A NOTE ON QUANTIZATION OF WEINBERG-SALAM UNIFIED FIELD THEORY*

By N. G. Deshpande

University of Oregon, Institute of Theoretical Science, Eugen, Oregon 97403, USA

and P. Kielanowski

Institute of Theoretical Physics, Warsaw University**

(Received June 15, 1978)

Equivalence of the Lagrangian formulation of Hsu and Sudarshan to the conventional Fujikawa, Lee and Sanda formulation of Weinberg-Salam model is demonstrated.

There exist several approaches to quantization of gauge theories [1–7]. The method presented by Hsu and Sudarshan [1] for Weinberg-Salam model consists in fixing the gauge by use of Lagrangian multipliers and then restoring unitarity by an appropriate choice of fictitious Lagrangian. The conventional approach by Fujikawa, Lee and Sanda [2] adds a fictitious Lagrangian, following t'Hooft [8], to restore gauge invariance. Thus in Hsu-Sudarshan approach gauge invariance has to be proved, while in the conventional approach unitarity must be demonstrated. The two formulations, which are complementary one to another, lead to the same Feynman rules except for the fictitious parts which apparently differ. In this note we show the equivalence of these parts.

The fictitious Lagrangian contributes to the generating function

$$I = \int [d\phi_a^\dagger]\,[d\phi_a]\exp\{i\int \phi_a^\dagger(x)M_{ab}\phi_b(x)d^4x\}. \tag{1}$$

Here M is an operator and can be written as $M = G^{-1}+\gamma$, and ϕ_a are complex scalar fields to be treated as fermions in the sense that closed loops have minus signs assigned to them. Then it is easy to show

$$I \equiv \exp\{\mathrm{Tr}\log M\} \equiv \det M. \tag{2}$$

---

* Work supported in part by U.S. National Science Foundation Grant No GF-42060.

** Address: Instytut Fizyki Teoretycznej UW, Hoża 69, 00-681 Warszawa, Poland.

The generating function is invariant under the transformations:

A.

$$\phi_a \to \Lambda_{ab}\phi_b, \quad \phi_a^\dagger \to \phi_b^\dagger[\Lambda^{-1}]_{ba}, \quad M \to \Lambda M \Lambda^{-1}; \tag{3}$$

special cases are

a) multiplying a row of matrix M by a number and the corresponding column by its inverse;

b) changing labels $1 \rightleftarrows 2$

B.

$$\phi_a \to \phi_a^\dagger, \quad \phi_a^\dagger \to \phi_a, \quad M \to M^\dagger, \tag{4}$$

(both matrix and operator transpose). Now the G^{-1} are identical in both approaches. For the sake of clarity we explicitly write down the matrix γ in two cases

$$[\gamma]_{\mathrm{FLS}} = \left[\begin{matrix} -ig\partial^\mu(Z_\mu \cos\theta + A_\mu \sin\theta) - \dfrac{gM}{2\xi} S^0, & 0, \\ 0, & ig\partial^\mu(Z_\mu\cos\theta + A_\mu\sin\theta) - \dfrac{gM}{2\xi}\overline{S^0}, \\ iG\sin\theta\partial^\mu W_\mu^-, & -iG\sin\theta\partial^\mu W_\mu^+, \\ ig\partial^\mu W_\mu^- + \dfrac{GM_Z}{2\eta} S^-, & -ig\partial^\mu W_\mu^+ + \dfrac{GM_Z}{2\eta} S^+, \end{matrix}\right.$$

$$\left.\begin{matrix} -ie\cos\theta\partial^\mu W_\mu^+ + \dfrac{eM\cos\theta}{\xi} S^+, & -ie\cos\theta\cot\theta\partial^\mu W_\mu^+ - \dfrac{GM\cos\theta\cos 2\theta}{2\xi} S^+ \\ ie\cos\theta\partial^\mu W_\mu^- + \dfrac{eM\cos\theta}{\xi} S^-, & ie\cos\theta\cot\theta\partial^\mu W_\mu^- - \dfrac{GM\cos\theta\cos 2\theta}{2\xi} S^- \\ 0, & 0 \\ 0, & -\dfrac{GM_Z}{2\sqrt{2}\,\eta}(S^0 + \overline{S^0}) \end{matrix}\right] \tag{5}$$

$$[\gamma]_{\mathrm{HS}} = \left[\begin{matrix} i(eA_\mu - G\cos^2\theta Z_\mu)\partial^\mu - \dfrac{GM\cos\theta}{2\xi}\overline{S^0}, & 0, \\ 0, & -i(eA_\mu - G\cos^2\theta Z_\mu)\partial^\mu - \dfrac{GM\cos\theta}{2\xi} S^0, \\ -ie\partial^\mu W_\mu^-, & ie\partial^\mu W_\mu^+, \\ iG\cos^2\theta\partial^\mu W_\mu^- + \dfrac{GM}{2\xi} S^-, & -iG\cos^2\theta\partial^\mu W_\mu^+ + \dfrac{GM}{2\xi} S^+, \end{matrix}\right.$$

$$\left.\begin{matrix} -ieW_\mu^+\partial^\mu, & iG\cos^2\theta W_\mu^+\partial^\mu + \dfrac{GM}{2\eta}S^+ \\ ieW_\mu^-\partial^\mu, & -iG\cos^2\theta W_\mu^-\partial^\mu + \dfrac{GM}{2\xi}S^- \\ 0 \quad , & 0 \\ 0 \quad , & -\dfrac{GM_Z}{2\sqrt{2}\eta}(S^0+\overline{S^0}) \end{matrix}\right] \tag{6}$$

Using relations between coupling constants g, g', e and G in the Weinberg-Salam model [9, 10] we obtain

$$[\gamma]_{HS} = \left[\begin{matrix} -ig(A_\mu\sin\theta+Z_\mu\cos\theta)\partial^\mu - \dfrac{gM}{2\xi}\overline{S^0}, & 0 & , \\ 0 & , \ ig(A_\mu\sin\theta+Z_\mu\cos\theta)\partial^\mu - \dfrac{gM}{2\xi}S^0, \\ -ie\partial^\mu W_\mu^- & , \ ie\partial^\mu W_\mu^+ & , \\ -ie\cot\theta\partial^\mu W_\mu^- + \dfrac{GM}{2\xi}S^- & , \ ie\cot\theta\partial^\mu W_\mu^+ + \dfrac{GM}{2\xi}S^+ & , \end{matrix}\right.$$

$$\left.\begin{matrix} iG\sin\theta\cos\theta W_\mu^+\partial^\mu, & ig\cos\theta W_\mu^+\partial^\mu + \dfrac{GM_Z}{2\eta}\cos\theta S^+ \\ -iG\sin\theta\cos\theta W_\mu^-\partial^\mu, & -ig\cos\theta W_\mu^-\partial^\mu + \dfrac{GM_Z}{2\eta}\cos\theta S^- \\ 0 \quad , & 0 \\ 0 \quad , & -\dfrac{GM_Z}{2\sqrt{2}\eta}(S^0+\overline{S^0}) \end{matrix}\right] \tag{7}$$

Taking the transpose of $(G^{-1}+\gamma)_{HS}$, changing labels $1 \leftrightarrows 2$, and multiplying rows and columns by suitable constants we get a form which differs from $(G^{-1}+\gamma)_{FLS}$ only by quantities like $[\partial^\mu W_\mu^+ + (iM/\xi)S^+]$. Such quantities are zero in the gauge we are working in [2]. Thus the theories are identical.

REFERENCES

[1] J. P. Hsu, E. C. G. Sudarshan, *Phys. Lett.* **51B**, 349 (1974).
[2] K. Fujikawa, B. W. Lee, A. I. Sanda, *Phys. Rev.* **D10**, 2923 (1972).
[3] R. P. Feynman, *Acta Phys. Pol.* **24**, 697 (1963).
[4] B. S. DeWitt, *Relativity, Groups and Topology*, Blackie and Son Ltd. 1964.
[5] V. N. Popov, L. N. Faddev, Fermilab preprint NAL-THY-57, 1972.

[6] L. N. Faddev, V. N. Popov, *Phys. Lett.* **25B**, 29 (1967).
[7] I. Białynicki-Birula, *Simplified Quantization of Gauge Theories*, Warsaw University, 1976.
[8] G. t'Hooft, *Nucl. Phys.* **B33**, 173 (1971).
[9] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); **26**, 1688 (1972).
[10] A. Salam, *Elementary Particle Theory*, N. Svartholm Editor, Almquist and Forlag A. B., Stockholm 1968.