ON THE POLARIZATION OF THE FINAL NUCLEUS IN THE (N, α) KNOCK-ON REACTION

By M. Kozłowski

Institute of Experimental Physics, Warsaw University*

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In this paper the polarization of the final nucleus in the (N, α) knock-on reaction is studied. Starting from the density matrix formalism the polarization transfer coefficients are computed. These coefficients describe the transfer of density matrix of the nucleon scattered on α particles. The capture of a polarized nucleon by the final nucleus gives the polarization vector of that nucleus.

1. *Introduction*

The motivation for the study of the knock-on mechanism of nuclear reactions can be stated as follows. First of all it was shown [1–3] that the knock-on mechanism is the leading mechanism in the preequilibrium emission of nuclear particles, especially α particles. The Feynman triangular diagram describes the amplitude of the knock-on reaction. The investigations of the structure of that diagram can give new information concerning the amplitude of the preequilibrium reaction. For example, this diagram gives automatically the asymmetry of the angular distributions of the emitteb particles in the preequilibrium nuclear reaction.

In this paper the polarization fo the final nucleus in the (N, α) knock-on reaction studied. Starting from the density matrix formalism the polarization transfer coefficients are computed. These coefficients describe the transfer of density matrix of the nucleon scattered on α particles. The capture of a polarized nucleon by the final nucleus gives the vector polarization of that nucleus.

One of the methods for studying this polarization is the investigation of the polarization of the nucleons emitted from the final nucleus. If during the knock-on reaction the IAR (isobaric analog state) is filled, then the products of the decay of IAR, the nucleons, can have vector polarization.

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* Address: Instytut Fizyki Doświadczalnej, Uniwersytet Warszawski, Hoża 69, 00-681 Warszawa, Poland.

2. *The non-dynamical description of knock-on reaction*

The knock-on reaction

$$2+W \to 3+1 \tag{1}$$

can be visualized as in Fig. 1. Particle 2 strikes particle 3′ in the field of nucleus W. In the second stage particle 2′ is captured by the core R to form the final nucleus 1. The

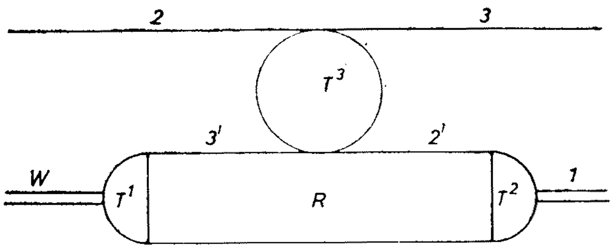

Fig. 1. The triangular diagram for the knock-on reaction

primes above the particles 2′ and 3′ denote that for these particles the off-shell effects can play an important role.

In order to describe the nondynamical structure of the amplitude of the reaction (1) we will use the mixed helicity-magnetic number basis for the spin states of particles and nucleus [4, 5]. For the heavy target nucleus W, the amplitude of the reaction (1) in the center of mass system of the particle W can be written as follows:

$$\begin{aligned} T_{\lambda_3 M_1 \lambda_2 M_W} &= \sum_{\lambda_3' \lambda_2' \lambda_R} \int T^3_{\lambda_2' \lambda_3 \lambda_2 \lambda_3'} T^1_{\lambda_3' \lambda_R M_W} T^2_{M_1 \lambda_2' \lambda_R} P_R P_{2'} P_{3'} d^3 p_{2'} dE_{2'} \\ &= \sum_{\lambda_3' \lambda_2' \lambda_R} \int T^3_{\lambda_2' \lambda_3 \lambda_2 \lambda_3'} T^1_{\lambda_3' \lambda_R M_W} T^2_{M_1 \lambda_2' \lambda_R} \\ &\quad \times [(p_R^2 - 2m_R E_R)(p_{2'}^2 - 2m_{2'} E_{2'})(p_{3'}^2 - 2m_{3'} E_{3'})]^{-1} d^3 p_{2'} dE_{2'}. \end{aligned} \tag{2}$$

In Eq. (2) the λ_i denote the helicities for the particles i and M_i denote the magnetic quantum numbers. The summation is carried out over the helicities of unobserved particles. The integration is carried out over the kinetic energy-momentum of the virtual particle 2′. The amplitude $T^3_{\lambda_2' \lambda_3 \lambda_2 \lambda_3'}$ denotes the amplitude for the quasi-elastic scattering

$$2+3' \to 3+2'. \tag{3}$$

The amplitude $T^1_{\lambda_3' \lambda_R M_W}$ and $T^2_{\lambda_2' \lambda_R M_1}$ are the amplitudes for the virtual decays

$$W \to 3'+R, \qquad 2'+R \to 1. \tag{4}$$

Let us suppose that the polarization of the initial state can be described by the density matrix $\varrho^{\mathrm{i}}_{\lambda_2 M_W \bar{\lambda}_2 \bar{M}_W}$. Then the density matrix for the nucleus 1 can be written as follows [4]

$$\frac{d\sigma}{d\omega}(2+W \to 3+1)\varrho^{1}_{M_1 M_1'}$$

$$= \sum_{\lambda_3\lambda_2} \sum_{\bar{\lambda}_2} \sum_{\bar{M}_W} \sum_{M_W} T_{\lambda_3 M_1 \lambda_2 M_W} \varrho^{\mathrm{i}}_{\lambda_2 M_W \bar{\lambda}_2 \bar{M}_W} \bar{T}_{\lambda_3 M_1' \bar{\lambda}_2 \bar{M}_W}, \tag{5}$$

where

$$\frac{d\sigma}{d\omega}(2+W \to 3+1)$$

$$= \sum_{\lambda_3\lambda_2} \sum_{\bar{\lambda}_2} \sum_{\bar{M}_W} \sum_{M_W M_1} T_{\lambda_3 M_1 \lambda_2 M_W} \varrho^{\mathrm{i}}_{\lambda_2 M_W \bar{\lambda}_2 \bar{M}_W} \bar{T}_{\lambda_3 M_1 \bar{\lambda}_2 \bar{M}_W}. \tag{6}$$

The decay amplitude for the process $W \to 3+R$ can be written as follows [5]

$$T^{1}_{\lambda_3' \lambda_R M_W} = N^{S_W} \bar{D}^{S_W}_{M_W, \lambda_3' - \lambda_R}(\Omega_{3'}) A^{S_W}_{\lambda_3' \lambda_R},$$

$$N^{S_W} = [(2S_W+1)/4\pi]^{1/2}. \tag{7}$$

In Eq. (7) Ω_3 ,denotes the Euler angles of the relative momentum of particles $3'$ and R in the center of mass system of the nucleus W. For the process $2'+R \to 1$ we assume that the reaction amplitude $T^2_{M_1 \lambda_2' \lambda_R}$ is Hermitian. This assumption means that we neglect the final state interaction in the decay vertex $1 \to 2'+R$ [4]. Bearing in mind this assumption we obtain [5]:

$$T^2_{M_1 \lambda_2' \lambda_R} = N^{S_1} D^{S_1}_{M_1 \lambda_2' - \lambda_R}(\Omega_{2'}) \bar{A}^{S_1}_{\lambda_2' \lambda_R}, \qquad N^{S_1}_{S} = [(2S_1+1)/4\pi]^{1/2}. \tag{8}$$

Substitution of (7) and (8) into Eq. (2) gives

$$T_{\lambda_3 M_1 \lambda_2 M_W} = N^{S_W} N^{S_1} \int \sum_{\lambda_3' \lambda_2'} \sum_{\lambda_R} T^3_{\lambda_2' \lambda_3 \lambda_2 \lambda_3'}$$

$$A^{S_W}_{\lambda_3' \lambda_R} \bar{A}^{S_1}_{\lambda_2' \lambda_R} \bar{D}^{S_W}_{M_W \lambda_3' - \lambda_R}(\Omega_{3'}) D^{S_1}_{M_1 \lambda_2' - \lambda_R}(\Omega_{2'})$$

$$[(p_R^2 - 2m_R E_R)(p_{2'}^2 - 2m_{2'} E_{2'})(p_{3'}^2 - 2m_{3'} E_{3'})]^{-1} d^3 p_{2'} dE_{2'}. \tag{9}$$

Finally from Eq. (9) we obtain for the density matrix elements of particle 1

$$\frac{d\sigma}{d\omega}(2+W \to 3+1)\varrho^{1}_{M_1 M_1'}$$

$$= \frac{(2S_1+1)(2S_W+1)}{16\pi^2} \sum_{\lambda_3 \lambda_2 \bar{\lambda}_2} \sum_{M_W \bar{M}_W} \sum_{\lambda_3' \lambda_2'} \sum_{\lambda'_3 \lambda'_2} \sum_{\lambda_R \lambda_R'} \Big(\int T^3_{\lambda_2' \lambda_3 \lambda_2 \lambda_3'}$$

$$A^{S_W}_{\lambda_3' \lambda_R} \bar{A}^{S_1}_{\lambda_2' \lambda_R} \bar{D}^{S_W}_{M_W, \lambda_3' - \lambda_R}(\Omega_{3'}) D^{S_1}_{M_1 \lambda_2' - \lambda_R}(\Omega_{2'}) P_R P_{2'} P_{3'} d^3 p_{2'} dE_{2'} \Big)$$

$$\varrho^{\mathrm{i}}_{\lambda_2 M_W \bar{\lambda}_2 \bar{M}_W} \Big(\int \bar{T}^3_{\lambda'_2 \lambda_3 \bar{\lambda}_2 \lambda'_3} \bar{A}^{S_W}_{\lambda'_3 \lambda_R'} A^{S_1}_{\lambda'_2 \lambda_R'}$$

$$D^{S_W}_{\bar{M}_W' \lambda'_3 - \lambda_R'}(\Omega_{3'}) \bar{D}^{S_1}_{M_1' \lambda'_2 - \lambda_R'}(\Omega_{2'}) P_{R'} P'_{2'} P'_{3'} d^3 p'_{2'} dE'_{2'}. \tag{10}$$

If only particle 2 is polarized then the density matrix of the initial state can be written as follows [4]

$$\varrho^{i}_{\lambda_2 M_W;\bar{\lambda}_2 \bar{M}_W} = \frac{1}{2S_W+1}\,\delta_{M_W \bar{M}_W}\varrho^{i}_{\lambda_2\bar{\lambda}_2}. \tag{11}$$

Substitution of (11) into Eq. (10) gives:

$$\frac{d\sigma}{d\omega}(2+W \to 3+1)\varrho^{1}_{M_1 M_1'} = \frac{2S_1+1}{16\pi^2}\sum_{\lambda_3\lambda_2\bar{\lambda}_2}\sum_{M_W}\sum_{\lambda_{3'}\lambda_{2'}}\sum_{\lambda'_{3'}\lambda'_{2'}}$$
$$\sum_{\lambda_R\lambda_R'}\left(\int T^{3}_{\lambda_{2'}\lambda_3\lambda_2\lambda_{3'}}A^{S_W}_{\lambda_{3'}\lambda_R}\bar{A}^{S_1}_{\lambda_{2'}\lambda_R}\bar{D}^{S_W}_{M_W\lambda_{3'}-\lambda_R}(\Omega_{3'})D^{S_1}_{M_1\lambda_{2'}-\lambda_R}(\Omega_{2'})\right.$$
$$\left.P_R P_{2'}P_{3'}d^3p_{2'}dE_{2'}\right)\varrho^{i}_{\lambda_2\bar{\lambda}_2}\left(\int \bar{T}_{\lambda'_{2'}\lambda_3\bar{\lambda}_2\lambda'_{3'}}\bar{A}^{S_W}_{\lambda'_{3'}\lambda'_R}A^{S_1}_{\lambda'_{2'}\lambda'_R}\right.$$
$$\left.D^{S_W}_{M_W\lambda'_{3'}-\lambda_R'}(\Omega_{3'})\bar{D}^{S_1}_{M_1'\lambda'_{2'}-\lambda_R'}(\Omega_{2'})P'_R P'_{2'}P'_{3'}d^3p'_{2'}dE'_{2'}\right). \tag{12}$$

For the completely unpolarized initial state we obtain for the density matrix of particle 1

$$\frac{d\sigma}{d\omega}(2+W \to 3+1)\varrho^{1}_{M_1 M_1'} = \frac{2S_1+1}{16\pi^2(2S_2+1)}\sum_{\lambda_3\lambda_2}\sum_{M_W}\sum_{\lambda_{2'}\lambda_{3'}}\sum_{\lambda_R\lambda'_R}$$
$$\sum_{\lambda'_{2'}\lambda'_{3'}}\left(\int T^{3}_{\lambda'_{2}\lambda_3\lambda_2\lambda_{3'}}A^{S_W}_{\lambda_{3'}\lambda_R}\bar{A}^{S_1}_{\lambda_{2'}\lambda_R}\bar{D}^{S_W}_{M_W\lambda_{3'}-\lambda_R}(\Omega_{3'})\right.$$
$$\left.D^{S_1}_{M_1\lambda_{2'}-\lambda_R}(\Omega_{2'})P_R P_{2'}P_{3'}d^3p_{2'}dE_{2'}\right)\left(\int \bar{T}^{3}_{\lambda'_{2'}\lambda_3\lambda_2\lambda'_{3'}}\bar{A}^{S_W}_{\lambda'_{3'}\lambda'_R}A^{S_1}_{\lambda'_{2'}\lambda'_R}\right.$$
$$\left.D^{S_W}_{M_W\lambda'_{3'}-\lambda'_R}(\Omega_{3'})D^{S_1}_{M'_1\lambda'_{2'}-\lambda_R'}(\Omega_{2'})P'_R P'_{2'}P'_{3'}d^3p'_{2'}dE'_{2'}\right). \tag{13}$$

From now on we will assume that the scattering process $2+3' \to 3+2'$ can be considered as quasi-free elastic scattering. The amplitude for the quasi-free elastic scattering can be written as follows:

$$T^{3}_{\lambda'_{2'}\lambda_3\lambda_{2'}\lambda'_{3'}} = A_{\lambda'_{2'}\lambda_3\lambda_{2'}\lambda'_{3'}}\delta(\Omega_{p_{2'}}-\Omega_{2'}). \tag{14}$$

If we use the reduction formulae for the $D^{S_i}_{M\lambda_i}(\Omega)$ function, we obtain

$$\bar{D}^{S_W}_{M_W\lambda_{3'}-\lambda_R}(\Omega_{3'})D^{S_W}_{M_W\lambda'_{3'}-\lambda_R'}(\Omega_{3'}) = \sum_{L_1}(-1)^{M_W-\lambda_{3'}+\lambda_R}\langle S_W S_W - M_W,\, M_W|L_1 0\rangle$$
$$\langle S_W S_W -\lambda_{3'}+\lambda_R,\, \lambda'_{3'}-\lambda'_R|L_1\alpha\rangle D^{L_1}_{0\alpha}(\Omega_{3'}),$$
$$D^{S_1}_{M_1\lambda_{2'}-\lambda_R}(\Omega_{2'})\bar{D}^{S_1}_{M_1'\lambda'_{2'}-\lambda_R'}(\Omega_{2'}) = \sum_{L_2}(-1)^{M_1'-\lambda'_{2'}+\lambda_R'}\langle S_1 S_1 M_1,\, -M_1'|L_2\beta\rangle$$
$$\langle S_1 S_1\lambda_{2'}-\lambda_R,\, -\lambda'_{2'}+\lambda_{R'}|L_2\gamma\rangle D^{L_2}_{\beta\gamma}(\Omega_{2'}), \tag{15}$$

where

$$\alpha = -\lambda_{3'}+\lambda_R+\lambda'_{3'}-\lambda'_R,\qquad \beta = M_1-M_1',\qquad \gamma = \lambda_{2'}-\lambda_R-\lambda'_{2'}+\lambda'_R.$$

Substitution of (15) into Eq. (13) gives

$$\frac{d\sigma}{d\omega}(2+W \to 3+1)\varrho^{1}_{M_1M_1'} = \frac{2S_1+1}{16\pi^2(2S_2+1)} \sum_{\lambda_3\lambda_2} \sum_{M_W} \sum_{\lambda_2'\lambda_3'}$$

$$\sum_{\lambda_R} \sum_{\lambda'_{2'}} \sum_{\lambda'_{3'}} \sum_{L_1L_2} (-1)^{M_W-\lambda'_{3'}+\lambda_R+M_1'-\lambda'_{2'}+\lambda_{R'}} \langle S_W S_W - M_W, M_W | L_1 0\rangle$$

$$\langle S_W S_W - \lambda_{3'} + \lambda_R, \lambda'_{3'} - \lambda'_R | L_1 \alpha\rangle \langle S_1 S_1 M_1, -M_1' | L_2 \beta\rangle$$

$$\langle S_1 S_1 \lambda_{2'} - \lambda_R, -\lambda'_{2'} + \lambda_{R'} | L_2 \gamma\rangle \iint A_{\lambda_{2'}\lambda_3\lambda_2\lambda_{3'}} \bar{A}_{\lambda'_{2'}\lambda_3\lambda_2\lambda'_{3'}}$$

$$A^{S_W}_{\lambda_{3'}\lambda_R} \bar{A}^{S_W}_{\lambda'_{3'}\lambda_{R'}} \bar{A}^{S_1}_{\lambda_{2'}\lambda_R} A^{S_1}_{\lambda'_{2'}\lambda'_R} P_R P_{2'} P_{3'} P'_R P'_{2'} P'_{3'}$$

$$D^{L_1}_{0\alpha}(\Omega_{3'}) D^{L_2}_{\beta\gamma}(\Omega_{2'}) d^2 p_{2'} d^2 p'_{2'} dE_{2'} dE'_{2'}. \tag{16}$$

Formula (16) gives the non-dynamical description of the knock-on reaction for the unpolarized initial state. All the dynamical information such as energy dependence of the vertices are included in the reduced amplitudes $A^{S}_{\lambda_i \ldots \lambda_f}$. However, in many instances we can obtain important information about the cross sections, polarization, etc. without knowledge of the above mentioned dynamical information.

Let us consider the knock-on of α particles by fast nucleons. Recently growing interest in experimental as well as theoretical investigations of these reactions is observed [6–8]. In the papers [3, 8] the knock-on mechanism is considered as the basic mechanism of the preequilibrium emission of α particles during the (N, α) reactions. The investigations of (N, α) reactions are limited only to the study of the angular distributions and energy spectra of α particles. In our paper we will consider polarization and alignment of the final nuclei in reactions of the type (N, α).

In the case of (N, α) reactions we assume that particle 3 is identical to the free particle: $3' = 3 = \alpha$, $S_3 = \lambda_3 = 0$. Eq. (16) can be written as follows

$$\frac{d\sigma}{d\omega}(2+W \to \alpha+1)\varrho^{1}_{M_1M_1'} = \frac{2S_1+1}{16\pi^2(2S_2+1)} \sum_{\lambda_2\lambda_2'} \sum_{M_W} \sum_{\lambda_R\lambda_R'} \sum_{L_1L_2}$$

$$(-1)^{M_W+\lambda_R+M_1'-\lambda'_{2'}+\lambda_{R'}} \langle S_W S_W - M_W, M_W | L_1 0\rangle \langle S_W S_W \lambda_R, -\lambda_{R'} | L_1 \alpha\rangle$$

$$\langle S_1 S_1 M_1, -M_1' | L_2 \beta\rangle \langle S_1 S_1 \lambda_2' - \lambda_R, -\lambda_{2'} + \lambda_{R'} | L_2 \gamma\rangle$$

$$D^{L_1}_{0\alpha}(\Omega_{3'}) D^{L_2}_{\beta\gamma}(\Omega_{2'}) \iint A_{\lambda_{2'}\lambda_2} \bar{A}_{\lambda'_{2'}\lambda_2} A^{S_W}_{\lambda_R} A^{S_W}_{\lambda_{R'}}$$

$$\bar{A}^{S_1}_{\lambda_{2'}\lambda_R} A^{S_1}_{\lambda'_{2'}\lambda_R'} P_R P_{2'} P_{3'} P'_{2'} P'_{3'} P'_R d^2 p'_{2'} d^2 p_{2'} dE_{2'} dE'_{2'}. \tag{17}$$

We can define the density matrix for particle 2′ when particle 2 is unpolarized [4]:

$$\varrho^{2'}_{\lambda_{2'}\lambda'_{2'}} \frac{d\sigma}{d\omega}(2+\alpha' \to \alpha+2') = \frac{1}{2S_2+1} \sum_{\lambda_2} A_{\lambda_{2'}\lambda_2} A_{\lambda'_{2'}\lambda_2}. \tag{18}$$

In (18) the λ_i have the meaning of the helicity quantum number in the center of mass system of the particles 2 and α'. Substitution of (18) into Eq. (17) gives

$$\frac{d\sigma}{d\omega}(2+W \to \alpha+1)\varrho^{1}_{M_1M_1'} = \frac{2S_1+1}{16\pi^2}\sum_{\lambda_2\lambda_2'M_W}\sum_{\lambda_R\lambda_R'}$$

$$\sum_{L_1L_2}(-1)^{M_W+\lambda_R+M_1'-\lambda'_2'+\lambda_R'}\langle S_WS_WM_W,\ -M_W|L_10\rangle\,\langle S_WS_W\lambda_R,\ -\lambda_{R'}|L_1\alpha\rangle$$

$$\langle S_1S_1M_1,\ -M_1'|L_2\beta\rangle\,\langle S_1S_1\lambda_{2'}-\lambda_R,\ -\lambda_{2'}'+\lambda_{R'}|L_2\gamma\rangle$$

$$\iint \varrho^{2'}_{\lambda_2'\lambda'_2'}\frac{d\sigma}{d\omega}(2+\alpha' \to 2'+\alpha)A^{S_W}_{\lambda_R}\overline{A}^{S_W}_{\lambda_{R'}}\overline{A}^{S_1}_{\lambda_2'\lambda_R}A^{S_1}_{\lambda'_2'\lambda_{R'}}$$

$$P_{2'}P_{3'}P_RP'_{2'}P'_{3'}P'_Rd^2p_{2'}dE_{2'}d^2p'_{2'}dE'_{2'}. \tag{19}$$

It is convenient to define the decay matrix elements

$$B_{\lambda_R\lambda_R'} = \sum_{M_W}\sum_{L_1}(-1)^{M_W+\lambda_R}\langle S_WS_W\lambda_R,\ -\lambda_{R'}|L_1\alpha\rangle\,\langle S_WS_W-M_W,\ M_W|L_10\rangle$$

$$A^{S_W}_{\lambda_R}\overline{A}^{S_W}_{\lambda_{R'}}D^{L_1}_{0\alpha}(\Omega_{3'}),$$

$$F^{\beta}_{\lambda_2'\lambda'_2'\lambda_R\lambda_R'} = \sum_{L_2}(-1)^{M_1'+\lambda_R'-\lambda_2'}\langle S_1S_1M_1,\ -M_{1'}|L_2\beta\rangle\,\langle S_1S_1\lambda_{2'}-\lambda_R,\ -\lambda'_{2'}+\lambda_{R'}|L_2\gamma\rangle$$

$$\overline{A}^{S_1}_{\lambda_2'\lambda_R}A^{S_1}_{\lambda'_2'\lambda_{R'}}D^{L_2}_{\beta\gamma}(\Omega_{2'}). \tag{20}$$

Due to symmetries of the Clebsch-Gordan coefficients we obtain for the matrix elements $B_{\lambda_R\lambda_R'}$

$$B_{\lambda_R\lambda_R'} = \sum_{M_W}\sum_{L_1(\text{even})}(-1)^{M_W+\lambda_R}\langle S_WS_W\lambda_R,\ -\lambda_{R'}|L_1\alpha\rangle$$

$$\langle S_WS_W-M_W,\ M_W|L_10\rangle A^{S_W}_{\lambda_R}\overline{A}^{S_W}_{\lambda_{R'}}D^{L_1}_{0\alpha}(\Omega_{3'}). \tag{21}$$

Taking into account the following well known property of Clebsch-Gordan coefficients

$$\langle S_WS_W-M_W,\ M_W|00\rangle = \frac{1}{(2S_W+1)}(-1)^{-S_W-M_W} \tag{22}$$

we obtain from Eq. (21)

$$B^{S_W}_{\lambda_R\lambda_R'} = |A^{S_W}_{\lambda_R}|^2\delta_{\lambda_R\lambda_R'} \equiv B^{S_W}_{\lambda_R}. \tag{23}$$

To obtain the final form of the formula (23) the orthogonality of the Clebsch-Gordan coefficients was used. For a parity conserving decay we have

$$|A^{S_W}_{\lambda_R}|^2 = |A^{S_W}_{-\lambda_R}|^2 \tag{24}$$

and also

$$B^{S_W}_{\lambda_R} = B^{S_W}_{-\lambda_R}. \tag{25}$$

If we use Eq. (25), the formula for the density matrix $\varrho^1_{M_1M_1'}$ can be written as follows

$$\frac{d\sigma}{d\omega}(2+W \to \alpha+1)\varrho^1_{M_1M_1'} = \frac{2S_1+1}{16\pi^2}\sum_{\lambda_2'}\sum_{\lambda'_2'\lambda_R}\sum_{L_2}(-1)^{M_1'+\lambda_R'-\lambda_2'}$$

$$\langle S_1S_1M_1, -M_1'|L_2\beta\rangle \langle S_1S_1\lambda_{2'}-\lambda_R, -\lambda_{2'}'+\lambda_R|L_2\gamma\rangle$$

$$D^{L_2}_{\beta\gamma}(\Omega_{2'})\iint \varrho^{2'}_{\lambda_2'\lambda'_2'}\frac{d\sigma}{d\omega}(2+\alpha' \to \alpha+2')G'B^{S_W}_{\lambda_R}\overline{A}^{S_1}_{\lambda_2'\lambda_R}A^{S_1}_{\lambda'_2'\lambda_R}, \tag{26}$$

where

$$G' = P_RP_{2'}P_{3'}P_R'P_{2'}'P_{3'}'d^2p_{2'}d^2p_{2'}'dE_{2'}dE_{2'}'.$$

Eq. (26) describes the density matrix elements for the final nucleus with spin S_1. The general formula for the density matrix of a particle with spin S_1 can be written as follows [4]:

$$\varrho^{S_1}_{M_1M_1'} = (2S_1+1)^{-1}\sum_{\Lambda=0}^{2S_1}\sum_{\mu=-\Lambda}^{\Lambda} q^{\Lambda}_{\mu}\{Q^{\Lambda}_{\mu}\}_{M_1M_1'}, \tag{27}$$

where the matrix elements of the polarization tensors are defined as follows:

$$\{Q^{\Lambda}_{\mu}\}_{M_1M_1'} = (-1)^{S_1-M_1'}(2S_1+1)^{1/2}\langle S_1S_1M_1', -M_1|\Lambda\mu\rangle.$$

After simple rearrangement of Eq. (26) we obtain

$$\varrho^{S_1}_{M_1M_1'} = (2S_1+1)^{-1}\sum_{L_2=0}^{2S_1} q^{L_2}_{\beta}\{Q^{L_2}_{\beta}\}_{M_1M_1'}, \tag{28}$$

where $(\beta = M_1-M_1')$

$$q^{L_2}_{\beta} = \left[\frac{d\sigma}{d\omega}(2+W \to \alpha+1)16\pi^2(2S_1+1)^{-3/2}\right]^{-1}\sum_{\lambda_2'\lambda'_2'\lambda_R}(-1)^{S_1+\lambda_R-\lambda_2'}$$

$$\langle S_1S_1\lambda_{2'}-\lambda_R, -\lambda_{2'}'+\lambda_R|L_2\gamma\rangle D^{L_2}_{\beta\gamma}(\Omega_{2'})$$

$$\iint \varrho^{2'}_{\lambda_2'\lambda'_2'}\frac{d\sigma}{d\omega}(2+\alpha' \to \alpha+2')B^{S_W}_{\lambda_R}\overline{A}^{S_1}_{\lambda_2'\lambda_R}A^{S_1}_{\lambda'_2'\lambda_R}G'. \tag{29}$$

In the subsequent discussion we will consider the (N, α) reaction on double even target nuclei. Moreover we will assume that during the (N, α) reaction the $(T-\alpha)$ nucleus rests in its ground state. Due to this assumption we obtain $\lambda_R = 0$ and Eq. (29) can be written as follows

$$q^{L_2}_{\beta} = \left[\frac{d\sigma}{d\omega}(2+W \to \alpha+1)16\pi^2(2S_1+1)^{-3/2}\right]^{-1}$$

$$\sum_{\lambda_2'\lambda'_2'}(-1)^{S_1-\lambda_2'}\langle S_1S_1\lambda_{2'}, -\lambda_{2'}'|L_2\gamma\rangle D^{L_2}_{\beta\gamma}(\Omega_{2'})$$

$$\iint \varrho^{2'}_{\lambda_2'\lambda'_2'}\frac{d\sigma}{d\omega}(2+\alpha' \to \alpha+2')\overline{A}^{S_1}_{\lambda_2'}A^{S_1}_{\lambda'_2'}G, \tag{30}$$

where $G = BG'$. The density matrix $\varrho^{2'}_{\lambda_2'\lambda'_2'}$ can be written as follows

$$\varrho^{2'}_{\lambda_2'\lambda'_2'} = (2S_2+1)^{-1/2} \sum_{n=0}^{2S_2} \sum_{m=-n}^{n} (-1)^{S_2-\lambda'_2'} q^n_m(S_2) \langle S_2 S_2 \lambda_{2'}, -\lambda'_{2'} | nm \rangle. \tag{31}$$

After the substitution of (31) into Eq. (30) we obtain for the $q^{L_2}_\beta$

$$q^{L_2}_\beta = C \sum_{\lambda_2'\lambda'_2'} \sum_{n=0}^{2S_2} \sum_{\gamma=-n}^{n} (-1)^{S_1-\lambda_2'+S_2-\lambda'_2'} \langle S_2 S_2 \lambda_{2'}, -\lambda'_{2'} | n\gamma \rangle \langle S_1 S_1 \lambda'_2, -\lambda'_{2'} | L_2\gamma \rangle$$

$$D^{L_2}_{\beta\gamma}(\Omega_{2'}) \iint \frac{d\sigma}{d\omega} (2+\alpha' \to \alpha+2') \overline{A}^{S_1}_{\lambda_2'} A^{S_1}_{\lambda'_2'} G q^n_\gamma(S_2), \tag{32}$$

where

$$C = \left[\frac{d\sigma}{d\omega} (2+W \to \alpha+1) 16\pi^2 (2S_1+1)^{-3/2} (2S_2+1)^{1/2} \right]^{-1}.$$

Let us define the polarization transfer coefficients $X^{L_2 n}_{\beta\gamma}$ [9]

$$X^{L_2 n}_{\beta\gamma} = C \sum_{\lambda_2'\lambda'_2'} (-1)^{S_1-\lambda_2'+S_2-\lambda'_2'} \langle S_2 S_2 \lambda_{2'}, -\lambda'_{2'} | n\gamma \rangle$$

$$\langle S_1 S_1 \lambda_{2'}, -\lambda'_{2'} | L_2\gamma \rangle D^{L_2}_{\beta\gamma}(\Omega_{2'}) \iint \frac{d\sigma}{d\omega} (2+\alpha' \to \alpha+2') \overline{A}^{S_1}_{\lambda_2'} A^{S_1}_{\lambda'_2'} G. \tag{33}$$

Using Eq. (33) one can write the polarization coefficients for the final nucleus 1 in a compact form

$$q^{L_2}_\beta = \sum_{n\gamma} X^{L_2 n}_{\beta\gamma} q^n_\gamma(S_2). \tag{34}$$

Eq. (34) shows that all polarization observables for the nucleus 1 can be derived from the quantities $X^{L_2 n}_{\beta\gamma}$.

It is easily shown from the symmetry properties of the amplitudes $A^S_{\lambda_i\lambda_f}$ and the symmetry properties of the Clebsch-Gordan coefficients that conservation of parity in the reaction implies:

$$\overline{X}^{L_2 n}_{-\beta-\gamma} = (-1)^{\gamma-\beta} X^{L_2 n}_{\beta\gamma}. \tag{35}$$

Furthermore for the polarization coefficients we obtain

$$X^{11}_{\beta 1} = D^1_{\beta 1}(\Omega_{2'}) A^{11},$$

$$X^{11}_{\beta 0} = D^1_{\beta 0}(\Omega_{2'}) B^{11}, \tag{36}$$

where

$$B^{11} = 2C(-1)^{S_1-1/2} \langle \tfrac{1}{2} \tfrac{1}{2} \tfrac{1}{2}, -\tfrac{1}{2} | 10 \rangle \langle S_1 S_1 \tfrac{1}{2}, -\tfrac{1}{2} | 10 \rangle \iint \frac{d\sigma}{d\omega} (2+\alpha' \to \alpha+2') |A^{S_1}_{1/2}|^2 G,$$

$$A^{11} = C\eta(-1)^{S_2+1/2} \langle S_1 S_1 \tfrac{1}{2}, \tfrac{1}{2} | 11 \rangle \iint \frac{d\sigma}{d\omega} (2+\alpha' \to \alpha+2') |A^{S_1}_{1/2}|^2 G. \tag{37}$$

The parameter η denotes the product of the parities $\eta(\mathrm{T}-\alpha)\,\eta(\mathrm{T}-\alpha+n)$.

Let us compute the polarization vector of the final nucleus with spin S_1. From Eq. (34) we obtain

$$q^1_\beta = \sum_{n\gamma} X^{1n}_{\beta\gamma} q^n_\gamma = X^{10}_{\beta 0} + \sum_{\gamma=-1}^{1} X^{11}_{\beta\gamma} q^1_\gamma(S_2). \tag{38}$$

It is easily shown from the symmetry properties of Clebsch-Gordan coefficients that when parity is conserved the $X^{L_2 n}_{\beta\gamma}$ obeys the relation

$$X^{L_2 n}_{\beta 0} = 0 \quad \text{for } n+L_2 \text{ odd.} \tag{39}$$

Bearing in mind Eq. (39) we obtain for the components of the polarization vector q^1_β

$$q^1_\beta = \sum_\gamma X^{11}_{\beta\gamma} q^1_\gamma(S_2). \tag{40}$$

If we pass to "cartesian" basis the components of the polarization vector can be written as follows [4]:

$$\xi_x = \frac{1}{\sqrt{2}}(q^1_{-1} - q^1_1) = \sqrt{2}\,\mathrm{Re}\,(\sum_\gamma X^{11}_{-1\gamma} q^1_\gamma(S_2)),$$

$$\xi_y = \frac{-i}{\sqrt{2}}(q^1_1 + q^1_{-1}) = \sqrt{2}\,\mathrm{Im}\,(\sum_\gamma X^{11}_{-1\gamma} q^1_\gamma(S_2)),$$

$$\xi_z = q^1_0 = \mathrm{Re}\,(\sum_\gamma X^{11}_{0\gamma} q^1_\gamma(S_2)). \tag{41}$$

The polarization coefficients $q^1_\gamma(S_2)$ are measured in the moving reference frame $(x'y'z')$ (see Fig. 2). The axes $(x'y'z')$ are obtained from the rest frame (XYZ) of the nucleus W by the Lorentz transformation $R(\varphi 90)L_z(v)$ [4]. The quasi-free scattering process $S_2+0 \to 0+S_2$ can lead to the polarization vector which has the following components in the reference frame $(x'y'z')$ [4]:

$$\xi_{x'}(S_2) = \xi_{z'}(S_2) = 0, \quad \xi_{y'}(S_2) \neq 0. \tag{42}$$

The result (42) means that the polarization vector $\xi(S_2)$ has the direction of the y'-axis and hence is perpendicular to the plane of reaction. Relation (42) can be written also for the $q^1_\gamma(S_2)$

$$q^1_1(S_2) = q^1_{-1}(S_2) = i\xi_{y'}/\sqrt{2}\,, \quad q^1_0(S_2) = 0. \tag{43}$$

Using (43) in Eq. (41) one finds

$$\xi_x = -A^{11}\xi_{y'}(S_2)\sin\varphi, \quad \xi_y = A^{11}\xi_{y'}\cos\varphi, \quad \xi_z = 0. \tag{44}$$

From Eq. (44) we conclude that the polarization vector of the final nucleus is perpendicular to the plane of reaction (see Fig. 2). Bearing in mind Eq. (37), we obtain from Eq. (44)

$$\xi_x = C\eta\langle S_1 S_1 \tfrac{1}{2}, \tfrac{1}{2}\,|11\rangle \sin\varphi$$

$$\iint \frac{d\sigma}{d\omega}(2+\alpha \to \alpha+2')\,|A^{S_1}_{1/2}|^2 G\xi_{y'}(S_2), \tag{45}$$

$$\xi_y = -C\eta\langle S_1 S_1 \tfrac{1}{2}, \tfrac{1}{2}|11\rangle \cos\varphi$$

$$\iint \frac{d\sigma}{d\omega}(2+\alpha \to \alpha+2')\,|A^{S_1}_{1/2}|^2 G\xi_{y'}(S_2), \qquad \xi_z = 0.$$

Let us suppose that the final nucleus with spin S_1 is in a quasi-stationary state, viz., in an isobaric analog state (IAR). The IAR decays with emission of a nucleon with spin 1/2. The decay of the IAR belongs to the class of the parity conserving decays,

$$S_1 \to \tfrac{1}{2} + S_3, \tag{46}$$

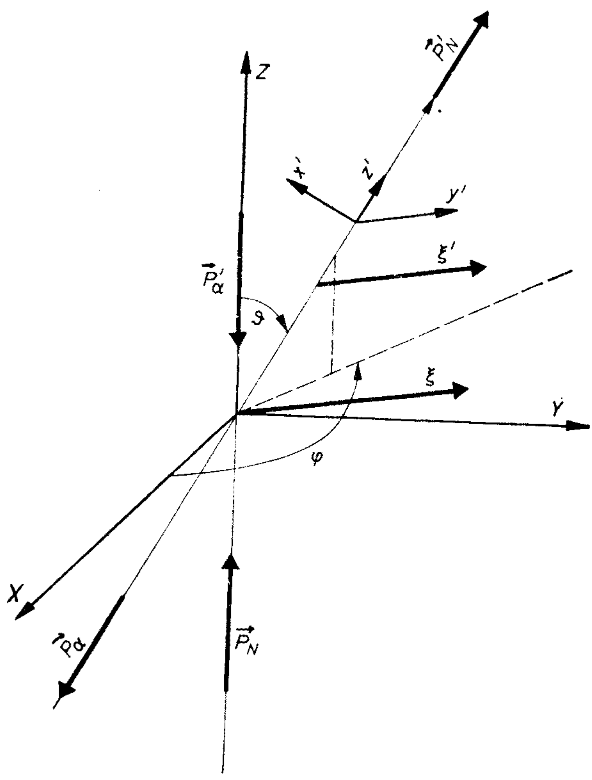

Fig. 2. The reference frame $x'y'z'$ obtained from the XYZ frame by the Lorentz transformation $R(\varphi\vartheta 0)L_z(v)$

where S_3 denotes the spin of the final stable nucleus with excitation energy lower than the threshold for particle emission. For simplicity we assume $S_3 = 0$. Let us consider the polarization of the emitted nucleon in the set of reactions $A(\text{N}, \alpha)B^*(C, \tilde{\text{N}})$:

$$0+\tfrac{1}{2} \to 0+S_1 \nearrow {}^{1/2+0}. \tag{47}$$

The polarization of the nucleus with spin S_1 is described by the polarization coefficients $q^{L_2}_\beta$. Let us denote the components of the polarization vector for the emitted nucleon by (p_x, p_y, p_z). It was demonstrated (see for example [5]) that the components of the polarization vector (p_x, p_y, p_z) can be written as follows:

$$\frac{dW}{d\omega_1} p_x = \frac{1}{2\pi}(2S_1+1)^{1/2}(-1)^{S_1-1/2} \sum_{L_2(\text{odd}),\beta} \langle S_1 S_1 \tfrac{1}{2}, \tfrac{1}{2} | L_2 1\rangle$$

$$\mathrm{Re}\,[q_\beta^{L_2} \bar{D}^{L_2}_{\beta,-1}(\Omega) C_{1/2} \bar{C}_{-1/2}],$$

$$\frac{dW}{d\omega_1} p_y = \frac{1}{2\pi}(2S_1+1)^{1/2}(-1)^{S_1+1/2} \sum_{L_2(\text{odd}),\beta} \langle S_1 S_1 \tfrac{1}{2}, \tfrac{1}{2} | L_2 1\rangle$$

$$\mathrm{Re}\,[i q_\beta^{L_2} \bar{D}^{L_2}_{\beta 1}(\Omega) C_{1/2} \bar{C}_{-1/2}],$$

$$\frac{dW}{d\omega_1} p_2 = \frac{1}{2\pi}(2S_1+1)^{1/2}(-1)^{S_1-1/2} \sum_{L_2,\beta} \langle S_1 S_1 \tfrac{1}{2}, -\tfrac{1}{2} | L_2 0\rangle q_\beta^{L_2}$$

$$\bar{D}^{L_2}_{\beta 0}(\Omega)\,[|C_{1/2}|^2 - (-1)^{L_2}|C_{-1/2}|^2]. \tag{48}$$

Because in the decay $S_1 \to 1/2+0$ the conservation of parity was assumed, we obtain (bearing in mind Eqs (48) and (34))

$$\frac{dW}{d\omega_1} p_x = \frac{1}{2\pi}(2S_1+1)^{1/2}\eta|C_{1/2}|^2 \sum_{L_2(\text{odd}),\beta} \sum_{n\gamma} \langle S_1 S_1 \tfrac{1}{2}, \tfrac{1}{2} | L_2 1\rangle$$

$$\mathrm{Re}\,[\bar{D}^{L_2}_{\beta,-1}(\Omega) X^{L_2 n}_{\beta\gamma} q^n_\gamma],$$

$$\frac{dW}{d\omega_1} p_y = -\frac{1}{2\pi}(2S_1+1)^{1/2}\eta|C_{1/2}|^2 \sum_{L_2(\text{odd}),\beta} \sum_{n\gamma} \langle S_1 S_1 \tfrac{1}{2} \tfrac{1}{2} | L_2 1\rangle$$

$$\mathrm{Re}\,[i\bar{D}^{L_2}_{\beta,-1}(\Omega) X^{L_2 n}_{\beta\gamma} q^n_\gamma],$$

$$\frac{dW}{d\omega_1} p_z = \frac{1}{2\pi}(2S_1+1)^{1/2}(-1)^{S_1-1/2}|C_{1/2}|^2 \sum_{L_2(\text{odd}),\beta} \sum_{n\gamma} \langle S_1 S_1 \tfrac{1}{2}, -\tfrac{1}{2} | L_2 0\rangle \bar{D}^{L_2}_{\beta 0}(\Omega) q^n_\gamma. \tag{49}$$

In Eq. (49) $dW/d\omega_1$ denotes the angular distribution of the nucleons produced in the decay $S_1 \to 1/2+0$,

$$\frac{dW}{d\omega_1} = \sum_{L_2(\text{even}),\beta} a_{L_2\beta} Y^{L_2}_\beta(\theta, \varphi), \qquad a_{L_2\beta} = b_{L_2} q^{L_2}_\beta,$$

$$b_{L_2} = \left(\frac{(2S_1+1)(2L_2+1)}{\pi}\right)^{1/2} \langle S_1 S_1 \tfrac{1}{2}, -\tfrac{1}{2} | L_2 0\rangle (-1)^{S_1-1/2}|C_{1/2}|^2. \tag{50}$$

From (39) we have

$$q^{L_2}_\beta = X^{L_2 0}_{\beta 0} = 0 \quad \text{for } L_2 \text{ odd}. \tag{51}$$

The selection rules (51) impose definite restrictions on the polarization of the emitted nucleon. The polarization vector p of the nucleon is not equal zero only if $q_{\gamma}^{1} \neq 0$. For the observation of the polarized nucleon which goes from parity conserving decay of the polarized final nucleus the following inequality must be fulfilled

$$\iint \frac{d\sigma}{d\omega}(2+\alpha' \to \alpha+2')\,|A_{1/2}^{S_1}|^2 G q_{\gamma}^{1} \neq 0. \tag{52}$$

It seems that the following conclusion can be formulated. During the (N, α) knock-on reaction the final nucleus has the polarization vector perpendicular to the reaction plane. The measurement of this polarization can be done, for example, by investigating the polarization of the particle products in the decay: IAR $\to$ nucleon + stable final nuclei.

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