of microwave techniques.

4. *Limiting observation possibilities*

The estimation of observation possibilities of above detection method requires additional assumptions. At first we assume that the detector waveguide and microwave switches are lossless elements; lack of losses in the motion of photons allows one to carry on their interaction with a gravitational wave through an arbitrarily long time. The second assumption is that in experiment there are no factors which disturb the phase distribution of photons during the circulation period and in the phase detection process.

Let us determine the power which one obtains as a result of interference of photons. This will allow us to settle the role of noises of a microwave receiver used in the experiment.

During the detection we try to keep a constant number of photons $\bar{n}$ being in interaction. It follows from previous considerations, particularly from the expression (3.6), that at interferometer output we have

$$\bar{n}_\Delta = \bar{n}\psi_\Delta^2 \tag{4.1}$$

photons with two frequency ranges $\omega_R = \omega_F \pm \omega_r$. Thus energy which we obtain as a result of interference equals

$$W_\Delta = \hbar\omega_F n_\Delta. \tag{4.2}$$

The energy of thermal noise of microwave receiver collected in this time is

$$W_R = kT_R\eta, \tag{4.3}$$

where T_R is the equivalent temperature of receiver noise and η is a coefficient dependent on the detection method applied. Limiting conditions of signal registration take place, when detection energy W_Δ becomes comparable with the energy W_R,

$$W_\Delta = W_R. \tag{4.4}$$

It follows from this condition and from the expression (3.3), in which the wave amplitude can be expressed through the energy flux

$$S = \frac{c^3}{32\pi G} |h_0|^2 \omega_g^2, \tag{4.5}$$

that minimal flux of gravitational radiation, limited by thermal noise of a microwave receiver, is

$$S = \frac{4c^3}{\pi G h} \frac{c^2 v_g}{v_p^3} \frac{kT_R \eta}{N^3 \omega_r \bar{n} t^2}. \tag{4.6}$$

For small T_R we can expect in the detector the quantum noise of phase. Number of photons and magnitude of phase are complementary quantities. Thus the accuracy of phase measurement in each quantum system is finite. The interferometer-receiver system fulfills an uncertainty relation of the form

$$\Delta\psi = \frac{1}{2\sqrt{\bar{n}}}, \tag{4.7}$$

where $\bar{n}$ is the number of photons in the measurement. The limiting range of the detector is obtained from the condition

$$\Delta\psi = \psi_A \tag{4.8}$$

and thus

$$S = \frac{2c^3}{\pi G} \frac{c^2}{v_p^2} \frac{\eta}{N^2 \bar{n} t^2}. \tag{4.9}$$

The expressions (4.6) and (4.9) are similar, only the thermal energy of the receiver kT_R has been replaced by $h\omega_F$. The quantum phase noise begins to play a role when

$$\omega_F > \frac{k}{h} T_R. \tag{4.10}$$

For the frequency $\nu_F = 10^{10}$ Hz (microwave range) a role of the phase noise becomes essential when $T_R = 0.5$ K. This temperature is lower than the temperature of noise of the best maser amplifiers operating in this frequency range. Thus the phase noise is not an essential limitation here. From (4.10) it results that for photons near the optical range with $\nu_F = 10^{14}$ Hz, the photon noise does play an important role. Since from (2.12) $\nu_F \sim \nu_r$, there are quantum limitations of detectable gravitational flux in the optical frequency range.

5. *Method of continuous interference*

The condition most difficult to fulfil is a sufficiently low photon absorption. The attenuation for the best switches is at least of the order of 10 per cent.

The absorption of photons by the detector circuit causes a considerable shortening of their time of interaction with a gravitational wave. Therefore a detector without switches would be considerably better. It is possible to construct such a detector if we can realize continuous detection instead of pulse detection. For this purpose one can lead out a small stream of photons from two points of the detector waveguide, Fig. 2. These photons can give continuously the detected signal after sending them to the interferometer. A constant

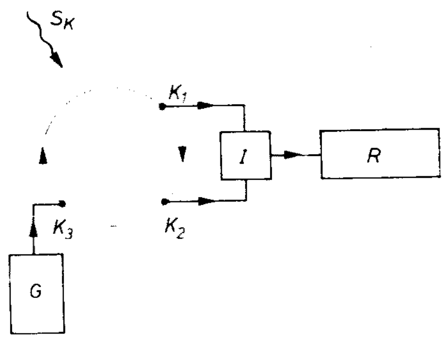

Fig. 2. Gravitational detector with continuous realization of the interference process. G — source of photons, K_1, K_2, K_3 — directional waveguide couers with a small coupling coefficient, I — interferometer system, R — microwave receiver

number of photons in circulation can be assured by an external source of photons. Besides the waveguide the only cause of photon losses in the detector system with continuous operation is a detecting process. We will prove that the optimal number of photons to be led out for detection is comparatively small. The probability that a photon travels a waveguide segment of length x equals [4]

$$p(x) = \exp\{-\alpha_{0F}x\}, \tag{5.1}$$

where α_{0F} is a quantity equal to the classical attenuation coefficient of the waveguide. The waveguide attenuation coefficient for one circulation is equal $\alpha_F = 2\pi r\alpha_{0F}$. The total attenuation coefficient for one circulation, taking into account loss of photons destined for detection, is

$$\alpha = \alpha_F + \alpha_\Delta, \tag{5.2}$$

where α_Δ is connected with the detected photons. It results from these considerations that during one circulation $\bar{n}(1-e^{-\alpha})$ photons leave the detector circuit ($\bar{n}$ is the number of photons in circulation) and the same number must be introduced into their place. The photons can be introduced in two ways: either by amplification with the help of an amplifier placed in the detector waveguide circuit or by injection from an external source (Fig. 3).

Let us consider the first possibility. Insted of photons which have already made a number of circulations in the gravitational wave field, new photons which do not carry any information about the wave are inserted. They can contribute to the detection only after a number of circulations, provided they are not absorbed before. Both the absorption and the creation of photons are stochastic processes, therefore the number of photons in the detector is a stationary stochastic variable $\hat{n}(t)$ with a mean value $\langle\hat{n}(t)\rangle = \bar{n}$. To describe the information meaning of photons in the detection process, it is convenient to consider groups of photons with exactly the same number of circulations around the

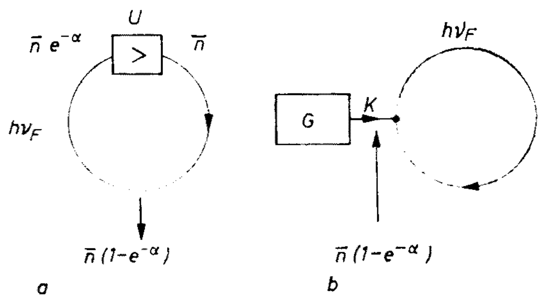

Fig. 3. Two methods of keeping the constant number of photons in the waveguide of a detector with continuous interference. a) detector system equiped with photon amplifier U, b) system with the external source of photons G

waveguide during the interaction with the gravitational wave. In this way a vector stochastic variable is formed

$$\{\hat{n}(1, t), \hat{n}(2, t), \ldots, \hat{n}(r, t), \ldots\}, \tag{5.3}$$

for which the mean value of its r-th component $\langle\hat{n}(r, t)\rangle = \bar{n}(r)$ and expresses the mean multiplicity of the group of photons which performed r circulations. Let us determine the power obtained in the detector output as a result of the interference, which continuously takes place. The right-hand side of Eq. (4.2) can be represented also as a function of the number circulations

$$P_\Delta(r) = P_0\psi_0^2\bar{n}r^2. \tag{5.4}$$

Now we can see the physical sense of the power equation. $P_0 = h\omega_F\omega_r/4\pi$ is the power carried by a single photon during one circulation and $\psi_0 = \pi v_p N h_0/2c$ indicates the mean phase displacement of each photon, which takes place in this time. This expression is true when all $\bar{n}$ photons have the same number of circulations r. When there are groups of photons which have some multiplicity distribution $\bar{n}(r)$ as a function of the number of circulations, detection power will be proportional to the second distribution moment M_2. In the interference process $\bar{n}(1-e^{-\alpha_\Delta}) = \bar{n}\gamma_\Delta$ photons take part: the power obtained in this process equals

$$P_\Delta = P_0\psi_0^2\bar{n}M_2\gamma_\Delta, \tag{5.5}$$

where

$$M_2 = \frac{1}{\bar{n}}\sum_{r=0}^{\infty} r^2\bar{n}(r).$$

One can prove that distribution of photons in the considered detector is such that

$$\bar{n}(r) = \bar{n}(1-e^{-\alpha})e^{-r\alpha}. \tag{5.6}$$

Thus $M_2 = e^{\alpha}/(e^{\alpha}-1)^2$. Equation (5.5) can be written also in the form

$$P_{\Delta} = \delta S.$$

The coefficient

$$\delta = P_0 \psi_0^2 M_2 \gamma_{\Delta} \tag{5.7}$$

expresses the effective surface of photon gravitational detector for a monochromatic radiation. Similarly as in the case of the pulse detector, we obtain the minimum of gravitational field flux from the condition

$$P_{\Delta} = kT_{\mathrm{R}}\eta_{\mathrm{R}}. \tag{5.8}$$

From expressions (5.5), (5.8) and (4.5) we obtain

$$S = \frac{c^3}{\pi^2 G\hbar} \frac{c^2 v_{\mathrm{g}}}{v_{\mathrm{p}}^3} \frac{kT_{\mathrm{R}}\eta_{\mathrm{R}}}{N^3 \bar{n} Q_{\alpha}}. \tag{5.9}$$

The function

$$Q_{\alpha} = M_2 \gamma_{\Delta} \tag{5.10}$$

describes summarily processes connected with losses of photons. This function has a maximum for some value of α_F/α_{Δ} and then the detector sensitivity is also maximal. In the range $\alpha \ll 1$

$$Q_{\alpha} = \frac{\alpha_{\Delta}}{(\alpha_F + \alpha_{\Delta})^2} \tag{5.11}$$

and the optimum relation is $\alpha_F/\alpha_{\Delta} = 1$. This means that only 1/2 of the photons absorbed in the waveguide takes part in the detection process. We should remember that the above expressions hold only when the stochastic variable $\hat{n}(t)$ is stationary. The variable $\hat{n}(t)$ becomes stationary after the characteristic time $t_n = 2M_1/v_r$ from the moment of switching on the detector; $M_1 = e^{-\alpha}/(1-e^{-\alpha})$ is the first moment of the distribution function $\bar{n}(r)$. For this reason the bandwidth of detector $I(\omega)$ is of the order

$$B_{\Delta} \approx \frac{1}{t_n} = \frac{v_r}{M_1} = \tfrac{1}{2} v_r \alpha. \tag{5.12}$$

The method described above can be used to register very small fluxes, especially in the case when superconductive materials are used for the construction of waveguide walls. For example, for a superconductive rectangular microwave waveguide with the sides ratio b and for the TE_{01} mode excited, the resulting average number of photons is

$$\bar{n} = \frac{v_{\mathrm{g}}^7}{c^5} \frac{E_0^2}{\xi^2 p Z_0 \hbar N^3 v_r^4},$$

where E_0 is the amplitude of the electric field, Z_0 is the impedance of the medium inside the waveguide. The group velocity $v_g = c\sqrt{1-\xi^2}$ and $\xi = \pi c/b\omega_F$. If we use photons with frequency $\nu_F = 50$ MHz, the observation frequency $\nu_g = 10$ MHz ($r = 6$ m), the electric field amplitude $E_0 = 10^{10}$ V/m, $\bar{n}$ reaches a value 10^{38}. This means that for the equivalent receiver temperature $T_R = 1$ K and the resultant attenuation coefficient $\alpha = 2 \cdot 10^{-13}$ it would be possible to observe the flux $S \sim 10^{-14}$ W/m^2 after a period of signal fixing $t_n = 10^6$ s.

Sensitivity of the detectors with continuous action is also limited because of quantum effects. To estimate the sensitivity, we should construct an expression similar to (5.5) expressing the level of the noise power of quantum origin as a function of the interferometer output. In this way the noise power connected with the group $\bar{n}(r)$ of photons can be written in the form

$$P_N(r) = P_0 \Delta\psi_r^2 \bar{n}(r) \gamma_\Delta, \tag{5.13}$$

where $\Delta\psi_r$ is the quantum uncertainty of phase due to measurement of the phase angle during detection of the signal made by this group of photons. This uncertainty according to (4.7) is

$$\Delta\psi_r = \frac{1}{2\sqrt{\bar{n}(r)}}. \tag{5.14}$$

Thus

$$P_N(r) = \tfrac{1}{4} P_0 \gamma_\Delta \tag{5.15}$$

and we obtain an important result: the level of noise power of quantum origin of the detector system equipped with an external photon generator does not depend on the multiplicity of photons $\bar{n}(r)$. The relation between the signal power and the noise power, according to (5.5) and (5.15), is

$$F = 4\bar{n}\psi_0^2 M_2. \tag{5.16}$$

Thus, when $F = 1$, the minimal radiation flux is

$$S = \frac{c^3}{32\pi^2 G} \frac{c^2}{v_p^2} \frac{\omega_r^2}{N^2 \bar{n} M_2}. \tag{5.17}$$

The minimal flux calculated on the basis of (5.17) is of the order of $2 \cdot 10^{-17}$ W/m^2.

6. *Comparison with other detection methods*

The estimations obtained above, concerning the minimum detectable radiation flux, differ not only from the observation possibilities in the case of mechanical detectors but differ also from the estimations obtained for other detection methods, including those using interaction of the gravitational field with the variable electromagnetic field.

Differences associated with various procedures are undoubtedly worthy of mention and that is why we will make a short review of the methods described up to now.

The first group of methods is described by Braginsky et al. [1, 2] and Grishchuk and Sazin [3].

The resonator configuration considered there has at least two proper vibration modes, which differ in frequency, for example: $\omega_n > \omega_m$. In such a configuration a gravitational wave of frequency $\omega_g = \omega_n - \omega_m$, can cause a transfer of energy from one mode of vibration to the other. Quanta of higer energy ω_n are created at the cost of gravitational energy.

The time dependent electromagnetic field amplitude of mode n consists of the initial amplitude $\varphi_n{}^{(0)}$ and a component of gravitational origin $\varphi_n{}^{(G)}$. If $\varphi_n{}^{(0)} = 0$ the energy increase of the mode n is

$$W_\Delta \sim [\varphi_n{}^{(G)}]^2 \sim t^2$$

and detection is made at "zero level", which allows one to detect the energy

$$W_\Delta = kT_R.$$

The quadratic t dependence of W does not hold for $\varphi_n{}^{(0)} \neq 0$. The energy change is now

$$W_\Delta \sim \Delta[\varphi_n{}^{(0)} + \varphi_n{}^{(G)}]^2 \sim \varphi_n{}^{(0)}\varphi_n{}^{(G)} \sim t,$$

which is small against the background energy $W_0 \sim [\varphi_n(0)]^2$. Signal determination condition has here the form

$$W_\Delta = 2\sqrt{W_0 kT_R}\,.$$

For both reasons this case is less advantageous then the previous one.

The second group of methods is based on the interaction of a gravitational radiation wave with wave packets circulating in a circular or other resonator which fulfils certain conditions [2]. The interaction of wave packets causes their energy to change at the rate

$$W_\Delta / W_0 \sim \omega_g t h_0.$$

That is why we can measure this change. However the rule

$$W_\Delta = 2\sqrt{W_0 kT_R}$$

again should be applied here. It is clear from the considerations above that it is convenient to get rid of the constant energy level in the detection process. Substraction of the electromagnetic fields, obtained at two waveguide points before an energy measurement, is made upon them, and is the only effective method here. Then their relative phase dependences play a fundamental role. A detection made in such a way was discussed above. Because during resonance there is a relative phase shift between both wave packets

$$\psi_\Delta \sim \omega_F t h_0,$$

we obtain a rate of energy change

$$W_\Delta \sim W_0 \psi_\Delta^2 \sim t^2$$

at "zero level". The essence of the interference detection is an amplification effect due to the release of some previously stored electromagnetic field energy that takes place as a result of interaction of the gravitational field with the detector. In other words, this interaction is sort of a controlling mechanism triggering off stored energy. The detection energy is proportional to t^2 while the interaction energy is linear in t; this is a consequence of the fact that the detection energy at the output of the interferometer results mainly from the electromagnetic field of the waveguide and is not the energy absorbed directly from the gravitational field.

Because of this interaction mechanism, the cross-section of the detector for interaction with gravitational radiation has here a little different sense than generally accepted. It can reach a large value on the basis of expression (5.7)

$$\sigma = \pi^2 l_g^2 N^2 \bar{n} Q_\alpha, \quad \text{where} \quad l_g = \sqrt{\frac{Gh}{c^3}}. \tag{6.1}$$

The dimensionless number

$$\alpha^* \sim \frac{l_g^2}{\lambda_g^2} N^3 \bar{n} Q_\alpha \tag{6.2}$$

characterizes the detector (it is a proportionality coefficient between the detection energy and the gravitational field flowing through the detector's geometrical surface).

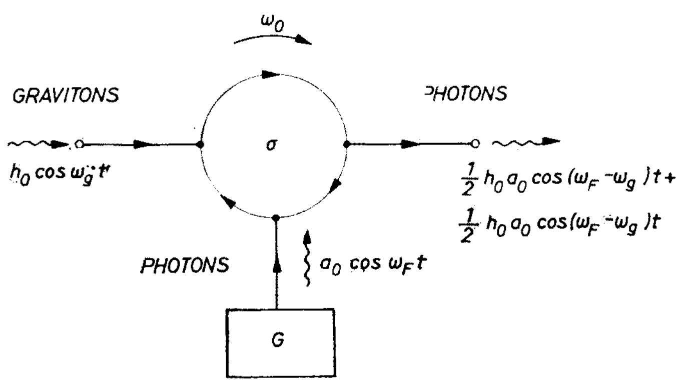

Fig. 4. Heterodyne properties of the photon gravitational detector

From the experimental point of view, the detection of interference has one more remarkable property which allows one to rank it in the class of heterodyne systems (Fig. 4). According to expression (3.6) the amplitude of the detector output signal

$$\varphi_\Delta \sim h_0 \varphi_0 \cos \omega_g t \cdot \cos \omega_F t \tag{6.3}$$

is a product of the amplitudes of the gravitational field and the electromagnetic field. The analogy to the heterodyne systems considered in circuit theory has a wider range. The gravitational photon detector has four characteristics of heterodyne systems:

1) cooperating with its microwave receiver must be tuned to one of two output frequencies $\omega_F + \omega_g$ and $\omega_F - \omega_g$;

2) frequency response of the cooperating receiver should be, considering a maximum of noise matching, equal to the heterodyne detector frequency response;

3) in the circuit there exists a noise process which limits the amplitude of registered signals. It is a quantum noise of phase;

4) properties of the detector can be expressed in the form of the transmittance function of the detector.

Property 1 is especially important from the experimental point of view, because in this case the detector is considerably less dependent on various disturbances. The only reason for the appearance of a coherent signal on the interferometer output can be a coherent gravitational interaction with a frequency determined exactly by the circulation period of photons in the waveguide.

The influence of the individual noise of the microwave receiver, losses of photons and photon noise discussed above do not exhaust all the physical phenomena taking place in the detector circuit. Under real experimental conditions the influence of the following factors should also be expected:

1) instability of frequency of the photon source;
2) changes in dimensions of the detector waveguide caused by thermic influences;
3) waveguide's mechanical vibrations;
4) thermic radiation through the waveguide walls;
5) gyroscopic effect caused by detector rotation in the waveguide plane;
6) quality of the waveguide walls.

Leaving the detailed consideration of each of the above points for individual study, we will only add that all of the factors mentioned above can be made negligibly small, so that finally the limit of sensitivity in the experiment is determined by the energy of receivers individual noise.

7. *Conclusions*

We conclude that astrophysical sources of gravitational radiation could not be detected by means of electromagnetic detectors of gravitational waves. This is mainly due to an unsatisfactory spectral range of the detectors as limited by the present technique. However, construction of these detectors would help in the investigation of laboratory gravitational wave sources.

Properties of an electromagnetic field as the gravitational radiation emitter are satisfactorily described in literature. Microwave resonator sets filled by suitably synchronized electromagnetic fields could be a source of gravitational radiation with strong directional properties and power sufficient for detecting by the method described above. Doroshkevich, Novikov and Polnaryov pointed out an interesting method of generating gravitational radiation. They take two wave packets located on opposite sides of a toroidal waveguide, as a simulation of a relativistic "dumb bell" which produces the synchrotron radiation [7]. Electromagnetic devices could be simple laboratory sources of coherent and directional gravitational radiation in the meter wave range.

Due to similar toroidal electromagnetic field configuration in the detector described above, the "dumb bell" set is suitable because of a possibility of using two identical waveguide devices for two points in the experiment. The gravitational radiation power emitted by the waveguide of this mode is approximately

$$P_0 \sim \frac{Gh^2}{4\pi^2 c^5} \frac{\omega_r^4 N^2 \bar{n}^2}{\Delta^{2/3}},$$

where $N = 2\omega_F/\omega_r$, ω_F is the average frequency of the packets, $\bar{n}$ is the number of photons, Δ is the angular width of the packets on the waveguide circumference. The emission frequency is twice as big as the circulation frequency of the packets $\omega_r = 2\omega_0$ and it is possible to change it by changing the waveguide radius or its electrical parameters. Limitations concerning $\bar{n}$ are the same as in the detector. Using the same waveguide of $N = 10$ and $\bar{n} = 10^{38}$, radiation power is of the order of $P_0 \sim 10^{-12}$ W which makes possible gravitational communication over distance of 10 m by detecting fluxes $S \sim 10^{-14}$ W/m^2. Thus, there is a possibility of testing the gravitational theory. This is associated, however, with considerable technical difficulties. The experiment, for instance shown above, is not easy, especially because of the big volume of microwave resonators that have to be cooled. The principle of detection described above could be used also if one could use other kinds of particles or quasiparticles to create a coherent beam in the toroidal waveguide. In each case, the arising phase oscillation due to the beams and gravitational interaction could be measured by interference of two particle beams which originate from two points separated by 1/4 of a waveguide circumference. The arising energy of this interference is proportional to the flux of gravitational radiation penetrating the waveguide plane.

However, it is possible to obtain such a propagation condition for these particles, that the increase of the phase oscillation against time t is much faster than a linear one. An example of such a possibility could be a detector using the skin acoustical wave which moves through an acoustical medium deformed by the interacting gravitational wave. Due to the parametrical interaction, the amplitude of phase oscillation of phonons increases like t^2 [8].

If similar effect existed in electrodynamics, the detector power would be proportional to the fourth distribution moment $P_\Delta \sim M_4 \gamma_\Delta \sim \alpha^{-3}$ which increases the detector cross--section by a factor of α^{-2}. This effect occurs when photons are moving in a dispersive medium ($N \gg 1$) and the group delay of two wave packets separated by 1/4 of the waveguide circumference increases with t^2: therefore its phase deviation increases with t^2 too.

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