COMMENT ON AN UNCORRELATED JET MODEL WITH QUANTUM STATISTICS

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We perform a careful analysis of an uncorrelated jet model with Bose-Einstein statistics proposed by Kripfganz.

Recently, the influence of Bose-Einstein (BE) statistics on pion production has been analysed [1] in the framework of an uncorrelated jet model (UJM).

The distribution function of a system of N particles is determined by the available level density [2]

$$\Omega_N(Q) = \sum_{\{n_a\}} \delta^{(4)}(Q - \sum_a n_a p_a)\delta_{\rm K}(N - \sum_a n_a). \tag{1}$$

The occupation numbers $\{n_a\}$ indicate how many particles have the four-momentum p_a in the state under consideration. A UJM with BE statistics is obtained by specifying the single-particle momentum density when the sum over p_a is replaced by an integration

$$\sum_p \to B \int \frac{d^3p}{2p_0} f(p_{\rm T}), \tag{2}$$

with a given transverse-momentum cut-off function $f(p_{\rm T})$ and the strength parameter B.

The model thus formulated is claimed to predict the existence of pronounced positive correlations between like pions nearby in momentum space. This result is obtained under a high-energy approximation that $\Omega_N(Q)$ behaves as

$$\tilde{\Omega}_N(Q) = \sum_{\{n_a\}} \delta^{(4)}\left(Q - \sum_a n_a p_a\right)\delta_{\rm K}\left(N - \sum_a n_a\right)\prod_a \frac{1}{n_a!}, \tag{3}$$

which is the conventional UJM with Boltzmann statistics [3] as $Q^2 \to \infty$.

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We have made a careful analysis of the model given by Eqs. (1) and (2) and found that the approximation

$$\Omega_N(Q) \simeq \tilde{\Omega}_N(Q) \quad \text{as} \quad Q^2 \to \infty \tag{4}$$

is very crude; it does not include the most important features of the model. The correct predictions of the model are that the total average multiplicity, the single-particle rapidity distribution, and the second correlation moment show a power-like behaviour with increasing energy,

$$\langle n \rangle \sim \frac{(Q^2)^{1/2}}{\log Q^2}, \quad \frac{1}{\sigma}\frac{d\sigma}{dy} \sim (Q^2)^{1/2}, \quad f_2 \sim Q^2, \tag{5}$$

in contrast to the predictions of the model (3), which are

$$\langle \tilde{n} \rangle \sim \tfrac{1}{2} B \log Q^2, \quad \frac{1}{\sigma}\frac{d\sigma}{dy} \sim \text{const}, \quad \tilde{f}_2 \sim -\log Q^2. \tag{6}$$

A straightforward way to see the difference betwene $\Omega(Q)$ and $\tilde{\Omega}(Q)$ is to use the energy--momentum sum rule and the assumption

$$\frac{\Omega(Q-kp)}{\Omega(Q)} \sim (1-kx_0)^{\varrho(Q^2)-1}; \quad k = 1, 2, \ldots, \tag{7}$$

where $x_0 = 2p_0/(Q^2)^{1/2}$.

We find that (1) gives

$$\varrho(Q^2) = \tfrac{1}{4} B \log Q^2 \tag{8}$$

and (3) gives

$$\tilde{\varrho}(Q^2) = \tfrac{1}{2} B. \tag{8'}$$

More rigorous analysis [4] yields the same results and thus justifies the assumption (7).

To estimate $\Omega_N(Q)$ for large Q^2-values, we apply the method of Khinchin [5]. The following behaviour of $\Omega_N(Q)$ for large Q^2-values has been found

$$\Omega_N(Q) \simeq \tilde{\Omega}_N(Q) \frac{1}{\langle k^2 \rangle} \sum_{M=1}^{N} \frac{1}{M!} (\tfrac{1}{2} B \log Q^2)^{M-N} c_M(N, K_{\max}), \tag{9}$$

where

$$c_M(N, K_{\max}) = \lim_{z \to 0} \partial_z^N \left(\sum_{k=1}^{K_{\max}} \frac{z^k}{k} \right)^M \tag{10}$$

and

$$K_{\max} \sim (Q^2)^{1/2}.$$

Here $\langle k^2 \rangle$ is connected with the dispersion of the average number of particles inside the BE-cluster in the following way:

$$\langle k^2 \rangle = 1 + \left(\frac{\text{dispersion}}{\langle k \rangle} \right)^2 \leqslant Q^2. \tag{11}$$

$\tilde{\Omega}_N(Q)$ is given by [6]

$$\tilde{\Omega}_N(Q) \simeq \frac{1}{2} B \frac{\sigma_0^2}{Q^2} \frac{(\frac{1}{2} B \log Q^2)^{N-1}}{N!}. \tag{12}$$

The total density of states leading to (7) is then

$$\Omega(Q) = \sum_N \Omega_N(Q) \simeq \frac{\sigma_0^2 (Q^2)^{\frac{1}{2} B \log Q^2}}{\langle k^2 \rangle Q^2 \log Q^2}. \tag{13}$$

In conclusion we may say that the model (1) represents an extreme case among UJM with quantum statistics [4]. The predicted behaviour of the total average multiplicity nearly saturates the upper limit allowed by the energy-momentum conservation law.

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