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LOW AND HIGH-ENERGY PROPERTIES OF SCATTERING AMPLITUDES SATISFYING GEOMETRICAL SCALING

BY E. PIETARINEN

Research Institute for Theoretical Physics, University of Helsinki

AND R. WIT

Institute of Computer Science, Jagellonian University, Cracow*

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It is shown that the ratio of the slope of the diffraction peak to the total cross-section at infinite energy, $B(\infty)/\sigma_{tot}(\infty)$, is essentially given by the low and intermediate energy behaviour of the scattering amplitude satisfying geometrical scaling (GS).

The unifying strength of the geometrical scaling assumption was already pointed out in a rather comprehensive way [1–3]. In this note we would like to extend the phenomenological consequences following from the hypothesis of GS showing that, to a large extent, the asymptotic value of the ratio of the slope of the diffraction peak to the total cross-section for the Pomeron amplitude is given by the low and intermediate energy contributions in a rather non-trivial way.

Consider the s–u crossing symmetric Pomeron amplitude $C^+(s, t)$; following the notation of Ref. [1] we have for large s in the case of GS

$$2MC^+(s, t) = isR^2(s)\Phi(tR^2(s)). \tag{1}$$

Both $R^2(s)$ and $\Phi(tR^2)$ are "real" analytic functions of their arguments. They are chosen in such a way that $\Phi(0) = 1$. Eq. (1) yields immediately

$$2M \frac{\partial C^+(s, t)}{\partial t} = isR^4(s)\Phi'(tR^2),$$

and if Im $C^+(s, t)$ dominates near the forward direction then for $s \to \infty$

$$\Phi'(0) = \frac{B(\infty)}{2\sigma_{tot}(\infty)}$$

* Address: Instytut Fizyki UJ, Reymonta 4, 30-059 Kraków, Poland.

since

$$B(s) = \frac{\partial}{\partial t} \ln |C^+(s, t)|^2_{t=0} = 2R^2(s)\Phi'(0).$$

Using a more convenient notation we may write

$$C^+(s, 0) = ip[a(p) - id(p)], \quad p \geqslant 0,$$
$$C^{+\prime}(s, 0) = ip[a(p) - id(p)]^2 \Phi'(0),$$

where $d(p)$ and $a(p)$ are real functions. Then

$$\varrho(p) \equiv \frac{\text{Re } C^+(s, 0)}{\text{Im } C^+(s, 0)} = \frac{d(p)}{a(p)}$$

and

$$\varrho'(p) = \frac{\text{Re } C^{+\prime}(s, 0)}{\text{Im } C^{+\prime}(s, 0)} = \frac{2\varrho}{1-\varrho^2} \tag{2}$$

independently from any *assumption* about the rate of growth of $R^2(s)$. This phase relation appears to be crucial for the validity of the following sum rule [4]

$$\frac{4}{\pi}\int_0^\infty dpQ'(p, 0) + \frac{m}{M\pi}\int_0^\infty \frac{dpQ(p, 0)}{(p^2+m^2)^{1/2}[(p^2+m^2)^{1/2}+m]} = \frac{B(\infty)}{\sigma_{\text{tot}}(\infty)}, \tag{3}$$

where $Q(p, t) = \text{Re } T(p, t)/[\text{Re } T(p, t)^2 + \text{Im } T(p, t)^2]$. The function $T(p, t)$ is related to the scattering amplitude $C^+(s, t)$ by the formula

$$T(p, t) = C^+(s, t) - C^+_{\text{Born}}(s, t) - C^+(s_0, t) - \gamma(t),$$

where $\gamma(t)$ is chosen in such a way that at the threshold ($s = s_0 = (M+m)^2$)

$$T(p = 0, 0) < 0.$$

In this note we would like to present the results following from the evaluation of the low and intermediate energy contributions to the integrals in Eq. (3) using the new phase shift analysis results for πN scattering [5]. For this aim we proceed in the following way. The integration regions are split into four subdomains: $0 \leqslant p \leqslant 0.5$ GeV/c, 0.5 GeV/c $\leqslant p \leqslant 4$ GeV/c, 4 GeV/$c \leqslant p \leqslant 10$ GeV/c and 10 GeV/$c \leqslant p < \infty$. The first two subdomains contain many experimental data allowing to calculate $T'(p, 0)$ in a rather accurate way. In particular, the first subdomain contains the well known (3/2, 3/2) resonance. Between 4 GeV/c and 10 GeV/c we do not have so many accurate estimates for $T'(p, 0)$ and therefore the numerical estimates for this subdomain should be taken with caution.

The results of our numerical calculations are presented in Table I. For definiteness we have taken $\gamma(0) = 50$ and $\gamma'(0) = 0$, ensuring in this way numerical stability, in particular for small values of p.

TABLE I

Numerical estimates for different terms in Eq. (3)

Eq. (3)	$0 \leqslant p \leqslant 0.5$	$0.5 \leqslant p \leqslant 4$	$4 \leqslant p \leqslant 10$	$B(\infty)/\sigma_{tot}(\infty)$
First term	-0.044	0.211	0.106	$0.37\ \mathrm{GeV}/c^{-2}/\mathrm{mb}$
Second term	-0.012	-0.001	0.0	

In principle the numbers presented for the first term could reveal some Δt dependence, since $\Delta C^{+}/\Delta t$ $(t = 0)$ was, of course, evaluated numerically. A direct check shows that this effect does not exceed 3% of our estimates. In general it seems that the phase shift results are well suited for an analysis like ours. Due to an extensive use of analyticity in both s and t variables as well as crossing symmetry a good fit to data can be obtained and this simplifies considerably our model independent low-energy calculations.

As seen from Table I the second term of Eq. (3) gives almost negligible contributions. The first one, however, reveals interesting features. Due to the well known (3/2, 3/2) resonance we observe in the first subdomain very strong *cancellations* (the relevant real parts of the considered amplitudes change the signs). Thus essentially the ratio $B(\infty)/\sigma_{tot}(\infty)$ is given by the integral over momenta from 0.5 up to 10 GeV/c. Above 10 GeV/c the integrals to a large extent are model dependent. A detailed analysis of their contributions would be beyond the scope of this paper. It could be, however, quite interesting to see the way in which intermediate and high-energy cancellations develop in order to satisfy Eq. (3). If, on the other hand, the GS property is also a low energy phenomenon [6], relations like Eq. (3) could prove to be quite useful as self-consistency constraints connecting via analyticity and crossing the high- and low-energy domains.

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