EXACT SOLUTIONS FOR A VECTOR MESON IN QUANTIZED FIELD OF A MONOCHROMATIC ELECTROMAGNETIC PLANE WAVE

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The exact solutions for a vector meson in a quantized plane-wave external field are obtained. The expressions for 4-momentum of the particle and wave system are derived for each of three particle spin states.

The exact solutions of the Klein-Gordon and Dirac equations for particles in a quantized field of a monochromatic electromagnetic plane wave with linear polarization have been found for the first time by Berson [1]. Later these solutions have been generalized to the case of an arbitrarily polarized electromagnetic wave [2]. Some different aspects of the problem have been considered in Refs [3–5].

One encounters great mathematical difficulties trying to obtain the exact wavefunctions describing particles with higher spins in an external electromagnetic field. The covariant method [6, 7] for deriving the wavefunctions of particles with arbitrary spin in a classical electromagnetic plane-wave field enables us to avoid, to a certain extent, the difficulties in solving similar problems for a quantized electromagnetic field. The present authors used this method to obtain the wavefunctions for a vector meson (spin 1) in a quantized field of a linearly polarized monochromatic electromagnetic plane wave.

We start by considering (see [8]) the equation

$$\left[\beta_l(\partial_l - ie_0A_l) + \frac{i}{2}(\kappa_1 P + \kappa_2 \overline{P})\beta_{ln}F_{ln} + m\right]\psi = 0. \tag{1}$$

Here β_l are the Duffin-Kemmer matrices, $\beta_{ln} = \beta_l\beta_n - \beta_n\beta_l$ and κ_1, κ_2, respectively, are the particle parameters expressed in terms of the static anomalous magnetic moment (AMM) and electric quadrupole moment (EQM); P and $\overline{P}$ are the projective operators denoting, respectively, the vector and tensor parts of the 10-component wavefunction ψ,

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$\partial_l = \partial/\partial x_l$, $x = (x_l) = (\boldsymbol{x}, it)$, $l = 1, 2, 3, 4$. The vector potential A_l satisfies the Lorentz gauge $\partial_l A_l = 0$ and is expressed through the creation c^+ and annihilation c^- operators of photons with the 4-momentum $k = (\boldsymbol{k}, ik_0)$ and polarization e':

$$A = \frac{e'}{(2k_0 V)^{1/2}}(c^- e^{i\varphi} + c^+ e^{-i\varphi}), \quad \varphi = kx = k_l x_l, \quad k^2 = 0. \tag{2}$$

The $c^\pm$ operators in (2) are taken in the coordinate representation [9]

$$c^\pm = (1/\sqrt{2})(\xi \pm \partial_\xi).$$

According to [1] the wavefunction is defined as follows

$$\psi(x, \xi) = U\chi, \quad U = \exp\{i[qx + \tfrac{1}{2}\varphi(\partial_\xi^2 - \xi^2)]\}, \tag{3}$$

where q is a constant 4-vector. We use (1) to derive an equation for the unknown function $\chi(\xi)$

$$\hat{k}\chi'' + 2i(\kappa_1 P + \kappa_2 \bar{P})[\hat{k}\hat{a}]\chi' - 2i(i\hat{c} + m)\chi = 0 \tag{4}$$

with the help of the operator identity

$$U^+(\xi \pm \partial_\xi)U = e^{\mp i\varphi}(\xi \pm \partial_\xi), \quad U^+ = \exp\{-i[qx + \tfrac{1}{2}\varphi(\partial_\xi^2 - \xi^2)]\}.$$

Here

$$[\hat{k}\hat{a}] = \hat{k}\hat{a} - \hat{a}\hat{k}, \quad \hat{a} = a_l\beta_l, \quad a = e'/(k_0 V)^{1/2},$$

$$c = b - \tfrac{1}{2}\xi^2 k, \quad b = q - e_0\xi a, \quad \chi' = d\chi/d\xi.$$

The function χ as well as the result of action of an arbitrary operator $\hat{r}$ can be written in the form [7]

$$\chi = \begin{pmatrix} u \\ \alpha \end{pmatrix}, \quad \hat{r}\chi = \begin{pmatrix} \alpha r \\ [u \cdot r] \end{pmatrix}, \tag{5}$$

where u is a 4-vector, $\alpha = (\alpha_{ln}) = -\tilde{\alpha}$ is an antisymmetric tensor of the second rank with six independent components, $[u \cdot r] = ([u \cdot r]_{ln}) = (u_l r_n - r_l u_n) = u \cdot r - r \cdot u$ is the alternating dyad, $\alpha r = (\alpha_{ln} r_n)$. Equation (4) is thus reduced to a system of two equations

$$\alpha'' k + 2\alpha c + 2im\kappa_1[k \cdot a]u' - 2imu = 0, \tag{6}$$

$$\alpha = m^{-1}\{(i/2)[k \cdot u]'' + i[c \cdot u] + \kappa_2([\alpha' a \cdot k] + [a \cdot \alpha' k])\}. \tag{7}$$

From (7) we can derive the tensor α. Indeed, multiplying (7) by a and differentiating it with respect to ξ, we obtain

$$\alpha' a = i(2m)^{-1}(au)''' k + \varrho_2[(\alpha k a)a - a^2\alpha k]'', \quad \varrho_2 = \kappa_2/m. \tag{8}$$

Similarly, we derive

$$\alpha'' k a = -i\mu m^{-1}(du)'', \quad \mu = kq, \quad d = a - (ab)\mu^{-1}k, \tag{9}$$

$$\alpha'' k = im^{-1}[\tfrac{1}{2}(ku)'' k + [c \cdot u]k - \mu\varrho_2(du)k]''.$$

From (8) and (9) we have

$$[\alpha' a \cdot k] = im^{-1} \{[[c \cdot u]a \cdot k] + \varrho_2(\mu(du)\,[k \cdot a] + a^2[k \cdot [c \cdot u]\,k])'\}'.$$

In the same manner we obtain

$$[\alpha' k \cdot a] = im^{-1}\{\tfrac{1}{2}(ku)''\,[k \cdot a] + [[c \cdot u]k\,.\,a] + \mu\varrho_2(du)'[a \cdot k]\}'.$$

As a result, we have

$$\alpha = im^{-1}\{\tfrac{1}{2}[k \cdot u]'' + [c \cdot u] + \varrho_2(\tfrac{1}{2}(ku)''[a \cdot k] + [[c \cdot u]a \cdot k] + [a \cdot [c \cdot u]k])' + \varrho_2^2(2\mu(du)\,[k \cdot a] + a^2[k \cdot [c \cdot u]k])''\}. \tag{10}$$

The tensor α is expressed through the unknown vector u and the constant 4-vectors k, a, q. The substitution of (10) into Eq. (6) results in a fourth-order equation for the unknown vector u

$$\tfrac{1}{2}(ku)^{(4)}k + [c \cdot u]''k + [k \cdot u]''c + 2([c \cdot u]c - m^2u + m\kappa_1[k \cdot a]u') + \mu\varrho_2[(ku)'''d - (du)'''k] + 2\varrho_2^2\{-2\mu^2(du)''d + \mu[u \cdot d]'c + a^2(\mu[u \cdot c]''k + ([c \cdot u]''kc)k) + ((au)\,[c \cdot k])'c + ((ku)\,[a \cdot c])'c\} = 0. \tag{11}$$

Subsequent multiplication of this equation by the vector k gives the relations

$$ku'' + 2\,[gku - cu - \mu\varrho_2(du)'] = 0, \quad g = \mu^{-1}(c^2 + m^2),$$

which along with (11) provide the second-order equation for the unknown vector $u(\xi)$

$$[(c - gk)u]'k + [c \cdot u]''k + [k \cdot u]''c + 2[(cu)c - \mu gu + m\kappa_1[k \cdot a]u'] + 2\varrho_2\{\mu[(c - gk)u]'d + \mu[u \cdot d]'c + ((au)\,[c \cdot k])'c + ((ku)\,[a \cdot c])'c\} + 2\varrho_2^2[-\mu^2(du)''d + a^2([c \cdot u]''kc)k + \mu a^2[u \cdot c]''k] = 0. \tag{12}$$

According to [7] we introduce the following basic vectors

$$k,\, d,\, e = c - \mu^{-1}c^2k, \qquad v = (v_l) = (i\mu^{-1}\varepsilon_{lmnr}k_m a_n q_r). \tag{13}$$

The vector u is expanded now as follows

$$u = \eta_1 k + \eta_2 d + \eta_3 e + \eta_4 v. \tag{14}$$

Here the coefficients η_1, η_2, η_3, η_4 are the unknown scalar functions of ξ. These functions satisfy the system of four equations resulting from (12)

$$\eta_1 = (1/2)\eta_3'' + g\eta_3 - a^2\varrho_2\eta_2', \tag{15}$$

$$\eta_2'' + (e_0a^2\varrho_2 + g)\eta_2 - \tau_-\eta_3' = 0, \tag{16}$$

$$\eta_3'' + (e_0a^2\varrho_1 + g)\eta_3 + a^2m^{-1}\tau_-\eta_2' = 0, \tag{17}$$

$$(1 - \mu a^2\varrho_2^2)\eta_4'' + (-e_0a^2\varrho_2 + g)\eta_4 = 0, \tag{18}$$

where $\varrho_1 = \kappa_1/m$, $\tau_- = e_0 - \kappa_+$, $\kappa_+ = m(\kappa_1 + \kappa_2)$.

Equation (18) gives

$$\eta_4 = C_1\beta^{-1/2}H_n(\sigma)\exp(-\sigma^2/2). \tag{19}$$

Here

$$\beta = 1-\mu a^2\varrho_2^2, \quad \sigma = \beta^{-1/4}\zeta, \quad \zeta = \tau(\xi+\kappa),$$
$$\tau^4 = 1-e_0^2a^2/\mu, \quad \kappa = e_0qa/\mu\tau^4, \quad n = 0, 1, 2, \ldots .$$

$H_n(\sigma)$ are the Hermite polynomials and C_1 is the normalization constant. From the finiteness of the solution of (18) we have

$$q^2 = -m^2+2\mu\tau^2[(n+\tfrac{1}{2})\beta^{1/2}-\tau^2\kappa^2/2+e_0a^2\varrho_2/2\tau^2]. \tag{20}$$

Due to (5), (10), (14), (19) we obtain a unique solution of the problem

$$\chi_1 = \begin{pmatrix} im\eta_4 v \\ \alpha_1 \end{pmatrix},$$

$$\alpha_1 = [v\cdot(c-e_0a^2\varrho_2k)]\eta_4+\mu\varrho_2[d\cdot v]\eta_4'+(\tfrac{1}{2}-\mu\varrho_2^2a^2)\,[v\cdot k]\eta_4''. \tag{21}$$

Now (16) and (17) ought to be solved in order to construct two other solutions. With (16) and (17) solved we readily obtain from (15) the function η_1. Below we deal with some particular cases because the solution of the system for a general case is very cumbersome in practice.

1. Neglecting the particle interaction due to the particle extra moments, i. e. considering $\kappa_1 = \kappa_2 = 0$, we find from (16) and (17)

$$\begin{pmatrix} \eta_2 \\ \eta_3 \end{pmatrix}^{\pm} = C_{\pm}\begin{pmatrix} \pm im^{1/2}/|a| \\ 1 \end{pmatrix} H_n(\zeta)\exp[-\tfrac{1}{2}(\zeta^2\pm ie_0|a|m^{-1/2}\xi)], \tag{22}$$

$$q^2 = -m^2+2\mu\tau^2(n+1/2-\tau^2\kappa^2/2+e_0^2a^2/8m\tau^2). \tag{23}$$

2. Let us consider the neutral vector particle ($e_0 = 0$). In this case the particle interaction with the electromagnetic field is due solely to the extra moments κ_1 and κ_2. From (16) and (17), with $e_0 = 0$, we have

$$\begin{pmatrix} \eta_2 \\ \eta_3 \end{pmatrix}^{\pm} = C_{\pm}\begin{pmatrix} \pm im^{1/2}/|a| \\ 1 \end{pmatrix} H_n(\xi)\exp[\tfrac{1}{2}(\pm i|a|m^{-1/2}\kappa_+-\xi)\xi], \tag{24}$$

$$q^2 = -m^2+2\mu(n+\tfrac{1}{2}+a^2\kappa_+^2/8m). \tag{25}$$

The above results describe exactly the solution of the problem for the neutral vector meson.

3. It should be noted that the electric charge e_0 and particle extra moments enter, as additive parameters, the exact wavefunction of a vector meson interacting with a classical electromagnetic plane wave [10]. Equations (16) and (17) for the case of interaction with a quantized electromagnetic field contain nonadditive terms (with respect to these

parameters) which can be neglected provided the electromagnetic wave frequency is sufficiently large. From (16) and (17) under conditions $e_0 a^2 \varrho_i \ll 1$ $(i = 1, 2)$ we have

$$\begin{pmatrix} \eta_2 \\ \eta_3 \end{pmatrix}^{\pm} = C_{\pm} \begin{pmatrix} \pm i m^{1/2}/|a| \\ 1 \end{pmatrix} H_n(\xi) \exp\left[-\tfrac{1}{2}(\zeta^2 \pm i|a| m^{-1/2}\tau_-\xi)\right], \tag{26}$$

$$q^2 = -m^2 + 2\mu\tau^2(n + \tfrac{1}{2} - \tau^2\kappa^2/2 + a^2\tau_-^2/8m\tau^2). \tag{27}$$

It is worth noting that the smallness of the parameter $\lambda = e_0 a^2/m$ at high electromagnetic field frequencies enables us to express the unknown functions η_2 and η_3 in the form of an infinite series.

The 4-vector q represents the total momentum of the particle and electromagnetic field system. From (20), in particular, we find

$$q = p + \tau^2[(n + 1/2)\beta^{1/2} - \tau^2\kappa^2/2 + e_0 a^2 \varrho_2/2\tau^2]k,$$

$$p^2 = -m^2, \tag{28}$$

where p is the free particle energy-momentum 4-vector. In particular cases one can derive from (23), (25), (27) the expressions for q for each of the three spin states.

The wavefunctions of the vector meson in the classical electromagnetic wave field contain the tachyonic modes (see [11–13]). The problem of existence of tachyonic modes in the case of a quantized field has not yet been solved.

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