Vol. 1 (2008) *Acta Physica Polonica B Proceedings Supplement* No 1

A NEW (VERY) UNORTHODOX APPROACH TO EQUATIONS OF MOTION IN GENERAL RELATIVITY*

Ezra T. Newman

Department of Physics and Astronomy, University of Pittsburgh
Pittsburgh, PA 15260, USA

Gilberto Silva-Ortigoza

Facultad de Ciencias Físico Matemáticas
de la Universidad Autónoma de Puebla
Apartado Postal 1152, 72001 Puebla, Puebla, México

in collaboration with

Carlos Kozameh

FaMaF, Univ. of Cordoba, Cordoba, Argentina
kozameh@famaf.unc.edu.ar

(Received November 20, 2007)

From the study of the asymptotic behavior of the Einstein or Einstein–Maxwell fields, a rather unusual new structure was found. This structure which is associated with asymptotically shearfree null congruences, appears to have significant physical interest or consequences. More specifically it allows us to define, at future null infinity, the center-of-mass and center-of-charge with detailed equations of motion, for an interior gravitating-electromagnetic system. In addition it allows for a definition of total angular momentum with its evolution equations. Though at the present time the details remain obscure to us, nevertheless we feel that our version of equations of motion are closely related to the point of view of Myron Mathisson.

To Myron Mathisson who was far ahead of all of us and almost nobody knew it

PACS numbers: 03.30.De, 04.40.Nk

* Presented at the conference "Myron Mathisson: his life, work and influence on current research", Stefan Banach International Mathematical Center, Warsaw, Poland, 18–20 October, 2007 .

1. Introduction

It is well known in classical field theory that often local interior information can be found from the study of the asymptotic fields as, for example, in Maxwell theory, the total charge and multipole moments are obtained from different far-field components of the Maxwell tensor, while in General Relativity, (GR), the Bondi four-momentum [1] defined at future null infinity describes the total interior energy-momentum of a gravitating system. It is the purpose of this note to point out that a further asymptotic structure exists in GR that allows for a definition of center-of-mass and center-of-charge, with detailed equations of motion and a definition of total angular momentum with its own evolution equations. Though the calculations to obtain them in detail are lengthy, how they originate is relatively simple. One of the purposes of this note is to document this.

This structure, which arises for both asymptotically flat vacuum Einstein and Einstein–Maxwell fields, is associated with asymptotically shear-free null congruences and appears to have significant physical interest or consequences. Though we know of no *a priori* reason for this rather mysterious connection, it appears to be a fact. It is more fully discussed in the conclusion.

Since the material is unorthodox and out of the mainstream, we will proceed in a rather heuristic fashion and try to give an overview of the basic ideas. The details are being prepared for a more complete future presentation.

We start with a discussion of the asymptotic Bianchi identities (historically referred to as the Bondi supplementary conditions [1]) and the asymptotic Maxwell equations. By making a simple assumption and using the linearized equations, we easily see how the equations of motion originate. This assumption is then justified (using the properties of asymptotically shear-free null geodesic congruences) and then used in the full theory (with approximations) to obtain our equations of motion.

The justification is argued from the existence of a unique regular asymptotically shear-free null geodesic congruence in any asymptotically flat spacetime. Associated with this congruence is a unique complex analytic curve defined in the space of complex Poincare translations which has its action on complexified null infinity, *i.e.*, Penrose's (complexified) $\mathfrak{I}^+$. This curve then becomes, for its real part, the center of mass motion and for the imaginary part the angular momentum.

The physical situation we are dealing with is a complicated gravitating-electromagnetic system viewed from $\mathfrak{I}^+$, with a resulting Bondi asymptotic four-momentum. First of all the complex curve yields kinematic meaning to the Bondi four-momentum, in the sense of $P^a = Mv^a$ with v^a being the real part of world-line velocity. The Bondi energy-momentum expression and its evolution then yields explicit evolution equations for both the real and the imaginary parts of the curve. The imaginary part, which is interpreted as the spin-angular momentum, satisfies its own evolution equation while the real part yields equations of motion for a spinning particle — with spin coupling terms. Much of the physical identification arises from a comparison of the radiation terms — both electromagnetic and gravitational — with the variables associated with the complex world-line. We even obtain the classical radiation reaction terms with the 'correct' numerical coefficients without model building.

The approach to motion in GR that we use is completely outside of mainstream ideas and is therefore difficult to understand on first viewing. It is the reason we are giving heuristic description rather than giving a detailed presentation. To help, several points must be clarified at the start:

(a) We are dealing with the total system — a closed gravitating-electromagnetic system viewed from the far field. No external forces are acting on it.

(b) We are dealing with real space-times — but we are assuming that all the relevant fields and functions are analytic and can be extended at least a small way into the complex space-time.

(c) In some sense this work is "observational theory" since there was no *a priori* reason for the ideas developed here to yield anything of physical value or interest. The observation was that shear-free or asymptotically shear-free null geodesic congruences are of considerable importance in GR and (among other items [2]) that they are connected in an intrinsic way to equations of motion.

2. The asymptotic Bianchi identities

In the study of asymptotically flat space-times (Einstein or Maxwell–Einstein) after one integrates the radial behavior of the relevant Einstein equations [3] one ends with a set of equations that 'live' on null infinity or $\mathfrak{I}^+$. $\mathfrak{I}^+$, a three-dimensional null surface, S^2xR, coordinatized by the Bondi coordinates $(u,\zeta,\overline{\zeta})$, u real and $(\zeta,\overline{\zeta}\ )$ the complex stereographic coordinate on the S^2, represents the future endpoints of all null geodesics. The remaining variables are essentially the leading terms in different components of the Weyl tensor. The basic asymptotic relationship, (the peeling theorem [3]), using spin-coefficient notation, is

$$\psi_0 \ = \ \frac{\psi_0^0}{r^5} + 0(r^{-6})\,, \tag{1}$$

$$\psi_1 = \frac{\psi_1^0}{r^4} + 0(r^{-5}), \tag{2}$$

$$\psi_2 = \frac{\psi_2^0}{r^3} + 0(r^{-4}), \tag{3}$$

$$\psi_3 = \frac{\psi_3^0}{r^2} + 0(r^{-3}), \tag{4}$$

$$\psi_4 = \frac{\psi_4^0}{r} + 0(r^{-2}), \tag{5}$$

with similar results for the asymptotic Maxwell field

$$\phi_0 = \frac{\phi_0^0}{r^3} + 0(r^{-4}), \tag{6}$$

$$\phi_1 = \frac{\phi_1^0}{r^2} + 0(r^{-3}), \tag{7}$$

$$\phi_2 = \frac{\phi_2^0}{r} + 0(r^{-2}). \tag{8}$$

The leading terms satisfy the asymptotic Bianchi identities and asymptotic Maxwell equations [3]

$$(\psi_0^0)^{\cdot} = -\eth\psi_1^0 + 3\sigma\psi_2^0 + 3k\phi_0^0\overline{\phi}_2^0, \tag{9}$$

$$(\psi_1^0)^{\cdot} = -\eth\psi_2^0 + 2\sigma\psi_3^0 + 2k\phi_1^0\overline{\phi}_2^0, \tag{10}$$

$$(\psi_2^0)^{\cdot} = -\eth\psi_3^0 + \sigma\psi_4^0 + k\phi_2^0\overline{\phi}_2^0, \tag{11}$$

$$\psi_3^0 = \eth(\overline{\sigma})^{\cdot}, \tag{12}$$

$$\psi_4^0 = -(\overline{\sigma})^{\cdot\cdot}, \tag{13}$$

$$\overline{\Psi} = \Psi = \psi_2^0 + \eth^2\overline{\sigma} + \sigma(\overline{\sigma})^{\cdot}, \tag{14}$$

$$(\phi_0^0)^{\cdot} + \eth\phi_1^0 - \sigma\phi_2^0 = 0, \tag{15}$$

$$(\phi_1^0)^{\cdot} + \eth\phi_2^0 = 0, \tag{16}$$

where dot indicates u-derivatives, the asymptotic shear, (the null data),

$$\sigma = \sigma(u, \zeta, \overline{\zeta})$$

is an arbitrary complex spin-wt-2 function on $\mathfrak{I}^+$ and

$$k = \frac{2G}{c^4}. \tag{17}$$

Our idea is to first summarize known physical ideas from these equations and then with a simple assumption (in the linearized version) find further physical content in the equations.

Each of the terms will be expanded in spin-s tensor harmonics [4] as

$$\begin{aligned}
\phi_0^0 &= \phi_{0i}^0 Y_{1i}^1 + \phi_{0ij}^0 Y_{2ij}^1 + \dots\,, \\
\phi_1^0 &= Q + \phi_{1i}^0 Y_{1i}^0 + \phi_{1ij}^0 Y_{2ij}^0 + \,.\,, \\
\phi_2^0 &= \phi_{2i}^0 Y_{1i}^{-1} + \phi_{2ij}^0 Y_{2ij}^{-1} + \dots\,, \\
\psi_1^0 &= \psi_{1i}^0 Y_{1i}^1 + \psi_{1ij}^0 Y_{2ij}^1 + \dots\,, \\
\psi_2^0 &= \Upsilon + \psi_{2i}^0 Y_{1i}^0 + \psi_{2ij}^0 Y_{2ij}^0 + \dots\,.
\end{aligned} \tag{18}$$

First we point out that the $l = (0,1)$ components of the Mass Aspect, Ψ, contain the Bondi four momentum via

$$\Psi(u,\zeta,\overline{\zeta}) = \overline{\Psi}(u,\zeta,\overline{\zeta}) = -\frac{2\sqrt{2}G}{c^2}M - \frac{6G}{c^3}P^i Y_{1i}^0 + \dots \tag{19}$$

i.e., the $l = 0$ part is basically the mass while the $l = 1$ part is the total linear momentum of the system with the further property that the *mass and momentum are real*. In the linearized version of the mass aspect we would have

$$\psi_2^0\,(u,\zeta,\overline{\zeta}) = \overline{\psi}_2^0\,(u,\zeta,\overline{\zeta}) = -\frac{2\sqrt{2}G}{c^2}M - \frac{6G}{c^3}P^i Y_{1i}^0 + \dots\,. \tag{20}$$

Now considering the linearization (off Reissner–Nordstrom) of Eq. (10), obtaining

$$(\psi_1^0)^{\cdot} = -\eth\psi_2^0 + 2k\phi_1^0\overline{\phi}_2^0\,.$$

where $\phi_1^0 = Q$, the total coulomb charge and ϕ_2^0 is proportional to the complex (electric + i magnetic) dipole radiation fields written as $Q\,\ddot{\eta}_i$ with $\ddot{\eta}_i$ the "complex" center of charge. Looking only at the $l = 1$ part of this equation we have

$$\psi_{1i}^{0\,\cdot} = 2\psi_{2i}^0 + BQ^2\ddot{\eta}_i$$

or, for ease, hiding the known numerical factors in the A and B, we have

$$\psi_{1i}^{0\,\cdot} = AP_i + BQ^2\ddot{\eta}_i\,. \tag{21}$$

Now, roughly speaking, it would appear as if ψ_{1i}^0 could be interpreted as being proportional to the mass dipole moment (so that its derivative would look like Mv) and thus have the form

$$\psi_{1i}^0 = \alpha M\xi_i\,, \tag{22}$$

where ξ_i would be interpreted as the center of mass and α is an adjustable parameter to be determined. However, since ψ_{1i}^0 is in general complex, we will take ξ_i as a complex displacement vector depending on u. This now becomes our assumption: *we assume* that $\psi^0_{1i} = \alpha M \xi_i$. This leads to

$$P_i = \frac{\alpha}{A} M v_i - \frac{B}{A} Q^2 \ddot{\overline{\eta}}_i \,.$$

If $\alpha = A$, $\frac{B}{A} = \frac{2}{3} c^{-3}$ and $\eta_i = \xi_i$, we would have

$$\begin{aligned} P_i &= M v_i - \frac{2}{3} c^{-3} Q^2 (\overline{\xi}_i)^{\cdot\cdot} \,, \\ v_i &= \dot{\xi}_i = v_{\mathrm{R}i} + i v_{\mathrm{I}i} = \dot{\xi}_{\mathrm{R}i} + i \dot{\xi}_{\mathrm{I}i} \,. \end{aligned}$$

Using the conditions that M and P_i are real, leads to

$$\begin{aligned} P_i &= M v_{\mathrm{R}i} - \frac{2}{3} c^{-3} Q^2 (\xi_{\mathrm{R}i})^{\cdot\cdot} \,, \\ 0 &= M v_{\mathrm{I}i} + \frac{2}{3} c^{-3} Q^2 (\xi_{\mathrm{I}i})^{\cdot\cdot} \,, \end{aligned}$$

where the subscripts R and I denote the real and imaginary parts. The first of this pair is the well-known classical kinematic expression for the linear momentum of a charged particle that leads to the radiation reaction force. The second of the pair, [when $Q = 0$] expresses conservation of angular momentum if angular momentum was defined by

$$S_i = M c \xi_{\mathrm{I}i} \,.$$

(In the case of the Kerr metric, this, in fact, is the angular momentum.)

The equations of motion now follow immediately from the Eq. (11) by simply replacing the Bondi momentum by its kinematic expression, leading in the linearized version, to

$$M (v_{\mathrm{R}i})^{\cdot} = \frac{2}{3c^3} Q^2 (\xi_{\mathrm{R}i})^{\cdot\cdot\cdot} \,,$$

i.e., equations of motion with the classical radiation reaction force.

This heuristic derivation of the equations of motion involved, in addition to the linearization, the arbitrary introduction of the complex displacement vector ξ_i. In the next sections we will show that there is a very pretty geometric construction such that this complex vector arises naturally with the correct value for the constant α. This construction is then used in the full theory.

3. Justification: the geometric origin for the complex displacement

We begin a digression with a few words about null infinity, $\mathfrak{I}^+$. We are working with an arbitrary choice of Bondi coordinates $(u, \zeta, \overline{\zeta})$ and Bondi tetrad $(l, n, m, \overline{m})$ on $\mathfrak{I}^+$. (All our results are invariant under BMS transformations of the Bondi coordinates.) At each point of $\mathfrak{I}^+$we consider an arbitrary *null direction pointing backwards* into the space-time denoted by l^* and given by the null rotation around n^a,

$$\begin{aligned} l^{*a} &= l^a - L\overline{m}^a - \overline{L}m^a + \overline{L}Ln^a \,, \\ m^{*a} &= m^a - Ln^a \,, \\ n^{*a} &= n^a \,, \end{aligned} \tag{23}$$

where

$$L = L(u, \zeta, \overline{\zeta})$$

is, for the moment, an arbitrary angle field on $\mathfrak{I}^+$. The family of all such null directions, l^{*a}, defines the initial directions at $\mathfrak{I}^+$ of a null geodesic congruence going backwards into the space-time. If the function $L = L(u, \zeta, \overline{\zeta})$ satisfies the differential relation [5, 6]

$$\eth L + LL^{\cdot} = \sigma(u, \zeta, \overline{\zeta}) \tag{24}$$

then L determines a shear-free null geodesic congruence. We are interested only in the regular congruences, *i.e.*, those that do not contain geodesics that are tangent to $\mathfrak{I}^+$. The regular family of solutions L are determined by the following:

• Defining a potential for L, namely $\tau = T(u, \zeta, \overline{\zeta})$, with its inverse function, $u = X(\tau, \zeta, \overline{\zeta})$, by

$$L = -\frac{\eth T}{T^{\cdot}} \,. \tag{25}$$

$L(u, \zeta, \overline{\zeta})$ can be written parametrically (after taking implicit derivatives) as

$$\begin{aligned} L(u, \zeta, \overline{\zeta}) &= \eth_{(\tau)} X(\tau, \zeta, \overline{\zeta}) \,, \\ \tau &= T(u, \zeta, \overline{\zeta}) \end{aligned}$$

and Eq. (24) becomes

$$\eth^2_{(\tau)} X(\tau, \zeta, \overline{\zeta}) = \sigma(X, \zeta, \overline{\zeta}) \,. \tag{26}$$

The subscript (τ) means $\eth$ holding τ constant.

• The solutions [6,7] to Eq. (26) for $X(\tau,\zeta,\overline{\zeta})$ and hence also for $L(u,\zeta,\overline{\zeta})$, are determined by four arbitrary complex functions, ξ^a of the complex parameter τ, that we interpret as an arbitrary world-line in the space of complex Poincare translations acting on complexified $\mathfrak{I}^+$. The solution can be written in a spherical harmonic expansion as

$$\begin{aligned} u &= X(\tau,\zeta,\overline{\zeta}) \\ &= \xi^a(\tau)\widehat{l}_a(\zeta,\overline{\zeta}) + X_{l\geq 2}(\tau,\zeta,\overline{\zeta})\,, \end{aligned} \tag{27}$$

$$L(u,\zeta,\overline{\zeta}) = \eth_{(\tau)}X(\tau,\zeta,\overline{\zeta}) = \xi^a(\tau)\widehat{m}_a(\zeta,\overline{\zeta}) + \eth_{(\tau)}X_{l\geq 2}(\tau,\zeta,\overline{\zeta})\,, \tag{28}$$

where $\widehat{l}_a(\zeta,\overline{\zeta})$ are the first four spherical harmonics arranged as a Lorentzian null vector

$$\begin{aligned} \widehat{l}_a(\zeta,\overline{\zeta}) &= \frac{\sqrt{2}}{2}\left(1, \frac{\zeta+\overline{\zeta}}{1+\zeta\overline{\zeta}}, -i\frac{\zeta-\overline{\zeta}}{1+\zeta\overline{\zeta}}, \frac{-1+\zeta\overline{\zeta}}{1+\zeta\overline{\zeta}}\right)\,, \\ \widehat{m}_a(\zeta,\overline{\zeta}) &= \frac{\sqrt{2}}{2P}\left(0, 1-\overline{\zeta}^2, -i\left(1+\overline{\zeta}^2\right), 2\overline{\zeta}\right)\,, \end{aligned}$$

and $\widehat{m}_a = \eth\widehat{l}_a$. We see that the first four harmonics of X are arbitrary, while the higher, $l \geqq 2$, are determined by $\sigma(u,\zeta,\overline{\zeta})$.

We thus have:

- Theorem: *In any asymptotically flat space-time each member of the set of regular asymptotically shear-free null geodesic congruences is determined by an arbitrary complex word-line in the space of complex Poincare translations.* ■

  After it is made unique by a specific physical argument, complex world-line becomes the complex displacement vector of our heuristic discussion. In order to describe this argument we first note that the null rotation Eq. (23) changes the tetrad components of both the Weyl tensor and Maxwell tensor. For us the relevant changes are

$$\begin{aligned} \psi_1^{*0} &= \psi_1^0 - 3L\psi_2^0 + 3L^2\psi_3^0 - L^3\psi_4^0\,, \\ \phi_0^{*0} &= \phi_0^0 - 2L\phi_1^0 + L^2\phi_2^0\,. \end{aligned}$$

Remark 1 *If by chance we were considering a Robinson–Trautman space-time or a Type II twisting space-time, (with or without a Maxwell field), there would be a unique choice of the complex world-line (or its associated $L(u,\zeta,\overline{\zeta})$) such that*

$$\begin{aligned} \psi_1^{*0} &= 0\,, \\ \phi_0^{*0} &= 0\,. \end{aligned}$$

In an arbitrary asymptotically flat space-time such an $L(u, \zeta, \overline{\zeta})$ cannot be found.

Though we cannot, in general, make $\psi_1^{*0} = 0$ (nor the $\phi_0^{*0} = 0$), nevertheless using the freedom to freely choose the complex world-line, we can make *either* the $l = 1$ harmonic of ψ_1^{*0} or of ϕ_0^{*0} vanish. In other words, in general, when a Maxwell field is present, there are two distinct complex world-lines making, respectively, the $l = 1$ harmonic of ψ_1^{*0} or of ϕ_0^{*0} vanish. For the remainder of this work *we restrict the discussion* (when there is a Maxwell field) *to the case where the two world-lines coincide.*

Without a Maxwell field, there is no restriction. We thus choose $L(u, \zeta, \overline{\zeta})$ so that

$$\psi_1^{*0}|_{l=1} = 0 = \left(\psi_1^0 - 3L\psi_2^0 + 3L^2\psi_3^0 - L^3\psi_4^0\right)|_{l=1}$$

or determine ψ_{1i}^0 by

$$\psi_{1i}^0 = \left(3L\psi_2^0 - 3L^2\psi_3^0 + L^3\psi_4^0\right)|_{l=1}\,. \tag{29}$$

It is this condition that is the geometric justification for Eq. (22). From Eqs. (28) and (20), we immediately see the form of the first (or linear) term, *i.e.*,

$$(3L\psi_2^0)_i = \alpha M \xi_i$$

with the numerical factor, no longer arbitrary, but now explicitly determined. The full expression for ψ_{1i}^0 is far more complicated than it was in our heuristic description.

The plan of operation is to take Eq. (29) and use it in Eqs. (10) and (19) to determine the Bondi three-momentum which takes the kinematic form

$$P^i = M(\xi^i)^{\cdot} + \dots\,.$$

The required calculations are long and rather complicated with little hope of finding exact expressions. Approximations are necessary. Specifically we assume that we are doing perturbations off either Schwarzschild or Reissner–Nordstrom. The mass and charge are considered as zero order while other basic quantities are treated as small or first order. In non-linear expressions we keep only terms up to second order. In addition, in the spherical harmonic expansions we keep only the $l = 0, 1, 2$ harmonics. Extensive use is made of Clebsch–Gordon expansions. In the remainder of the paper we will simply report on the main results.

4. Results

We start with Eq. (27), *i.e.*, $X(\tau,\zeta,\overline{\zeta})$ written (with first order ξ^i) in the form

$$\begin{aligned} u &\equiv \frac{w}{\sqrt{2}} = X(\tau,\zeta,\overline{\zeta}) \\ &= \frac{1}{\sqrt{2}}\xi^0(\tau) - \frac{1}{2}\xi^i(\tau)Y^0_{1i}(\zeta,\overline{\zeta}) + \xi^{ij}(\tau)Y^0_{2ij}(\zeta,\overline{\zeta}) + \ldots, \qquad (30) \\ \xi^a &= (\xi^0,\xi^i(\tau)) = (\tau,\xi^i(\tau)), \qquad (31) \end{aligned}$$

and its inverse

$$\tau = T(u,\zeta,\overline{\zeta}) = w + \frac{\sqrt{2}}{2}\xi^i(w)Y^0_{1i}(\zeta,\overline{\zeta}) - \sqrt{2}\xi^{ij}(w)Y^0_{1ij}(\zeta,\overline{\zeta}) + \ldots \qquad (32)$$

with w real. The complex $\xi^i(w)$ is decomposed into its real and imaginary parts

$$\begin{aligned} \xi^a(w) &= \xi^a_{\mathrm{R}}(w) + i\xi^a_{\mathrm{I}}(w)\,, \\ \xi^{a\,\prime} &\equiv v^a(w) = v^a_{\mathrm{R}}(w) + iv^a_{\mathrm{I}}(w)\,. \end{aligned}$$

The freedom in the choice of $\tau \Rightarrow \tau^* = F(\tau)$ allows us to give $v^a_{\mathrm{R}}(w)$ the Lorentzian norm, $v^2_{\mathrm{R}} = 1$.

Note that we can simplify all the calculations by reversing the standard point of view where the free characteristic data for the gravitational field is the Bondi shear, $\sigma(u,\zeta,\overline{\zeta})$. We can, instead, take $X(\tau,\zeta,\overline{\zeta})$ as the free data and then determine the shear, parametrically, from

$$\eth^2_{(\tau)}X(\tau,\zeta,\overline{\zeta}) = \sigma(X,\zeta,\overline{\zeta})\,, \qquad (33)$$

implying

$$\begin{aligned} \sigma(u,\zeta,\overline{\zeta}) &= 24\xi^{ij}(\tau)Y^2_{2ij}\,, \\ u &= X(\tau,\zeta,\overline{\zeta})\,. \end{aligned}$$

This avoids having to integrate beginning with knowledge of $\sigma(u,\zeta,\overline{\zeta})$.
Remark 2 *Though u is the conventional Bondi time, it is more appropriate to use $w = \sqrt{2}u$, it being the retarded time. Derivatives with respect to u are denoted by dot, i.e.,* $(\dot{\ })$, while w derivatives are given by a prime, $(')$.
Remark 3 *Almost all expressions are first calculated with the complex τ parameter and then re-expressed in terms of the w via Eq. (32). It is this process that is lengthy.*

Our first task is to describe the relevant Maxwell components obtained from the integration of Eqs. (15) and (16), [9]

$$\begin{aligned}\phi_{1i}^0 &= \sqrt{2}Q\left[v^i(w)+i\frac{1}{2}\epsilon_{ijl}v^{l\prime}\xi^j+N^{i\prime}\right]+i\sqrt{2}Q\epsilon_{kji}v^{j\prime}\xi^k-\frac{2}{5}(\phi_{0ki}^{0\prime}\xi^k)^\prime \\ &\quad -\frac{72}{5}Qv^jv^{ij}+i\frac{24\sqrt{2}}{15}\epsilon_{lji}\phi_{0kj}^{0\prime\prime}\xi^{lk}-i\frac{24\sqrt{2}}{5}\epsilon_{jli}(\phi_{0ml}^{0\prime}\xi^{jm})^\prime\,, \\ \phi_{2i}^0 &= -2Q\left[v^{i\prime}+i\frac{1}{2}\epsilon_{ijl}(v^{l\prime}\xi^j)^\prime+N^{i\prime\prime}\right]-i2Q\epsilon_{kji}(v^{j\prime}\xi^k)^\prime+\frac{2\sqrt{2}}{5}(\phi_{0ki}^{0\prime}\xi^k)^{\prime\prime} \\ &\quad +\frac{72\sqrt{2}}{5}Q(v^jv^{ij})^\prime-\frac{48}{15}i\epsilon_{lji}(\phi_{0kj}^{0\prime\prime}\xi^{lk})^\prime+i\frac{48}{5}\epsilon_{jli}(\phi_{0ml}^{0\prime}\xi^{jm})^{\prime\prime}\,, \\ N^i &= \frac{6\sqrt{2}}{5}v^k\xi^{ki}-\frac{18\sqrt{2}}{5}v^{ki}\xi^k+i\frac{144}{5}\epsilon_{jmi}v^{kj}\xi^{mk}\,.\end{aligned}$$

Remark 4 *We have (for notational simplicity) totally abused standard notion. We allow the indices* $(i,j,k,l\ldots)$*, which are Euclidean, to be raised and lowered with impunity and allow repeated indices, e.g.,* $v^k\xi^{ki}$*, to indicate summation.*

Using these fields, with our approximations, Eq. (29) becomes

$$\begin{aligned}\psi_{1i}^0 &= 3\varUpsilon\left[\xi^i(w)+i\frac{1}{2}\epsilon_{kji}v^k\xi^j+N^i\right]+i\frac{3\sqrt{2}}{2}\epsilon_{lji}\psi_{2j}^0\xi^l-\frac{18}{5}\psi_{2ij}^0\xi^j \\ &\quad -i\frac{6\cdot 36\sqrt{2}}{5}\psi_{2kj}^0\xi^{kl}\epsilon_{lji}-\frac{6\cdot 18}{5}\psi_{2j}^0\xi^{ij}\,. \qquad (34)\end{aligned}$$

Substituting Eq. (34) in (10) leads, after considerable effort and use of the real part of reality conditions, (14), to the full kinematic expression for the linear momentum of the system.

$$\begin{aligned}P^k &= Mv_{\mathrm{R}}^k+\frac{M}{c}\left(v_{\mathrm{R}}^iv_{\mathrm{I}}^j-\xi_{\mathrm{I}}^iv_{\mathrm{R}}^{j\prime}-(\xi_{\mathrm{R}}^iv_{\mathrm{I}}^j)^\prime\right)\epsilon_{ijk}-\frac{2Q^2}{3c^3}v_{\mathrm{R}}^{k\prime} \\ &\quad +\frac{2Q^2}{3c^4}[2\xi_{\mathrm{I}}^iv_{\mathrm{R}}^{j\prime}-\xi_{\mathrm{R}}^iv_{\mathrm{I}}^{j\prime}+v_{\mathrm{R}}^iv_{\mathrm{I}}^j]^\prime\epsilon_{ijk}+\varPi^k\,, \qquad (35) \\ \varPi^k &= -\frac{M}{c}\left(\frac{6\sqrt{2}}{5}\left[8\left(\xi_{\mathrm{R}}^{ki}v_{\mathrm{R}}^i-\xi_{\mathrm{I}}^{ki}v_{\mathrm{I}}^i\right)+3\left(v_{\mathrm{R}}^{ki}\xi_{\mathrm{R}}^i-v_{\mathrm{I}}^{ki}\xi_{\mathrm{I}}^i\right)\right]^\prime\right. \\ &\quad \left.+\frac{144}{5}\left(v_{\mathrm{R}}^{il\prime}\xi_{\mathrm{I}}^{ij}+v_{\mathrm{I}}^{il\prime}\xi_{\mathrm{R}}^{ij}\right)\epsilon_{ljk}\right) \\ &\quad +\frac{Q^2}{3c^4}\left(\frac{18(6)\sqrt{2}}{5}\left(v_{\mathrm{R}}^{i\prime}\xi_{\mathrm{R}}^{ik}+v_{\mathrm{I}}^{i\prime}\xi_{\mathrm{I}}^{ik}\right)^\prime+\frac{96\sqrt{2}}{5}\left(v_{\mathrm{R}}^{ki}v_{\mathrm{R}}^i-v_{\mathrm{I}}^{ki}v_{\mathrm{I}}^i\right)^\prime\right.\end{aligned}$$

$$
\begin{aligned}
&-\frac{12\sqrt{2}}{5}\left(\xi_{\mathrm{R}}^{ki}v_{\mathrm{R}}^{i\prime}-\xi_{\mathrm{I}}^{ki}v_{\mathrm{I}}^{i\prime}\right)'+\frac{(36)\sqrt{2}}{5}\left(v_{\mathrm{R}}^{ki\prime}\xi_{\mathrm{R}}^{i}-v_{\mathrm{I}}^{ki\prime}\xi_{\mathrm{I}}^{i}\right)'\\
&+\frac{288}{5}\left(v_{\mathrm{R}}^{il\prime}\xi_{\mathrm{I}}^{ij}+v_{\mathrm{I}}^{il\prime}\xi_{\mathrm{R}}^{ij}\right)'\epsilon_{ljk}\Bigg)\\
&+\frac{Q}{3c^4}\left(\frac{24}{5\sqrt{2}c}\left(D_E^{ij\prime\prime}\xi_{\mathrm{I}}^{il}+D_M^{ij\prime\prime}\xi_{\mathrm{R}}^{il}\right)''\epsilon_{ljk}-\frac{1}{5c}\left(\xi_{\mathrm{R}}^{i}D_E^{ik\prime\prime}-\xi_{\mathrm{I}}^{i}D_M^{ik\prime\prime}\right)''\right.\\
&-\frac{24}{15\sqrt{2}c}\left(D_E^{ij\prime\prime\prime}\xi_{\mathrm{I}}^{il}+D_M^{ij\prime\prime\prime}\xi_{\mathrm{R}}^{il}\right)'\epsilon_{ljk}\\
&\left.-\frac{1}{5c}\left(v_{\mathrm{R}}^{i\prime}D_E^{ik\prime\prime}+v_{\mathrm{I}}^{i\prime}D_M^{ik\prime\prime}\right)+\frac{1}{5c}\left(v_{\mathrm{R}}^{i}D_E^{ik\prime\prime\prime}+v_{\mathrm{I}}^{i}D_M^{ik\prime\prime\prime}\right)\right)\\
&+\frac{1}{490c^6}\left(D_M^{ij\prime\prime}D_E^{il\prime\prime\prime}-D_E^{ij\prime\prime}D_M^{il\prime\prime\prime}\right)\epsilon_{ljk}-\frac{36\sqrt{2}\,c^2}{5G}\left(\xi_{\mathrm{R}}^{i}\xi_{\mathrm{R}}^{ik}+\xi_{\mathrm{I}}^{i}\xi_{\mathrm{I}}^{ik}\right)'\\
&+\frac{2(24)^2c^2}{5G}\left(v_{\mathrm{R}}^{ij}\xi_{\mathrm{I}}^{il}+\xi_{\mathrm{R}}^{ij}v_{\mathrm{I}}^{il}\right)\epsilon_{ljk}\,. \qquad (36)
\end{aligned}
$$

All non-linear terms involving the quadrupole terms are gathered into the Π^k.

The vanishing of the imaginary part of the reality condition yields the relations

$$
\begin{aligned}
J^{k\prime} \;=\;& \frac{2Q^2}{3c^3}\left(v_{\mathrm{R}}^{i\prime}v_{\mathrm{R}}^{j}+v_{\mathrm{I}}^{i\prime}v_{\mathrm{I}}^{j}\right)\epsilon_{ljk}+\frac{1}{90c^5}\left(D_E^{ij\prime\prime}D_E^{il\prime\prime\prime}+D_M^{ij\prime\prime}D_M^{il\prime\prime\prime}\right)\epsilon_{ljk}\\
&-\frac{(24)^2c^3}{5G}\left(\xi_{\mathrm{R}}^{il}v_{\mathrm{R}}^{ij}+\xi_{\mathrm{I}}^{il}v_{\mathrm{I}}^{ij}\right)\epsilon_{ljk}\,, \qquad (37)
\end{aligned}
$$

where J^k, identified (from the dynamics rather than through the conventional symmetry argument) as the total angular momentum, is given by

$$
\begin{aligned}
J^k \;\equiv\;& Mc\xi_{\mathrm{I}}^{k}+\frac{2Q^2}{3c^2}v_{\mathrm{I}}^{k}+M\left(\xi_{\mathrm{R}}^{i}v_{\mathrm{R}}^{j}-\xi_{\mathrm{I}}^{i}v_{\mathrm{I}}^{j}\right)\epsilon_{ijk}\\
&-\frac{2Q^2}{3c^3}\left(\xi_{\mathrm{R}}^{i}v_{\mathrm{R}}^{j\prime}+2\xi_{\mathrm{I}}^{i}v_{\mathrm{I}}^{j\prime}\right)\epsilon_{ijk}+K^k\,,\\
K^k \;=\;& -M\left(\frac{6\sqrt{2}}{5}\left[8\left(\xi_{\mathrm{R}}^{ki}v_{\mathrm{I}}^{i}+\xi_{\mathrm{I}}^{ki}v_{\mathrm{R}}^{i}\right)+3\left(v_{\mathrm{R}}^{ki}\xi_{\mathrm{I}}^{i}+v_{\mathrm{I}}^{ki}\xi_{\mathrm{R}}^{i}\right)\right]\right.\\
&\left.-\frac{144}{5}\left(v_{\mathrm{R}}^{il}\xi_{\mathrm{R}}^{ij}-v_{\mathrm{I}}^{il}\xi_{\mathrm{I}}^{ij}\right)\epsilon_{ljk}\right)\\
&-\frac{Q^2}{3c^3}\left(-\frac{18(6)\sqrt{2}}{5}\left(v_{\mathrm{R}}^{i\prime}\xi_{\mathrm{I}}^{ik}-v_{\mathrm{I}}^{i\prime}\xi_{\mathrm{R}}^{ik}\right)+\frac{96\sqrt{2}}{5}\left(v_{\mathrm{R}}^{ki}v_{\mathrm{I}}^{i}+v_{\mathrm{I}}^{ki}v_{\mathrm{R}}^{i}\right)\right.\\
&-\frac{12\sqrt{2}}{5}\left(\xi_{\mathrm{R}}^{ki}v_{\mathrm{I}}^{i\prime}+\xi_{\mathrm{I}}^{ki}v_{\mathrm{R}}^{i\prime}\right)+\frac{(36)\sqrt{2}}{5}\left(v_{\mathrm{R}}^{ki\prime}\xi_{\mathrm{I}}^{i}+v_{\mathrm{I}}^{ki\prime}\xi_{\mathrm{R}}^{i}\right)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{288}{5}\left(v_{\mathrm{R}}^{il}{}'\xi_{\mathrm{R}}^{ij}-v_{\mathrm{I}}^{il}{}'\xi_{\mathrm{I}}^{ij}\right)\epsilon_{ljk}\Bigg)\\
&+\frac{Q}{15c^4}\left(\xi_{\mathrm{R}}^{i}D_{M}^{ik}{}'''+\xi_{\mathrm{I}}^{i}D_{E}^{ik}{}'''+2v_{\mathrm{I}}^{i}D_{E}^{ik}{}''+4\sqrt{2}\left(2\xi_{\mathrm{R}}^{il}D_{E}^{ij}{}'''-2\xi_{\mathrm{I}}^{il}D_{M}^{ij}{}'''\right.\right.\\
&\left.\left.+3v_{\mathrm{R}}^{il}D_{E}^{ij}{}''-3v_{\mathrm{I}}^{il}D_{M}^{ij}{}''\right)\epsilon_{ljk}\right)-\frac{36\sqrt{2}\,c^3}{5G}\left(\xi_{\mathrm{I}}^{i}\xi_{\mathrm{R}}^{ik}-\xi_{\mathrm{R}}^{i}\xi_{\mathrm{I}}^{ik}\right). \qquad (38)
\end{aligned}
$$

D_E^{lj} and D_M^{ij} are respectively the electric quadrupole and magnetic quadrupole moments found from the $l=2$ radiation term in the solution of the Maxwell equations.

Eq. (37) describes the loss of angular momentum. These relations, though now quite complicated, do simplify considerably when the dynamic equations for P^i are used.

{In the absence of a Maxwell field, these equations for P and J simplify to

$$
\begin{aligned}
P^k &= Mv_{\mathrm{R}}^k+\frac{M}{c}(v_{\mathrm{R}}^i v_{\mathrm{I}}^j-\xi_{\mathrm{I}}^i v_{\mathrm{R}}^{j\prime}-(\xi_{\mathrm{R}}^i v_{\mathrm{I}}^j)')\epsilon_{ijk}+\Pi^k\,,\\
\Pi^k &= -\frac{M}{c}\left(\frac{6\sqrt{2}}{5}\left[8\left(\xi_{\mathrm{R}}^{ki}v_{\mathrm{R}}^i-\xi_{\mathrm{I}}^{ki}v_{\mathrm{I}}^i\right)+3\left(v_{\mathrm{R}}^{ki}\xi_{\mathrm{R}}^i-v_{\mathrm{I}}^{ki}\xi_{\mathrm{I}}^i\right)\right]'\right.\\
&\quad\left.+\frac{144}{5}(v_{\mathrm{R}}^{il\prime}\xi_{\mathrm{I}}^{ij}+v_{\mathrm{I}}^{il\prime}\xi_{\mathrm{R}}^{ij})\epsilon_{ljk}\right)-\frac{36\sqrt{2}\,c^2}{5G}\left(\xi_{\mathrm{R}}^i\xi_{\mathrm{R}}^{ik}+\xi_{\mathrm{I}}^i\xi_{\mathrm{I}}^{ik}\right)'\\
&\quad+\frac{2(24)^2c^2}{5G}\left(v_{\mathrm{R}}^{ij}\xi_{\mathrm{I}}^{il}+\xi_{\mathrm{R}}^{ij}v_{\mathrm{I}}^{il}\right)\epsilon_{ljk}\,.
\end{aligned}
$$

and

$$
\begin{aligned}
J^{k\prime} &= -\frac{(24)^2c^3}{5G}\left(\xi_{\mathrm{R}}^{il}v_{\mathrm{R}}^{ij}+\xi_{\mathrm{I}}^{il}v_{\mathrm{I}}^{ij}\right)\epsilon_{ljk}\,,\\
J^k &\equiv Mc\xi_{\mathrm{I}}^k+M\left(\xi_{\mathrm{R}}^i v_{\mathrm{R}}^j-\xi_{\mathrm{I}}^i v_{\mathrm{I}}^j\right)\epsilon_{ijk}+K^k\,,\\
K^k &= -M\left(\frac{6\sqrt{2}}{5}\left[8\left(\xi_{\mathrm{R}}^{ki}v_{\mathrm{I}}^i+\xi_{\mathrm{I}}^{ki}v_{\mathrm{R}}^i\right)+3\left(v_{\mathrm{R}}^{ki}\xi_{\mathrm{I}}^i+v_{\mathrm{I}}^{ki}\xi_{\mathrm{R}}^i\right)\right]\right.\\
&\quad\left.-\frac{144}{5}\left(v_{\mathrm{R}}^{il}\xi_{\mathrm{R}}^{ij}-v_{\mathrm{I}}^{il}\xi_{\mathrm{I}}^{ij}\right)\epsilon_{ljk}\right)-\frac{36\sqrt{2}\,c^3}{5G}\left(\xi_{\mathrm{I}}^i\xi_{\mathrm{R}}^{ik}-\xi_{\mathrm{R}}^i\xi_{\mathrm{I}}^{ik}\right)\,.
\end{aligned}
$$

In this form it is simpler to look for the physical meaning of many of the terms.}

Though we do not have a fundamental argument for the identification of J^k with angular momentum, there are several points worth noting.

- For the charged spinning particle metric [8] the spin angular momentum is given by
$$S^k = J^k \equiv Mc\xi_{\mathrm{I}}^k\,.$$

- The second term in J, namely
$$M\xi_{\mathrm{R}}^i v_{\mathrm{R}}^j \epsilon_{ijk} = \xi_{\mathrm{R}}^i p_{\mathrm{R}}^j \epsilon_{ijk} = (\boldsymbol{r}x\boldsymbol{p})_k$$
is the orbital angular momentum which has appeared naturally. The third term in J is identified as the precession of the spin vector.

- The second term in P^i, namely
$$\frac{M}{c}\xi_{\mathrm{I}}^{i\prime} v_{\mathrm{R}}^j \epsilon_{ijk} = c^{-2} S^{i\prime} v_{\mathrm{R}}^j \epsilon_{ijk} = c^{-2}(\boldsymbol{S}'x\boldsymbol{v})_k$$
is the Mathisson–Papapetrou term in the linear momentum.

- The angular momentum equations are in the form of a conservation law where there is very little freedom to move terms from one side to the other. Our (tentative) identification of J with angular momentum thus comes from dynamic considerations rather than the usual symmetry arguments.

To determine the equations of motion for ξ_{R}^i we go to Eq. (11). Reexpressing ψ_2^0 in terms of the mass aspect and looking only at the $l = 0, 1,$ terms, Eq. (11) becomes

$$\begin{aligned}
M' \;&=\; -\frac{G}{5c^7}\left(Q_{\mathrm{Mass}}^{ij\,\prime\prime\prime} Q_{\mathrm{Mass}}^{ij\,\prime\prime\prime} + Q_{\mathrm{Spin}}^{ij\,\prime\prime\prime} Q_{\mathrm{Spin}}^{ij\,\prime\prime\prime}\right) - \frac{2Q^2}{3c^5}\left(v_{\mathrm{R}}^{i\prime} v_{\mathrm{R}}^{i\prime} + v_{\mathrm{I}}^{i\,\prime} v_{\mathrm{I}}^{i\,\prime}\right) \\
&\quad -\frac{1}{180c^7}\left(D_E^{ij\,\prime\prime\prime} D_E^{ij\,\prime\prime\prime} + D_M^{ij\,\prime\prime\prime} D_M^{ij\,\prime\prime\prime}\right)\,, \qquad (39)\\
P^{k\,\prime} = F^k \;&\equiv\; \frac{2G}{15c^6}\left(Q_{\mathrm{Spin}}^{lj\,\prime\prime\prime} Q_{\mathrm{Mass}}^{ij\,\prime\prime\prime} - Q_{\mathrm{Mass}}^{lj\,\prime\prime\prime} Q_{\mathrm{Spin}}^{ij\,\prime\prime\prime}\right)\epsilon_{ilk} \\
&\quad -\frac{Q^2}{3c^4}\left(v_{\mathrm{I}}^{l\,\prime} v_{\mathrm{R}}^{i\prime} - v_{\mathrm{R}}^{l\prime} v_{\mathrm{I}}^{i\,\prime}\right)\epsilon_{ilk} \\
&\quad +\frac{Q}{15c^5}\left(v_{\mathrm{R}}^{j\,\prime} D_E^{jk\,\prime\prime\prime} + v_{\mathrm{I}}^{j\,\prime} D_M^{jk\,\prime\prime\prime}\right) \\
&\quad +\frac{1}{540c^6}\left(D_E^{lj\,\prime\prime\prime} D_M^{ij\,\prime\prime\prime} - D_M^{lj\,\prime\prime\prime} D_E^{ij\,\prime\prime\prime}\right)\epsilon_{ilk}\,, \qquad (40)
\end{aligned}$$

where we have identified the gravitational quadrupoles via

$$\xi^{ij} = \left(\xi_{\mathrm{R}}^{ij} + i\xi_{\mathrm{I}}^{ij}\right) = \frac{G}{12\sqrt{2}c^4}\left(Q_{\mathrm{Mass}}^{ij\prime\prime} + iQ_{\mathrm{Spin}}^{ij\prime\prime}\right)\,.$$

In addition

$$Q\left(\xi_{\rm R}^i + i\xi_{\rm I}^i\right) = D_E^i + iD_M^i$$

are the electric and magnetic dipole moments. With these identifications, the mass loss equation is the classical energy-loss by electromagnetic dipole and quadrupole radiation and gravitational quadrupole radiation.

Finally substituting P from Eq. (35), we obtain our equations of motion

$$\begin{aligned} Mv_{\rm R}^{k\prime} \ =\ & \frac{2Q^2}{3c^3}v_{\rm R}^{k\prime\prime} + \frac{M}{c}\left(\xi_{\rm R}^i v_{\rm I}^{j\prime} + \xi_{\rm I}^i v_{\rm R}^{j\prime}\right)'\epsilon_{ijk} \\ & - \frac{2Q^2}{3c^4}\left[2\xi_{\rm I}^i v_{\rm R}^{j\prime} - \xi_{\rm R}^i v_{\rm I}^{j\prime} + v_{\rm R}^i v_{\rm I}^j\right]''\epsilon_{ijk} - M'v_{\rm R}^k - \Pi^{k\prime} + F^k\,. \end{aligned}$$

These equations contain the classical radiation reaction term with its associated unstable behavior. The question naturally arises do the other terms suppress this unstable behavior? Though it is hard to work through this issue in the present situation, we note that if there was no suppression, the Einstein–Maxwell equations (in this case) would be unstable since the coinciding center of mass/charge world-line would undergo unlimited acceleration leading to unlimited electromagnetic radiation even if the quadrupoles were shut off. It is possible that there could be a theorem, from the Cauchy problem, on the general stability of this type of coupling of Maxwell theory to GR. In that case we would have the result that GR suppresses the classical electromagnetic runaway behavior.

5. Discussion and conclusions

We have argued here that there is considerably more information hidden in the asymptotic fields of the Einstein and Einstein–Maxwell equations than was conventionally believed. First we pointed out that in linear theory the simple introduction of a complex time-dependent displacement vector, $\xi^a(u)$, led in a heuristic manner to both a definition of angular momentum with its conservation law and a definition of center of mass with its evolution equations. The issue was where does this displacement vector come from? It must have a geometric meaning with invariance under the BMS group. Without any claim for its uniqueness, we have shown that a surprising but natural potential origin for the ξ^a lies in the properties of shear-free or asymptotically shear-free null geodesic congruences. If one comes to GR without a background, there is no *a priori* reason to suspect that shear-freeness of null geodesic congruences are of any importance. Nevertheless, over the years, first from the study of algebraically special metrics, with the beautiful Goldberg–Sachs theorem [10], then in Penrose's development of twistor theory and on to the theory of H-space, we have seen or suspected that there is something of fundamental importance in these congruences. We thus invoked the theorem [6] that every regular asymptotically shear-free null geodesic congruence in an asymptotically flat space-time is generated by a complex world-line in the space of the complex Poincare translations. By generalizing the idea from algebraically special metric, where the tangent vectors to the congruence are chosen so that a component of the Weyl tensor vanishes, to requiring just the $l = 1$ harmonic of the same Weyl tensor component to vanish we were able to obtain a unique regular congruence and complex world-line. The complex vector describing the world-line becomes $\xi^a(u)$, the displacement vector whose real part yields the center of mass world-line and whose imaginary part describes the spin angular moment.

The main defense for this construction of $\xi^a(u)$ lies in the final results, *i.e.*, in the precise equations of motion that are obtained. Everything follows from our requirement of the vanishing of a specific Weyl tensor component − with no further use of adjustable parameters or model building. For the vacuum case our results are general, with a Maxwell field we have considered only the special case where there is a non-vanishing total charge, Q, and where the two complex world-lines coincide. Though there are many metrics with this property it appears almost certain that this is a severe restriction on the class of Einstein–Maxwell solutions.

- There are many terms that are familiar from classical mechanics or electrodynamics that appear with the correct numerical coefficients.
- The radiation reaction term arises (with the correct numerical coefficients) without any model building or renormalization.
- The Mathisson–Papapetrou spin-velocity coupling term appears in the momentum.
- The spin angular momentum of the Kerr–Newman metric [8] (and all type II twisting metrics [9]) arises from the same geometric construction used here, it being a special case of the present construction.
- Orbital angular momentum, $\boldsymbol{rxp}$ and spin precession simply appear in the expression that has tentatively been identified as total angular momentum. This total angular momentum satisfies a conservation law with a well-defined unambiguous flux.
- When a Maxwell field is present we obtain the Dirac value of the gyromagnetic ratio, *i.e.*, $g = 2$.

- The radiation fields (both electric and magnetic dipole and quadrupoles) are exactly the same as found in the classical treatment. The gravitational radiation involved an identification of our variable with the gravitational quadrupole.

- There are predictions of new kinematic terms in the momentum and angular momentum as well as new spin-velocity interactions in the equations of motion. Many of these terms can be interpreted as gravitational radiation reaction. For example, there is a kinematic contribution to the mass from the quadrupole terms. Though all these affects are extremely small, they might in the future be detectable.

- There is even the possibility that we can see how gravitational interactions suppress the unpleasant classical instabilities from the electromagnetic radiation reaction.

- Our construction is invariant under the BMS group.

It is this list that give us confidence that we are on a correct path and that shear-free and asymptotically shear-free null geodesic congruences are indeed of considerable importance in basic physics.

In a future publication many of the details omitted here will be fully described.

This material is based upon work (partially) supported by the National Science Foundation under Grant No. PHY-0244513. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science. E.T.N. thanks the NSF for this support. G.S.O. acknowledges the financial support from CONACyT and Sistema Nacional de Investigadores (SNI-México).

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