ON QUANTUM ELECTRODYNAMICS IN AN EXTERNAL GRAVITATIONAL FIELD. II. DISCUSSION OF THE EFFECTS

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(Received January 26, 1978)

The S matrix constructed in Part I of this work is evaluated for processes which it includes. Some of them are discussed in more detail: pair creation and scattering in an external gravitational field, pair creation by a photon and creation of an electron-positron pair and a photon in an external gravitational field.

1. *Introduction*

With the construction of the S matrix terms (formulae (3.6), (3.9), (5.4) and (5.6) of the previous paper in this issue, hereafter referred to as I) the foundations are laid down for the study of the influence of an external gravitational field on quantum electrodynamical processes. The basis for the evaluation of these terms are gravitational fields in which quantized fields are in a flat spacetime for large spacelike and timelike distances. Now we turn to the evaluation of the S matrix for processes which it includes. In the course of these calculations we substitute for the "in"-operators the expansions (A.1)—(A.3) familiar from the Minkowski spacetime.

2. *The meson effects*

Substituting the expansion (A.1) into the S matrix (I–3.6) and normalizing the operator products one sees that the following processes are contained in (I–3.6) (the external gravita-

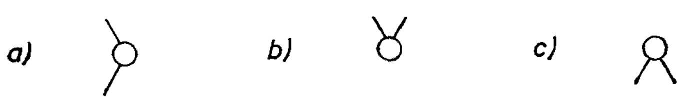

Fig. 1. Feynman diagrams for a) scattering, b) pair creation and c) pair annihilation of mesons in an external gravitational field

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tional field is symbolized by a circle and the external electromagnetic field by a cross; the time direction points out from below to above; all other conventions are as usual, see e.g. [2]) The calculation of the transition amplitudes ⟨final $|S|$ initial⟩ in momentum space leads to the following results:
for scattering (Fig. 1a)

$$\langle k'|S^{(1)}|k\rangle = \frac{ic}{8\pi}\frac{1}{\sqrt{\omega_k\omega_{k'}}}\left[f^{\mu\nu}(k-k')k_\mu k'_\nu + \frac{m_0^2c^2}{2\hbar^2}f(k-k')\right] \tag{2.1}$$

a nd for pair creation (Fig. 1b)

$$\langle kk'|S^{(1)}|0, \text{in}\rangle = \frac{ic}{8\pi}\frac{1}{\sqrt{\omega_k\omega_{k'}}}\left[-f^{\mu\nu}(k+k')k_\mu k'_\nu + \frac{m_0^2c^2}{2\hbar^2}f(k+k')\right]. \tag{2.2}$$

(The matrix element for the annihilation process simply follows from (2.2) by substituting $-(k+k')$ into the arguments of $f^{\mu\nu}$ and f.) If we specialize these transition amplitudes to the case of a weak Schwarzschild field with the aid of (I–2.8) we get

$$\langle k'|S^{(1)}|k\rangle = -\frac{i}{(8\pi)^2}\frac{c^2}{\hbar}\frac{M}{\omega_k}\frac{2\omega_k^2-m^2c^2}{\omega_k^2-m^2c^2}\frac{\delta(\omega_k-\omega_{k'})}{\sin^2\frac{\theta}{2}}, \quad \left(m \equiv \frac{m_0c}{\hbar}\right) \tag{2.3}$$

and

$$\langle kk'|S^{(1)}|0, \text{in}\rangle = -\frac{1}{(8\pi)^2}\frac{c^2}{\hbar}\frac{M}{\omega_k}\frac{2\omega_k^2-m^2c^2}{\omega_k^2-m^2c^2}\frac{\delta(\omega_k+\omega_{k'})}{\cos^2\frac{\theta}{2}}. \tag{2.4}$$

The argument of the delta function in (2.4) means that no pair creation can occur in a weak Schwarzschild field. In the framework of the S matrix theory this is a confirmation of the general perception that particles cannot be created by static gravitational fields. The delta function in (2.3) expresses the conservation of energy which has to be guaranteed because of the connection between invariance properties and conservation laws, of course, and which is the reason for the non-occurrence of pair creation.

Now we turn to the Robertson-Walker metrics (I–2.9) (respectively (I–3.7)). Instead of beginning directly with (2.1) (respectively (2.2)) using (I–3.10), we go back to the S matrix term (I–3.9) and, moreover, we consider the case of a stepwise expansion as given in (I–2.11). With the aid of the Fourier expansion (A.1) we obtain the S matrix up to the first order in Λ_2^0 (the superscript "0" shall indicate that in this model Λ_2 is stepwise constant) for the line element (I–3.7)[1]

$$S = 1 - \frac{i}{2}\frac{m_0^2c^4}{\hbar^2}\Lambda_2^0\int d^{(3)}k\left[\frac{\tau}{\omega_k}(a_k^\dagger a_k + a_k a_k^\dagger) + \frac{\sin 2\omega_k\tau}{2\omega_k^2}(a_k a_{-k} + a_k^\dagger a_{-k}^\dagger)\right]. \tag{2.5}$$

[1] The indices "in" at a_k^{in} etc. have been omitted for the sake of clearness.

If we start with the minimally coupled Klein-Gordon equation instead of the conformally invariant one, we get

$$S = 1 - \frac{ic^2}{2} \Lambda_2^0 \int d^{(3)}k k^2 \left[\frac{\tau}{\omega_k} (a_k^\dagger a_k + a_k a_k^\dagger) + \frac{\sin 2\omega_k \tau}{2\omega_k^2} (a_k a_{-k} + a_k^\dagger a_{-k}^\dagger) \right]. \tag{2.6}$$

The main difference between these two expressions is that (2.5) is proportional to the mass m_0 and (2.6) is not. The operator combinations occurring in (2.5) and (2.6) again indicate the processes represented in Fig. 1. Since particle creation can take place, the creation and annihilation operators for $t \to -\infty$ ($a_k^{\dagger\text{in}}$, a_k^{in}) and $t \to +\infty$ ($a_k^{\dagger\text{out}}$, a_k^{out}) differ but have to fulfil the same commutation relations. This difference between the operators is expressed by the S matrix:

$$a_k^{\text{out}} = S^{-1} a_k^{\text{in}} S. \tag{2.7a}$$

If we compare with the Bogoljubov transformation [6]

$$a_k^{\text{out}} = \alpha_k a_k^{\text{in}} + \beta_k a_{-k}^{\dagger\text{in}}, \tag{2.7b}$$

we see that the relation

$$\alpha_k \alpha_j^* - \beta_k \beta_j^* = \delta_{jk} \tag{2.8a}$$

holding between the Bogoljubov coefficients α_k and β_k, corresponds to the condition

$$[S^{-1} a_k^{\text{in}} S, S^\dagger a_{k'}^{\dagger\text{in}} (S^{-1})^\dagger] = \delta_{kk'} \tag{2.8b}$$

for the S matrix. This condition can be fulfilled only if the S matrix is unitary, $S^\dagger = S^{-1}$. With $S = 1 + \Lambda_2 S^{(1)}$ we get the connection between the S matrix and the Bogoljubov coefficients from (2.7),

$$\beta_k = \sum_{k'} \Lambda_2 [[S^{(1)}, a_{k'}^{\text{in}}], a_k^{\text{in}}] = \frac{im_0^2 c^4}{2\hbar^2} \Lambda_2^0 \frac{\sin 2\omega_k \tau}{\omega_k^2}, \tag{2.9}$$

$$\alpha_k = 1 - \sum_{k'} \Lambda_2 [[S^{(1)}, a_{k'}^{\text{in}}], a_k^{\dagger\text{in}}] = 1 - \frac{im_0^2 c^4}{\hbar^2} \Lambda_2^0 \frac{\tau}{\omega_k}, \tag{2.10}$$

where the results containing Λ_2^0 are valid for the S matrix (2.5). With the aid of these results we can represent the transition amplitudes for the processes given in Fig. 1, which could have been derived directly from (2.1) (respectively (2.2)), too, in the following manner:

meson scattering:

$$\langle k'|S|k\rangle = \frac{1}{\alpha_k} \langle 0, \text{in}|S|0, \text{in}\rangle \delta_{kk'}, \tag{2.11}$$

pair creation:

$$\langle kk'|S|0, \text{in}\rangle = \frac{\beta_k}{\alpha_k} \langle 0, \text{in}|S|0, \text{in}\rangle \delta_{-kk'}. \tag{2.12}$$

If we denote by P_c and P_s the probabilities for pair creation and scattering in the mode k,

$$P_c = \left|\frac{\beta_k}{\alpha_k}\right|^2 \quad \text{and} \quad P_s = \left|\frac{1}{\alpha_k}\right|^2$$

holds and the relation (2.8a) corresponds to $P_s = 1 - P_c$ which is compatible with $0 \leqslant P_c, P_s \leqslant 1$. Consequently scattering may be accompanied by pair creation in the same state.

The delta function in (2.11) and (2.12) expresses the conservation of momentum which has to be guaranteed because the spatial sections of the line elements (I–2.9) and (I–3.7) are Euclidean ones. The matrix element (2.11) gives the probability for the transition of the state $|k\rangle$ into $|k'\rangle$ caused by an expansion like (I–2.11). This probability is the larger the longer the expanded state lasts. This makes clear the proportionality to τ in (2.10).

Finally we notice that on the basis of the relations (2.7) it is possible to represent the complete S matrix for the influence of a gravitational field of the type (I–2.9) on a quantized Klein-Gordon field in terms of the Bogoljubov coefficients α_k and β_k. With the aid of the representation

$$\alpha_k = e^{i\gamma_\alpha} \cosh \vartheta_k \quad \text{and} \quad \beta_k = e^{i\gamma_\beta} \sinh \vartheta_k, \quad (\gamma \equiv \gamma_\alpha + \gamma_\beta) \tag{2.13}$$

of (2.8) and referring to Kamefuchi and Umezawa [3] we can write (compare [6])

$$S = \exp\left[\frac{i}{2}\int d^{(3)}k\gamma_\alpha(a^\dagger_{-k}a_{-k} + a_k a^\dagger_k)\right] \exp\left[\tfrac{1}{2}\int d^{(3)}k\vartheta_k(e^{i\gamma}a^\dagger_{-k}a^\dagger_k - e^{-i\gamma}a_k a_{-k})\right].$$

From (2.13) and (2.9), (2.10) the first two terms of the power series expansion in Λ_2^0 of this expression lead directly to (2.6). The applicability of the S matrix method carried out up to this point is not restricted to an isotropic expansion as (I–2.9). Without considerable difficulties one can generalize the method to an anisotropic expansion

$$ds^2 = \Omega_1^2(t)dx^2 + \Omega_2^2(t)dy^2 + \Omega_3^2(t)dz^2 - c^2dt^2, \tag{2.14}$$

where each of the functions $\Omega_i(t)$ has to fulfil the condition (I–2.10). If we assume that the expansion is stepwise in all the three directions as in (I–2.11) and that it begins and ends simultaneously for all directions, then we obtain for the Bogoljubov coefficient β_k describing the particle creation

$$\beta_k = \frac{ic^2}{6}\frac{\sin 2\omega_k\tau}{\omega_k^2}\sum_{i=1}^{3}\Lambda_{i,2}^0\left[(k^2 - 3k_i^2) + \frac{m_0^2c^2}{\hbar^2}\right],$$

which coincides with (2.9) in the isotropic case. Obviously, in the anisotropic case massless particles described by the conformally invariant Klein-Gordon equation can be created, too.

3. *The electron-positron processes*

The processes illustrated in the following refer to the influence of an external gravitational field on a Dirac field which is isolated from the Maxwell field. We go back to the S matrix terms (I–5.4b) and (I–5.6b). These terms are composed of the following processes:

Fig. 2. Feynman diagrams for a) scattering of an electron, b) electron-positron pair creation, c) electron-positron pair annihilation in an external gravitational field

We do not enter into the process of positron scattering. Using the expansions (A.2) we obtain the two essential matrix elements: electron scattering (Fig. 2a):

$$\langle +, \boldsymbol{p}', d' | S^{(1)} | +, \boldsymbol{p}, d \rangle = \frac{ic}{8\pi} \cdot \frac{1}{\sqrt{\omega_p \omega_{p'}}} \bar{u}^{(1,d')}(\boldsymbol{p}') \left[-f^{\mu\nu}(p-p') \right.$$

$$\left. \times \gamma_\nu (p_\mu + p'_\mu) + imf(p-p') \right] u^{(1,d)}(\boldsymbol{p}), \tag{3.1}$$

pair creation (Fig. 2b):

$$\langle +, \boldsymbol{p}, d; -, \boldsymbol{p}', d' | S^{(1)} | 0, \text{in} \rangle = \frac{ic}{8\pi} \frac{1}{\sqrt{\omega_p \omega_{p'}}} \bar{u}^{(1,d)}(\boldsymbol{p}) \left[f^{\mu\nu}(p+p') \right.$$

$$\left. \times \gamma_\nu (p_\mu - p'_\mu) + imf(p+p') \right] u^{(-1,d')}(\boldsymbol{p}'). \tag{3.2}$$

In the case of the weak Schwarzschild field the following expressions result from the preceding ones:

$$\langle +, \boldsymbol{p}, d; -, \boldsymbol{p}', d' | S^{(1)} | 0, \text{in} \rangle = \frac{iMc}{32\pi\hbar p^2} \frac{\delta(\omega_p + \omega_{p'})}{\cos^2 \frac{\theta}{2}} \bar{u}^{(1,d)}(\boldsymbol{p})$$

$$\left[-2\gamma^4 + \frac{imc}{\omega_p} \right] u^{(-1,d')}(\boldsymbol{p}') \tag{3.3}$$

and

$$\langle +, \boldsymbol{p}', d' | S^{(1)} | +, \boldsymbol{p}, d \rangle = -\frac{Mc}{32\pi\hbar p^2} \frac{\delta(\omega_p - \omega_{p'})}{\sin^2 \frac{\theta}{2}}$$

$$\times \bar{u}^{(1,d')}(\boldsymbol{p}') \left[-2\gamma^4 + \frac{imc}{\omega_p} \right] u^{(1,d)}(\boldsymbol{p}). \tag{3.4}$$

As expected the result (3.3) means that no electron-positron pair creation can occur in the weak Schwarzschild field.

If we consider the Robertson-Walker line element (I–3.7) we get for the case of a stepwise expansion, by substituting the operator expansions for the Dirac field into (I–5.6b)

$$S = 1 - imc^2 A_1^0 \int d^{(3)}p \sum_d \left[\frac{2m\tau}{\omega_p} (b_{p,d}^\dagger b_{p,d} + d_{p,d} d_{p,d}^\dagger) \right.$$

$$\left. + \frac{p}{\omega_p^2} \sin 2\omega_p \tau (b_{p,d}^\dagger d_{-p,-d}^\dagger + d_{-p,-d} b_{p,d}) \right]. \tag{3.5}$$

The connection between the S matrix and the Bogoljubov transformation yields

$$b_k^{\text{out}} = S^{-1} b_k^{\text{in}} S = \alpha_k b_k^{\text{in}} + \beta_k d_{-k}^{\dagger\text{in}}. \tag{3.6}$$

Taking into account the anticommutation relations for the operators of the Dirac field we obtain the Bogoljubov coefficients up to the first order in A_1:

$$\beta_k = -\sum_{k'} A_1 \{[S^{(1)}, b_{-k'}^{\text{in}}], d_k^{\text{in}}\} = -i \frac{mpA_1^0 c^2}{\omega_p^2} \sin 2\omega_p \tau, \tag{3.7}$$

$$\alpha_k = 1 - \sum_{k'} A_1 \{[S^{(1)}, b_{k'}^{\text{in}}], b_k^{\dagger\text{in}}\} = 1 - \frac{2im^2 A_1^0 \tau c^2}{\omega_p}. \tag{3.8}$$

The expressions containing A_1^0 refer to the S matrix (3.5) with a stepwise expansion. As in the case of the Klein-Gordon field we can represent the matrix elements (3.1) and (3.2) for Robertson-Walker metrics of the type (I–3.7) in the following manner: electron scattering:

$$\langle +, p', d' | S | +, p, d \rangle = \frac{1}{\alpha_k} \langle 0, \text{in} | S | 0, \text{in} \rangle \delta_{dd'} \delta(p - p'), \tag{3.9}$$

electron-positron pair creation:

$$\langle +, p, d; -, p', d' | S | 0, \text{in} \rangle = \frac{\beta_k}{\alpha_k} \langle 0, \text{in} | S | 0, \text{in} \rangle \delta_{d', -d} \delta(p + p'). \tag{3.10}$$

From this it is obvious that the probability for scattering (respectively pair creation) in one mode does not depend on what happens in other modes. By (3.9) again a forward scattering process is described. In connection with (3.10) the relation (3.7) means that in the case of an isotropic expansion no massless Dirac particles (neutrinos) can be created (cp. (2.9) for $m_0 = 0$). By considerations parallel to the Klein-Gordon field one can show that in the case of the Dirac field, scattering in a definite mode cannot be accompanied by pair creation in the same mode.

Unlike in (2.13) the anticommutator relations for the Dirac field can be represented by a real rotation in momentum space,

$$\alpha_k = \cos \vartheta_k \quad \text{and} \quad \beta_k = e^{i\varphi} \sin \vartheta_k.$$

Relying on investigations by Umezawa, Takahashi and Kamefuchi [9] we can give the complete S matrix in (3.6) the form

$$S = \exp\left[-2i\int d^{(3)}p \sum_d (\vartheta_1 T_1 + \vartheta_2 T_2)\right]. \tag{3.11}$$

Therein we have used the abbreviations

$$2T_1 \equiv b_k^{\dagger\text{in}} d_{-k}^{\dagger\text{in}} + d_{-k}^{\text{in}} b_k^{\text{in}}, \quad 2T_2 \equiv d_k^{\text{in}} d_{-k}^{\dagger\text{in}} + b_{-k}^{\dagger\text{in}} b_k^{\text{in}}$$

and

$$\vartheta^2 = \vartheta_1^2 + \vartheta_2^2, \quad \vartheta_2 - i\vartheta_1 = \vartheta e^{i\varphi}.$$

If we determine ϑ and φ from (3.7) and (3.8), the power series expansion of (3.11) up to the first order in Λ_1 coincides with (3.5).

After tedious calculations we obtain for an anisotropic expansion the result

$$\beta_k = \frac{c^2}{\omega_p^2}\sin 2\omega_p\tau \left\{\frac{\omega_p}{mpc}\frac{1}{\sqrt{p_1^2+p_2^2}}\left[pp_1p_2(\Lambda^0_{2,1}-\Lambda^0_{1,1}) + ip_1^2p_3d(\Lambda^0_{1,1}-\Lambda^0_{3,1})\right.\right.$$
$$\left.\left. + ip_2^2p_3d(\Lambda^0_{2,1}-\Lambda^0_{3,1})\right] - \frac{im}{3p}\sum_i \Lambda^0_{i,1}(p^2-3p_i^2) - \frac{ipm}{3}\sum_i \Lambda^0_{i,1}\right\}.$$

This result says that Dirac neutrinos ($m = 0$) can be created by anisotropically expanding gravitational fields. In the isotropic case this expression reduces to (3.7).

4. *The photon effects*

The S matrix term (I–5.4c) contains the following processes:

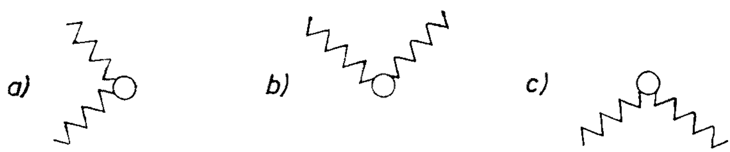

Fig. 3. Feynman graphs for a) scattering, b) pair creation and c) pair annihilation of photons in an external gravitational field

With the aid of (A.3) the matrix elements belonging to these processes can be proved to be:
for the photon scattering (Fig. 3a):

$$\langle k', \lambda'|S^{(1)}|k, \lambda\rangle = \frac{ic}{(2\pi)^3}\frac{1}{\sqrt{\omega_k\omega_{k'}}}\varepsilon^{(\lambda)}_{[\mu}(k)k_{\alpha]}[f^{\alpha\beta}(k-k')$$
$$\times\, \varepsilon^{[\mu(\lambda')}(k')k'_{\beta]} - \tfrac{1}{4}f(k-k')\varepsilon^{\mu(\lambda')}(k')k'^{\alpha}] \tag{4.1}$$

and for photon creation (Fig. 3b):

$$\langle \boldsymbol{k}, \lambda; \boldsymbol{k}', \lambda' | S^{(1)} | 0, \text{in} \rangle = \frac{ic}{(2\pi)^3} \frac{1}{\sqrt{\omega_k \omega_{k'}}} \varepsilon^{(\lambda)}_{[\mu}(\boldsymbol{k}) k_{\alpha]} [f^{\alpha\beta}(k+k')$$
$$\times \varepsilon^{[\mu(\lambda')}(\boldsymbol{k}') k'_{\beta} - \tfrac{1}{4} f(k+k') \varepsilon^{\mu(\lambda')}(\boldsymbol{k}') k'^{\alpha}]. \tag{4.2}$$

From this the following specializations result for the weak Schwarzschild field: for the photon creation

$$\langle \boldsymbol{k}, \lambda; \boldsymbol{k}', \lambda' | S^{(1)} | 0, \text{in} \rangle = -\frac{c^2 M}{\hbar \omega_k \pi (2\pi)^3} \varepsilon^{\mu(\lambda)}(\boldsymbol{k}) \varepsilon^{(\lambda')}_{\mu}(\boldsymbol{k}') \delta(\omega_k + \omega_{k'}) \tag{4.3}$$

and for the scattering process

$$\langle \boldsymbol{k}', \lambda' | S^{(1)} | \boldsymbol{k}, \lambda \rangle = -\frac{ic^2 M}{\hbar \omega_k \pi (2\pi)^3} \varepsilon^{\mu(\lambda)}(\boldsymbol{k}) \varepsilon^{(\lambda')}_{\mu}(\boldsymbol{k}') \cot^2 \frac{\theta}{2} \delta(\omega_k + \omega_{k'}). \tag{4.4}$$

This shows that the quanta of the considered fields are not created by static gravitational fields.

Concerning the Robertson-Walker metrics we remarked already in I that in (I–5.6) no term analogous to (I–5.4c) occurs. But if the expansion is anisotropic as in (2.14) we obtain

$$\beta_k = \frac{-ic^2}{6} (-1)^{\lambda} \frac{\sin 2\omega_k \tau}{\omega_k^2} \left[\sum_i A^0_{i,2}(k^2 - 3k_i^2) \right],$$

and in the isotropic case $\beta_k = 0$ follows correctly. Consequently, anisotropic expansion can cause important direction-dependent effects of particle creation or it can make it possible at all. We close this chapter comparing the matrix elements (2.3), (3.4) and (4.4) for scattering in the weak Schwarzschild field. From these matrix elements we obtain the following cross sections:
for mesons:

$$d\sigma = \frac{\kappa_0^2 M^2 c^4}{(8\pi)^2 k^4} (2k^2 + m^2) \frac{d\Omega}{\sin^4 \frac{\theta}{2}}, \tag{4.5a}$$

for electrons:

$$d\sigma = \frac{\kappa_0^2 M^2 c^4}{(4\pi)^2 p^4} \left[p^4 \cos^2 \frac{\theta}{2} + \frac{m^4}{4} + \frac{m^2 p^2}{4} \left(1 + 3\cos^2 \frac{\theta}{2} \right) \right] \frac{d\Omega}{\sin^4 \frac{\theta}{2}}, \tag{4.5b}$$

and for photons:

$$d\sigma = \frac{\kappa_0^2 M^2 c^4}{(4\pi)^2} \cot^4 \frac{\theta}{2} \, d\Omega. \tag{4.5c}$$

These cross sections have been given already by Mitskievich [5], but under specialization to the Schwarzschild field from the beginning, while we have derived them from a more general viewpoint including particle creation. For a vanishing rest mass of the particles and small deviation angles the same cross section results for all the particles considered,

$$d\sigma = \frac{\kappa_0^2 M^2 c^4}{(4\pi)^2} \frac{d\Omega}{\sin^4 \frac{\theta}{2}},$$

exhibiting the characteristic angle dependence of Rutherford's scattering formula. This result has to be expected because the static Schwarzschild field is the analog of the Coulomb field in electrodynamics.

In connection with the gravitational lens property of black holes during the last years classical cross sections have been evaluated on the basis of the geodesic equation [1, 8] which coincide with those given here. This is remarkable in so far as the derivation of the cross sections (4.5) is based on the idea of a wave field extended over the whole space while the classical results emerge from the concept of strongly localized rays (particle trajectories).

5. Corrections to the conventional quantum electrodynamics

The S matrix terms (I–5.4d...h) contain numerous processes which are all specified and discussed in [4].

As is well known, the term (I–5.4a) consists of 12 processes if one allows for a classical electromagnetic field apart from a quantized one. The symmetries of the Minkowski spacetime require the simultaneous fulfilment of the conservation laws for energy and momentum on what account nearly all of the 12 processes are forbidden in the conventional quantum electrodynamics. Because we did not presume any symmetries in the general ansatz for $f^{\mu\nu}$ we can expect that in a general external gravitational field all processes are allowed. In the special case of the Schwarzschild metric only the energy conservation law has to be fulfilled (static metric) while the purely time dependent Robertson-Walker metrics with Euclidean 3-space require conservation of the momentum only, but not of the energy. Here we select only two processes which show the character of the corrections to the conventional quantum electrodynamics coming from the gravitational field.

Example No. 1

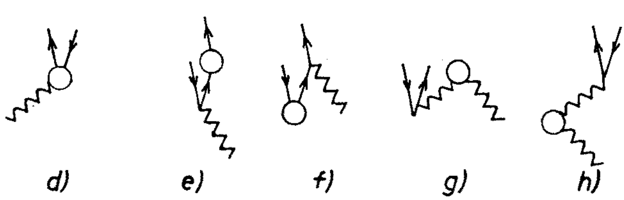

Fig. 4a. Pair creation by a photon in an external gravitational field

In general each of the S matrix terms (I–5.4d...h) contributes to this process, especially the process d) of first order. The corresponding process in the conventional quantum electrodynamics is [2]:

Fig. 4b. Pair creation by a photon in an external (time independent) electromagnetic field (cp. Fig. 4a, processes e, f)

The corresponding matrix element $\langle +, \boldsymbol{p}, d; -, \boldsymbol{p}', d'|S|\boldsymbol{k}, \lambda\rangle$ has essentially the following structure:

Fig. 4a: Processes (e), (f) for Robertson-Walker metrics with stepwise expansion:

$$\langle +, \boldsymbol{p}, d; -, \boldsymbol{p}', d'|S|\boldsymbol{k}, \lambda\rangle \sim e_0 A_1^0 \frac{\sin(\omega_p+\omega_{p'}-\omega_k)\tau}{\omega_p+\omega_{p'}-\omega_k}\,\delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}');$$

Fig. 4b: Time independent external electromagnetic field:

$$\langle +, \boldsymbol{p}, d; -, \boldsymbol{p}', d'|S|\boldsymbol{k}, \lambda\rangle \sim e_0^2 A_\mu^{\mathrm{ext}}(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{p}')\delta(\omega_k-\omega_p-\omega_{p'}).$$

From this the following can be read off: the process of pair creation by a photon occurs first in the second order of e_0 in the conventional quantum electrodynamics, *but with an external gravitational field it exists already as a first order process in* $e_0\kappa$, because the graph (d) of Fig. 4a contributes a nonvanishing term to the matrix element [4].

Example No. 2

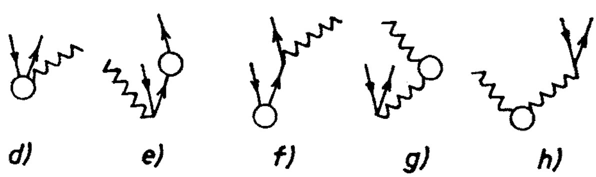

Fig. 5. Creation of an electron-positron pair and a photon in an external gravitational field

For a stepwise isotropic expansion (conformally flat metric) the corresponding matrix element reads [4]:

$$\langle +, \boldsymbol{p}, d; -, \boldsymbol{p}', d'; \boldsymbol{k}, \lambda|S|0, \mathrm{in}\rangle = -\frac{e_0 mc}{\hbar\sqrt{2\omega_k\omega_p\omega_{p'}}}\,\delta(\boldsymbol{p}+\boldsymbol{p}'+\boldsymbol{k})$$

$$\times \frac{\sin(\omega_p+\omega_{p'}+\omega_k)\tau}{\omega_p+\omega_{p'}+\omega_k}\,\bar{u}^{(1,d)}(\boldsymbol{p})\left[6-im\,\frac{i\gamma^\sigma(p'+k)_\sigma+m}{(p'+k)^2+m^2}+im\,\frac{i\gamma^\sigma(p+k)_\sigma-m}{(p+k)^2+m^2}\right]$$

$$\times \gamma^\varrho \varepsilon_\varrho^{(\lambda)}(\boldsymbol{k})u^{(-1,d')}(\boldsymbol{p}').$$

We restrict our discussion to conformally flat gravitational fields in which the graphs (g) and (h) of Fig. 5 give no contribution to the matrix element and in which, as is well known, no photons can be created according to (I–5.4c). It is an important result that the graphs (d)...(f) give nonvanishing contributions *such that photons can be created by conformally flat gravitational fields too but only together with an electron-positron pair.* In this sense we can speak of a "catalysing" effect of the Dirac field with respect to photon creation. This effect has to be expected because the coupling of the Maxwell-Dirac fields takes place by means of the charge which always is linked to the mass at one hand, and the gravitational field affects the electron mass (cp. (3.7)) at the other hand. In this way photons are coupled to the electron mass by means of the charge and thus are subjected to the influence even of conformally flat gravitational fields. Consequently, the process given by Fig. 5 explains a further possibility of photon creation by gravitational fields.

Of course (I–5.4) contains the following process, too (see Introduction to I):

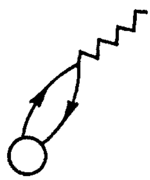

Fig. 6. Annihilation into a photon of an electron-positron pair which has been created by a gravitational field

6. *Summary*

A. Mutually isolated quantum fields under the influence of an external gravitational field

i) The S matrix theory shows with peculiar clearness that conformally flat gravitational fields do not create particles which are described by conformally invariant field equations. Moreover, in the framework of the considered models and approximations the S matrix theory confirms all exact results.

ii) The unitarity of the S matrix is strongly connected with the Bogoljubov transformation which in its turn is tied with the statistics belonging to the spin under consideration. Corresponding to this, scattering in a definite mode can be accompanied by pair creation in the same mode in the case of bosons and not in the case of fermions. The complete S matrix can be expressed by the Bogoljubov coefficients.

iii) Particles are created in pairs in so far as we consider mutually isolated quantum fields.

iv) The quantum field theoretic cross sections evaluated for the deviation of massless mesons and photons in the weak Schwarzschild field coincide with the classical ones.

B. Interacting quantum fields under the influence of an external gravitational field

i) Because an external gravitational field generally does not admit the Poincarè group as its symmetry group those processes which are forbidden in the Minkowski space-time are allowed in the general case.

ii) Processes which occur in the conventional quantum electrodynamics first in the second order are already possible in the first order if an external gravitational field is present.

iii) If the electromagnetic field couples with a Dirac current conformally flat gravitational fields can create photons.

APPENDIX

The S matrix terms (I–3.9), (I–5.4) and (I–5.6) describe the scattering of plane waves such that for the operators ϕ^{in}, A_μ^{in}, ψ^{in} and $\overline{\psi}^{\text{in}}$ the following operator expansions have to be inserted:

Klein-Gordon field:

$$\phi = \int d^{(3)}k \sqrt{\frac{m_0 c}{2\hbar}} \frac{c}{\omega_k} [a_k u_k e^{-i\omega_k t} + a_k^\dagger u_k^* e^{i\omega_k t}] \tag{A.1}$$

with

$$u_k = (2\pi)^{-3/2} \left(\frac{\omega_k}{c}\right)^{1/2} e^{ik_a x^a}, \quad \frac{\omega_k^2}{c^2} = k^2 + m^2.$$

Dirac field:

$$\psi = \sum_{d=-1}^{+1} \int d^{(3)}p [b_k u_k e^{-i\omega_k t} + d_k^\dagger v_k e^{i\omega_k t}], \tag{A.2}$$

$$\left.\begin{aligned} u_k &= \left(\frac{m_0 c^2}{\hbar \omega_p}\right)^{1/2} u^{(1,d)}(p) e^{ip_a x^a} \\ v_k &= \left(\frac{m_0 c^2}{\hbar \omega_p}\right)^{1/2} u^{(-1,d)}(p) e^{-ip_a x^a} \end{aligned}\right\} \frac{\omega_p^2}{c^2} = p^2 + m_0^2 c^2,$$

with respect to $u^{(\pm 1, d)}(p)$ see [7].

Maxwell field:

$$A_\mu = \sum_{\lambda=1}^{2} \int d^{(3)}k \sqrt{\frac{\hbar c}{2}} \frac{c}{\omega_k} [a_k u_{k\mu} e^{-i\omega_k t} + a_k^\dagger u_{k\mu}^* e^{i\omega_k t}] \tag{A.3}$$

with

$$u_{k\mu} = (2\pi)^{-3/2} \left(\frac{\omega_k}{c}\right)^{1/2} \varepsilon_\mu^{(\lambda)}(k) e^{ik_b x^b}, \quad \omega_k^2 = c^2 k^2.$$

The operator-valued expansion coefficients are constant.

The author wishes to thank Professors E. Schmutzer and G. Weber as well as Dr D. Kramer for numerous stimulating discussions.

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