COSMOLOGICAL IMPLICATIONS OF THE UNIFIED FIELD THEORY

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It is shown that the non-symmetric unified field theory of Einstein leads to a unique cosmological model which accounts well for the observed expansion of the universe. It is suggested that the theory may be empirically tested on the basis of its associated cosmology.

1. Introduction

It was shown in a previous article (Ref. [1]) that the metric tensor $a_{\mu\nu}$ of the non--symmetric unified field theory (e. g. Ref. [2]) can be obtained by equating Christoffel brackets formed from $a_{\mu\nu}$ to the symmetric part $\Gamma^{\lambda}_{\mu\nu}$ of the affine connection and regarding these equations as differential equations for the metric. As the result of this process the number of possible solutions of the field equations in the case of spherical symmetry is restricted to two of which only one appears to have physical meaning. Then, identifying the electromagnetic field tensor $f_{\mu\nu}$ with the skew symmetric part $\check{R}_{\mu\nu}$ of the generalised Ricci tensor, an identification which is virtually forced by the form of the solution, the electrostatic Coulomb law is seen to hold without modification. The corresponding, unique metric is

$$ds^2 = \omega\left(1+c\sqrt{\frac{r_0^2}{r^2}-1}\right)dt^2 - \frac{dr^2}{\left(1-\frac{r^2}{r_0^2}\right)\left(1+c\sqrt{\frac{r_0^2}{r^2}-1}\right)}$$
$$-r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{1}$$

where ω, c and r_0 are constants, the dimensionless c being related by the nature of the theory to the numerical strength of the electric charge. It is evident from (1) that the solution appears to be valid for

$$r < |r_0| \tag{2}$$

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only. Leaving for a moment a possible interpretation of this result, we should note that the apparent breakdown of the solution at $r = |r_0|$ is due exclusively to the particular choice of the radial co-ordinate. We shall show first that an alternative choice removes the singularity and that it can be made in such a way that the metric (1) approximates naturally to the Schwarzschild metric.

2. *Schwarzschild co-ordinates*

Let f be a function of r and let us write the relation (1) in the form

$$ds^2 = f^{-2}\left(\omega f^2\left(1+c\sqrt{\frac{r_0^2}{r^2}-1}\right)dt^2 - \frac{f^2 dr^2}{\left(1-\frac{r^2}{r_0^2}\right)\left(1+c\sqrt{\frac{r_0^2}{r^2}-1}\right)} - f^2 r^2 d\Omega^2\right), \tag{3}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Let g be a function of a new radial co-ordinate, say ϱ, such that

$$f^2\left(1+c\sqrt{\frac{r_0^2}{r^2}-1}\right) = g^2,$$

$$\frac{f dr}{\left(1-\frac{r^2}{r_0^2}\right)\left(1+c\sqrt{\frac{r_0^2}{r^2}-1}\right)} = \frac{d\varrho}{g}, \tag{4}$$

and $fr = \varrho$. Then

$$\frac{dr}{r^2\sqrt{1-\frac{r^2}{r_0^2}}} = \frac{d\varrho}{\varrho^2}, \tag{5}$$

whence

$$\varrho = \frac{r_0}{\sqrt{\frac{r_0^2}{r^2}-1}}, \quad f = \frac{1}{r_0}\sqrt{\varrho^2 + r_0},$$

$$g^2 = \frac{1}{r_0^2}(r_0^2+\varrho^2)\left(1+\frac{cr_0}{\varrho}\right). \tag{6}$$

With the new radial co-ordinate ϱ, the metric relation (1) becomes

$$ds^2 = \frac{r_0^2}{r_0^2+\varrho^2}\left[\left(1+\frac{\varrho^2}{r_0^2}\right)\left(1+\frac{cr_0}{\varrho}\right)d\tau^2 - \frac{d\varrho^2}{\left(1+\frac{\varrho^2}{r_0^2}\right)\left(1+\frac{cr_0}{\varrho}\right)} - \varrho^2 d\Omega^2\right], \tag{7}$$

where $\sqrt{\omega}t = \tau$. It clearly tends to the classical, Schwarzchild solution

$$ds^2 = \gamma d\tau^2 - \gamma^{-1} d\varrho^2 - \varrho^2 d\Omega^2, \quad \gamma = 1 - \frac{2m}{\varrho} \tag{8}$$

if we put

$$cr_0 = -2m \quad \text{and} \quad \varrho^2 \ll r_0{}^2. \tag{9}$$

3. Possible interpretation of the metric

It follows from the first of equations (6) that for small (with respect to r_0) values of r the two "radial" co-ordinates ϱ and r become approximately identical. According to (8) therefore, on the laboratory and astronomical (as distinct from cosmological!) scale, the predictions of the new theory are indistinguishable from those of classical electromagnetism and General Relativity. Electrostatics is still governed by the Coulomb law and the "tests" of general relativity are valid to a high degree of approximation. If our theory is correct, this constitutes an explanation why no local breakdown of the classical laws of macro-physics has ever been observed. In fact, one of the tests of general relativity, namely the gravitational red shift, holds accurately in the new interpretation. For a stationary (with respect to the observer) source of light, we have, from (7),

$$ds = \sqrt{1-\frac{2m}{\varrho}}\, d\tau \sim \left(1-\frac{m}{\varrho}\right) d\tau, \tag{10}$$

exactly.

Nevertheless, two important conclusions can be drawn from our results. The fact that we get a Schwarzschild solution on which the tests of General Relativity are based, for $r^2 \sim \varrho^2 \ll r_0^2$, indicates that the undertermined constant r_0 is of the order of a cosmological distance. Perhaps it should be identified with the radius of a finite universe. In that case, the characteristic mass $2m$, would become the "mass of distant stars" at least for large values of ϱ. We could then say with some justification that Coulomb law is valid throughout the universe. However, such an interpretation can only be tentative at this stage. All we can say with certainty is that r_0 is very large, at any rate, in comparison with the dimensions of the Solar System.

If, however, r_0 is not the radius of the whole universe, then we must distinguish physically between the "local" co-ordinate r and the "cosmological" co-ordinate ϱ. A surprising consequence as far as electromegnetism is concerned, follows. The solution, and hence also the influence of the (spherically symmetric) static field is real only up to a cut-off at $r = r_0$. Moreover, it is strictly a Coulomb field (Ref. [1], equation (5.9)). It is as if there existed a natural screening effect at that distance from the observer. On the other hand the effect is suppressed in the cosmological co-ordinates $(\varrho, \theta, \phi, \tau)$ (though, of course, the transformation between ϱ and r becomes singular at $r = r_0$).

It must be stressed that the theory proposed in this article and in Ref. [1] is very different from both General Relativity and the unified field theory of Einstein. It collapses into the former when skew symmetric part of the metric is removed from the structure of the theory. As it stands however, the metric and therefore geometry of the "background" Riemannian space are not determined by the field equations, as in General Relativity, but by a new "law" (Ref. [1], equations (3) or (4)). It is the remarkable correlation to the well-attested General Relativity, shown in the present work, which makes us confident that it may be the correct generalisation of the latter.

We shall investigate in the next section some of the properties of the background space and their "general relativistic" implications. It seems that confirmation or otherwise of the theory will rest on its cosmological meaning at least until more sophisticated, perhaps non-static, solutions of the field equations are found.

4. *The background space*

Let us now assume that the metric of the space-time (Riemannian) is given by equation (7) or

$$ds^2 = \left(1 - \frac{2m}{\varrho}\right) d\tau^2 - \frac{r_0^4 d\varrho^2}{(r_0^2 + \varrho^2)^2 \left(1 - \dfrac{2m}{\varrho}\right)} - \varrho^2 \left(1 + \frac{\varrho^2}{r_0^2}\right)^{-1} d\Omega^2 \tag{7'}$$

with $cr_0 = -2m$ and $2m < \varrho < +\infty$. For the purpose of calculation it is more convenient to introduce yet another coordinate, z, say,

$$(x^1, x^2, x^3, x^4) = (z, \theta, \phi, \tau)$$

by

$$\varrho = r_0 \tan z, \tag{11}$$

when

$$ds^2 = (1 + c \cot z) d\tau^2 - \frac{r_0^2 dz^2}{1 + c \cot z} - r_0^2 \sin^2 z d\Omega^2. \tag{12}$$

As z increases to $\pi/2$, ϱ, the "cosmological" radial coordinate tends to infinity. It is interesting to notice that when $z = \pi/2$, the metric becomes

$$ds^2 = d\tau^2 - r_0^2 (d\theta^2 + \sin^2 \theta d\phi^2). \tag{13}$$

This, of course, is a Minkowski metric whose spatial part is a Euclidean two-sphere of (constant) radius r_0. The surface of this sphere cannot be crossed by a signal sent by a "terrestrial" observer and can be regarded as the outermost boundary of the universe. What follows is based on the assumption that we have, so to say, left electromagnetic influence "far behind". Here again a difference from General Relativity may be observed. Cosmological models based on the latter, invariably involve some smoothing hypothesis about the distribution of matter, a "cosmological principle", for which the corresponding geometry is calculated from the field equations. We, on the other hand, have the metric provided for us, to be strict also by the field equations (and the "law" (4) of Ref. [1]), from which we can determine the properties or state of matter.

From equation (12) we easily find that the non-zero Christoffel brackets (of the second kind) are

$$\begin{Bmatrix} 1 \\ 11 \end{Bmatrix} = \frac{1}{2}\,\frac{c \operatorname{cosec}^2 z}{1+c \cot z},$$

$$\begin{Bmatrix} 1 \\ 22 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 33 \end{Bmatrix} \operatorname{cosec}^2 \theta = -\sin z \cos z(1+c \cot z),$$

$$\begin{Bmatrix} 1 \\ 44 \end{Bmatrix} = -\frac{\dot{c}}{2r_0^2} \operatorname{cosec}^2 z(1+c \cot z),$$

$$\begin{Bmatrix} 2 \\ 12 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 13 \end{Bmatrix} = \cot z, \quad \begin{Bmatrix} 2 \\ 33 \end{Bmatrix} = -\sin \theta \cos \theta,$$

$$\begin{Bmatrix} 3 \\ 23 \end{Bmatrix} = \cot \theta,$$

$$\begin{Bmatrix} 4 \\ 14 \end{Bmatrix} = -\frac{c \operatorname{cosec}^2 z}{2(1+c \cot z)}. \tag{14}$$

Let us now define the "Riemannian" Einstein tensor $G_{\mu\nu}$ by

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}\, a_{\mu\nu} R, \tag{15}$$

$a_{\mu\nu}$ being the metric tensor from (12) and the Ricci tensor $R_{\mu\nu}$, and Ricci invariant $R = a^{\mu\nu} R_{\mu\nu}$ being constant from the brackets (14). $G_{\mu\nu}$ is, of course, conserved identically

$$G^{\mu\nu}{}_{;\nu} \equiv 0, \tag{16}$$

if the covariant derivatives are constructed with the help of the Christoffel brackets (14) as components of the affine connection. Because of the "law"

$$\begin{Bmatrix} \lambda \\ \mu\nu \end{Bmatrix}_a = \Gamma^{\lambda}_{\underline{\mu\nu}},$$

((4), Ref. [1]), which enabled us to find the metric corresponding to the nonsymmetric structure of the unified field theory (Ref. [1]), equations (16) may be written also in the form

$$\overset{\mu\,\nu}{G^{00}}{}_{;\nu} = 0. \tag{17}$$

The non-vanishing components of the above Einstein tensor are

$$G_{11} = 1,$$
$$G_{22} = G_{33} \operatorname{cosec}^2 \theta = \sin^2 z(1+c \cot z), \tag{18}$$

and

$$G_{44} = -\frac{3}{r_0^2}(1+c \cot z)^2.$$

In view of the more general (than (17)) conservation law discussed in a separate publication (Ref. [3]) it may not be very meaningful to read too much physical significance into G_{11}, G_{22} and G_{33}. It is curious that the first component should be constant though G_{22} also tends to unity as z tends to $\pi/2$. This may imply that we have constant pressure "at infinity". On the other hand it is difficult to escape relating G_{44} to energy density D. Indeed, if as in General Relativity,

$$G_{44} = -\kappa a_{44} T_4^4 = -\kappa D a_{44},$$

then

$$\kappa D = \frac{3}{r_0^2}(1+c \cot z), \tag{19}$$

and tends to the general relativistic value for a spherically symmetric, static cosmology as z tends to $\pi/2$, that is as we approach what in the interpretation of a local observer is infinity.

We conclude this section by considering the geodesics at a cosmological distance from such an observer. If dots denote differentiation with respect to s and we suppress the ϕ co-ordinate, as we clearly may, by putting $\phi = \pi/2$, we get from (12)

$$\ddot{z} + \frac{c \operatorname{cosec}^2 z}{2(1+c \cot z)} \dot{z}^2 - \sin z \cos z(1+c \cot z)\dot{\theta}^2$$
$$- \frac{c}{2r_0^2}(1+c \cot z) \operatorname{cosec}^2 z \dot{\tau}^2 = 0,$$
$$\ddot{\theta} + 2 \cot z \dot{z}\dot{\theta} = 0, \tag{20}$$

and

$$\ddot{\tau} - \frac{c \operatorname{cosec}^2 z}{1+c \cot z} \dot{z}\dot{\tau} = 0,$$

together with the "first integral"

$$1 = (1+c \cot z)\dot{\tau}^2 - \frac{r_0^2}{1+c \cot z} \dot{z}^2 - r_0^2 \sin^2 z\dot{\theta}^2. \tag{21}$$

From the last two of equations (20)

$$\dot{\tau} = \frac{k}{1+c\cot z}, \qquad \dot{\theta} = \omega_0 \operatorname{cosec}^2 z, \tag{22}$$

where k and ω_0 are constants of integration. Hence, from (21)

$$\dot{z}^2 = \frac{k^2-1-c\cot z}{r_0^2} - \omega_0^2 \operatorname{cosec}^2 z(1+c\cot z), \tag{23}$$

For a signal therefore to reach observer infinity, we must have

$$k \geqslant \sqrt{1+r_0^2\omega_0^2}\,. \tag{24}$$

4. *Expansion of the universe*

The metric (7′) allows in certain cases expansion of the universe as far as a local observer is concerned and of course, only up to the absolute limit expressed by (13). It is of interest to see what these cases are and to compare the results with the Hubble expansion, say in a de Sitter space-time. To this end we consider again the geodesics of the last section, suppressing this time the angular , θ and ϕ dependence from the start. In the de Sitter case, we have, as is well known,

$$ds^2 = \left(1-\frac{r^2}{r_0^2}\right)dt^2 - \frac{dr^2}{1-\dfrac{r^2}{r_0^2}},$$

leading to

$$\ddot{r} = \frac{r}{r_0^2}, \tag{25}$$

(dots denoting differentiation with respect to s). We will now compare the (exact) formula (25), with the corresponding result obtained from

$$ds^2 = \left(1-\frac{2m}{\varrho}\right)d\tau^2 - \frac{d\varrho^2}{\left(1+\dfrac{\varrho^2}{r_0^2}\right)^2\left(1-\dfrac{2m}{\varrho}\right)}, \tag{26}$$

or

$$\ddot{\varrho} + \frac{m}{\varrho^2}\left(1+\frac{\varrho^2}{r_0^2}\right)^2 - \frac{2\varrho\dot{\varrho}^2}{r_0^2}\left(1+\frac{\varrho^2}{r_0^2}\right)^{-1} = 0. \tag{27}$$

Letting

$$u = \dot{\varrho}^2, \tag{28}$$

equation (27) becomes

$$\frac{d}{d\varrho}\frac{u}{\left(1+\frac{\varrho^2}{r_0^2}\right)^2} = -\frac{2m}{\varrho^2}, \tag{29}$$

on changing the independent variable from s to ϱ. Hence

$$\dot{\varrho}^2 = u = \left(k+\frac{2m}{\varrho}\right)\left(1+\frac{\varrho^2}{r_0^2}\right)^2, \tag{30}$$

and substituting into equation (27),

$$\ddot{\varrho}+\frac{m}{\varrho^2}\left(1+\frac{\varrho^2}{r_0^2}\right)^2 - \frac{2\varrho}{r_0^2}\left(k+\frac{2m}{\varrho}\right)\left(1+\frac{\varrho^2}{r_0^2}\right) = 0. \tag{31}$$

Hence for ϱ such that

$$2m \ll \varrho \ll r_0 \quad \text{and} \quad k > 0, \tag{32}$$

we obtaine approximately

$$\ddot{\varrho} = \frac{2k\varrho}{r_0^2}. \tag{33}$$

This is again a Hubble-like law of expansion. Except for its sign we cannot determine the constant k unless we indulge in some quesswork. In equation (25) r_0 is H^{-1} where H is the Hubble constant. We may perhaps be allowed to retain the same meaning for r_0 also in equation (33), that is as the value of ϱ when the velocity of recession of an extragalactic nebula becomes equal to the velocity of light $c(=1)$. The way in which the metric (26) was derived suggests that $2m$ represents the radius of the primeval atom. If we take density of matter as $10^{-28}-10^{-31}$ g cm^{-3}, and the radius as the distance of a Hubble horizon from the observer, inequality (32) becomes something like

$$\sim 10^{15}\ \text{cm} \ll \varrho \ll \sim 10^{30}\ \text{cm}. \tag{34}$$

In other words, according to (33), the Hubble law of expansion holds for relatively nearby galaxies.

It is curious to note that for galactic distances ($\varrho \ll r_0$), the expression (31) approximates very closely to Newtonian inverse square law that might hold as if the observer were located at the centre of the universe.

To get some idea what might be the actual value of k we must again change the independent variable in equation (30) from the proper times to the co-ordinate time τ. We have

$$\dot{\varrho}^2 = \dot{\tau}^2\left(\frac{d\varrho}{d\tau}\right)^2,$$

and, from (26),

$$\dot{\tau}^2 = \left(1-\frac{2m}{\varrho}\right)^{-1}\left[1+\left(1-\frac{2m}{\varrho}\right)^{-1}\left(1+\frac{\varrho^2}{r_0^2}\right)^{-2}\dot{\tau}^2\left(\frac{d\varrho}{d\tau}\right)^2\right]. \tag{35}$$

Eliminating $\dot{\tau}^2$ from equation (30), we find that

$$\frac{d\varrho}{d\tau} = \sqrt{\frac{k+2m/\varrho}{k+1}}\left(1-\frac{2m}{\varrho}\right)\left(1+\frac{\varrho^2}{r_0^2}\right). \tag{36}$$

Let us now suppose that

$$\varrho = r_0 \gg 2m \quad \text{when} \quad \frac{d\varrho}{d\tau} = 1, \tag{37}$$

so that r_0 is the actual radius of the universe, or the distance of the Hubble horizon from the observer. Then $k = 1/3$ and the (approximate) law of expansion becomes

$$\ddot{\varrho} = \frac{2\varrho}{3r_0^2} = H^2\varrho, \tag{33}$$

where H is the "corrected" Hubble constant. From (36) we now get for extragalactic distances

$$\frac{d\varrho}{d\tau} \approx \tfrac{1}{2}\left(1+\frac{3H^2}{2}\varrho^2\right).$$

Let us further suppose that $\varrho = 2m$ when $\tau = 0$. Then

$$\varrho = \frac{2m + \frac{\sqrt{6}}{3H}\tan\frac{\sqrt{6}\,H\tau}{4}}{1 - mH\sqrt{6}\tan\frac{\sqrt{6}\,H\tau}{4}}. \tag{38}$$

This represents an oscillating universe. However, as far as an observer is concerned, "infinity" is reached (presumably by a receding galaxy) after a time τ (from the initial "explosion") given by

$$mH\tan\frac{\sqrt{6}\,H\tau}{4} = 1/\sqrt{6},$$

or, approximately, when

$$\tau \sim \frac{2\pi}{H\sqrt{6}} \sim \frac{2}{H}. \tag{39}$$

It is difficult at this stage to speculate with any certainty whether this or double this time represents the actual period of oscillation of the universe. Clearly from our interpretation of r, it cannot become negative as the approximate formula (38) seems to require. Perhaps the modulus of the right hand side of (38) represents the actual, physical distance.

5. Conclusions

In this and the preceding articles (Refs [1, 3]) we have attempted a further reinterpretation of Einstein's nonsymmetric unified field theory based on the belief that it represents the correct extension of General Relativity to nongravitational fields (that is to electromagnetism, as the only other known macroscopic field; discussion of reasons for excluding nuclear fields is beyond the scope of our aims here). Our starting point was (Ref. [1]) solution of the metric problem. It is an additional postulate of the theory that the metric (tensor) should be determined from the differential equations obtained by equating the Christoffel brackets formed from the latter to the symmetric part of the affine connection derived from the field equations. Epistemologically this is no worse than regarding as the metric, the symmetric part of the fundamental tensor as Einstein would have it. The results obtained from our hypothesis however, are sufficiently surprising to suggest that we may be on the right track.

We saw (Ref. [1]) that one consequence of it was to limit the number of possible, static, spherically symmetric solutions of the field equations to just two. Moreover, we were unable to find physical meaning for one of these; it was somewhat fortuitous that this solution should be related to the presumably non-existent magnetic monopoles. (Strictly speaking this is an unwarranted conclusion, the theory merely implying that if magnetic monopoles do exist, they are either not spherically symmetric or have very unusual properties.) On the other hand, the second solution given by the metric (1) gave the exact, inverse square Coulomb law of force between static (symmetric) charges.

This in itself would be a considerable achievement as far as Einstein's theory is concerned. However, we have seen here that we can go further. If the integration constant r_0 can be interpreted as the radius of a finite universe then the cut-off of the electric field implied by the metric at this distance from an observer is not a serious problem. This then is the genesis of the cosmological interpretation of the metric described in the present work. We have seen that when the metric equation is written in the form (7), we obtain a cosmological model which has the remarkable property of reducing to a Schwarzschild space-time for a „local" mass source. We have already remarked that this fact coupled with the Coulomb law could be an explanation why it may be difficult to seek a local, that is a laboratory, or Solar-System-scale test of the unified field theory. It may be that discovery of cylindrically symmetric, or perhaps time-dependent solutions may reveal some peculiarity in, say, the structure of the Maxwell field which will lead to such a test. This however, is doubtful. Failing a local test we must look to galactic or cosmological consequences of the theory to seek its empirical confirmation. It is in this spirit that the present model is proposed.

It is interesting to note that the model of the universe based on the metric (7) arises apparently without appealing to a "cosmological principle". It may be held, of course, that such a principle is concealed in our interpretation of the cosmological co-ordinate ϱ. Even if this were granted however, it should be clear that we have here a very different kind of hypothesis from those habitually postulated in current cosmologies. We speculate on the interpretation of geometrical results instead of assuming a priori this or that distribution of matter. It is gratifying that we can derive conclusions which within admissible range of accuracy are not at variance with observational data. It is of course, another matter towards the boundaries of the visible world where we may expect our theory to be tested by a suitable analysis of current observations.

We should note in conclusion that a simple calculation of the Riemann-Christoffel tensor of the Riemannian space-time with the metric (7) shown that the relevant space-time does not admit hyperplanes of constant curvature. Hence our metric does not belong to the Robertson-Walker class of universes and can not be transformed into one of these.

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