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SIDES AND ANGLES OF THE UNITARY TRIANGLE*

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I review the present knowledge of the off-diagonal parameters of the Cabibbo–Kobayashi–Maskawa quark mixing matrix.

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1. Introduction

In the Standard Model (SM) of electroweak interactions, the couplings between quarks of different flavors are expressed by the elements of the Cabibbo–Kobayashi–Maskawa (CKM) [1] unitary matrix. For three quark families, the matrix consists of four independent real parameters, three Euler angles and a phase. This phase introduces non-trivial imaginary terms in the SM Lagrangian, inducing violation of Charge-Parity (CP) symmetry in the evolution and decay of hadrons. In the Wolfenstein parameterization, an expansion in powers of the Cabibbo angle $\lambda = 0.226 \pm 0.001$, the CKM matrix reads as

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda & 1-\lambda^2/2-iA^2\lambda^4\eta & A\lambda^2 \\ A\lambda^3(\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}.$$

The parameters $A, \bar{\rho} = \rho(1-\lambda^2/2), \bar{\eta} = \eta(1-\lambda^2/2)$, are of order unity. CP conservation would imply $\eta = 0$.

The CKM matrix has been the object of extensive studies in the last years, aimed at either consolidating the SM picture of quark transitions, or else revealing New Physics, as would be flagged by inconsistencies between the SM predictions and the experimental results. The object of these studies is to verify the unitary relation $V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0$, where V_{UD} is

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the element relating a $U(= u, c, t)$ type quark to a D $(= d, b)$ type quark. The relation is usually rewritten as

$$R_u + 1 + R_t = 0 \tag{1}$$

by dividing all its terms by the product $V_{cb}V_{cd}^*$, with $R_u = \frac{V_{ub}V_{ud}^*}{V_{cb}V_{cd}^*} \simeq (\bar{\rho}^2 + \bar{\eta}^2)e^{-i\gamma}$, and $R_t = \frac{V_{tb}V_{td}^*}{V_{cb}V_{cd}^*} \simeq ((1-\bar{\rho})^2 + \bar{\eta}^2)e^{-i\beta}$. Eq. (1) can be represented by a triangle with a unit side in the complex plane, the Unitary Triangle (UT). The other sides, $|R_u|$, $|R_t|$, and the three angles, $\beta = \arg(R_t), \gamma = \arg(R_u)$, and $\alpha = \pi - \gamma - \beta$[1] can be determined by measuring decays and time evolution of heavy-flavored hadrons or Kaons. In this review, I will concentrate on the measurement of B mesons properties affecting the UT.

2. The sides of the UT

Determination of $|R_u| = \frac{1}{\tan\lambda}\frac{|V_{ub}|}{|V_{cb}|}$. The branching ratio for inclusive semileptonic B decays can be written as $\mathcal{B}(B \to \ell^+\nu_\ell X) = \tau_B(|V_{cb}|^2\gamma_c + |V_{ub}|^2\gamma_u)^2$, where $\tau_B = 1.585 \pm 0.007$ ps [3] is the average B_q $(q = u, d)$ lifetime, and the terms γ_c, γ_u include contributions from the phase space and the matrix elements for $b \to c\ell\nu_\ell$ and $b \to u\ell\nu_\ell$ decays, respectively. Moments of the lepton energy,

$$M^n_{E(\ell)} = \langle E^n_\ell \rangle = \int_{m(\ell)}^{M_B/2} E^n_\ell dE_\ell \Bigg/ \int_{m(\ell)}^{M_B/2} dE_\ell \ ,$$

and of the mass of the hadron system recoiling against the leptons, $M^n_{M_X}$, are computed with an Operator Product Expansion [5] as a function of several parameters (quark masses: m_b, m_c, mean kinetic energy of the b-quark in the B meson: μ_π, chromomagnetic operator: μ_G, *etc.*) which are not computable in perturbative QCD. The 0^{th} order moment is the inclusive semileptonic partial decay width. Measurements of E_ℓ and M_X moments from the corresponding spectra can be used to determine the non perturbative parameters and CKM parameters. In practice, as $|V_{cb}| \gg |V_{ub}|$, inclusive measurements test in fact $b \to c$ decays.

Lepton energy moments are measured by CLEO, B-factories, and by DELPHI, hadronic mass moments are measured also by CDF [8]. While the measurement of E_ℓ spectra is straightforward, determining M_x is more difficult. B-factories select a sample of fully reconstructed B mesons with typical

[1] In the literature, these angles are often referred to also as ϕ_1, ϕ_2, and ϕ_3 respectively.
[2] Charge conjugate processes are throughout implied, unless the contrary is explicitly stated. With the symbol ℓ I mean only electron or muon.

efficiencies of 0.3–0.5 %. The remaining particles in the event ("recoil") are assigned to the other B. If a high-energy lepton is found, M_X^n is computed from all the recoil particles but the lepton. Kinematic constraints on the B mass, B energy and overall momentum are applied to reduce the effects of particle loss in the detector cracks, or due to $K_{\rm L}$ production. CDF, CLEO and DELPHI reconstruct $B \to \ell DX$ decays. The moments so measured are fitted with parametric functions obtained with different definitions of the b-quark mass [6], to determine the non-perturbative parameters and $|V_{cb}|$ [7]. The result $|V_{cb}| = (41.7 \pm 0.7) \times 10^{-3}$ is consistent within about two sigmas with the less precise result from exclusive $B \to D^{(*)}\ell\nu_\ell$ transitions [4].

To determine $|V_{ub}|$ tight cuts must be applied to enhance $b \to u$ transitions from the overwhelming $b \to c$ background, so that only a small portion of the phase space is in fact observed. This introduces additional sizable theoretical uncertainties when extrapolating the measurements to the whole phase space. The correction is computed using the same non perturbative parameters determined from $b \to c\ell\nu_\ell$ spectra (see above), and from $b \to s\gamma$ spectra (see [9]), in the "universal shape function" ansatz [10]. Theoretical errors are induced by uncertainties on the values of the parameters, by all the effects breaking the universality ansatz ("sub-leading shape functions"), and by four-fermions operators breaking B^+/B_d Isospin symmetry ("Weak Annihilation").

To reduce the size of theoretical uncertainties, experiments must relax their cuts as much as possible [8]. B-factories measure the inclusive electron energy spectrum in the ranges $E_e = 1.9/2.0$–$2.6\,\text{GeV}$: the upper value is the kinematic threshold for $B \to u$ transition and the lower value is a compromise between the need to reduce $B \to De\nu_e X$ decays background and the need to reduce sensitivity to theoretical uncertainties. An independent inclusive measurement is performed by BaBar: events are selected in a region of the E_e, q^2 plane, where $q^2 = (p_e + p_\nu)^2$ is the square of four momentum transferred to the leptons in the decay. The neutrino four momentum p_ν is inferred from the missing energy and the missing momentum in the event. In recoil analysis events are selected asking $E_\ell > 1$ GeV, and with loose cuts on $M_X(< 1.7\text{ GeV} \simeq M_D)$ or on the M_x, q^2 plane. The world average from inclusive measurements $|V_{ub}|^{\rm incl.} = (4.49 \pm 0.19_{\rm exp.} \pm 0.27_{\rm th.}) \times 10^{-3}$ is about two standard deviations larger than the values (see [4]) obtained from the measurements of the differential partial widths $d\Gamma(B \to \pi\ell\nu_\ell)/dq^2$ performed by BaBar, Belle, CLEO [8].

Determination of $|R_t| = \frac{1}{\sin\lambda}\frac{|V_{td}|}{|V_{cb}|}$. This side is determined from $B\bar{B}$ mixing. The probability that a particle produced as a B_q $(q = d, s)$ meson at $t = 0$ decays as its antiparticle at a later time t is $\Pi(B \to \bar{B})(t) = 1 - \cos(\Delta m_q t)$, where Δm_q is the difference of the masses of the two eigenstates of the weak Hamiltonian for B_q mesons[3]. The relation $\frac{\Delta m_d}{\Delta m_s} = \frac{1}{\xi}\frac{m_{Bd}}{m_{Bs}}\frac{|V_{td}|^2}{|V_{ts}|^2}$ permits to determine $|R_t|$ by replacing $|V_{ts}|$ with $|V_{cb}|$, as allowed by unitarity of the CKM matrix. Uncertainties in the calculation of the SU(3) breaking parameter $\xi = 1.24 \pm 0.06$ [11] are the main source of theoretical error on $|R_t|$. $B\bar{B}$ mixing frequencies are measured from the time-dependent rates of events where two equal flavor (B_qB_q or $\bar{B}_q\bar{B}_q$) are observed. B-mesons are tagged inclusively (from the electric charge of reconstructed Kaons or high-momentum leptons), or from the full reconstruction of some specific final state (*e.g.*, $B_d \to D^{*-}\ell^+\nu_\ell$, $B_d \to D^{*-}\pi^+/\rho^+$, $B_s \to D_S^{(*)-}\pi^+$, *etc.*), or else with partial reconstruction of some of these final states. The present values of $\Delta m_d = 0.505 \pm 0.005\,\mathrm{ps}^{-1}$ (mainly from B-factories) and of $\Delta m_s = 17.77 \pm 0.10 \pm 0.07\,\mathrm{ps}^{-1}$ as measured by CDF, [4], imply $|R_t| = 0.206 \pm 0.007_{\mathrm{exp}} \pm 0.008_{\mathrm{theo}}$.

The UT sides-view. The values of the two parameters, $\bar{\rho}, \bar{\eta}$, determining the UT can be determined from the measurements of $|R_t|$ and $|R_u|$; the corresponding prediction of the UT angles are compared with their direct measurements in Table I[4]. It should be noted that the measurements of the sides imply $\eta \neq 0$ at more than three standard deviations. This can be interpreted as the SM prediction of CP violation in B-decays prior to its direct observation.

3. The angles of the UT

The angles of the UT can be measured in principle from the interference between different processes leading to the same final state in CP-violating hadronic B decays. CP violation can show up in mixing ($\mathcal{P}(B_d \to \bar{B}_d) \neq \mathcal{P}(\bar{B}_d \to B_d)$), directly in the decay ($\mathcal{P}(B \to f) \neq \mathcal{P}(\bar{B} \to \bar{f})$), or in the interference of mixing and decay [14]. In this last case, CP violation is revealed by the time-dependent asymmetry $\mathcal{A}_{f\mathrm{CP}}(\Delta t)$:

$$\frac{\mathcal{P}(f_{\mathrm{CP}}) - \bar{\mathcal{P}}(f_{\mathrm{CP}})}{\mathcal{P}(f_{\mathrm{CP}}) + \bar{\mathcal{P}}(f_{\mathrm{CP}})} = S_{f\mathrm{CP}} \sin(\Delta m_d \Delta t) + C_{f\mathrm{CP}} \cos(\Delta m_d \Delta t)\,, \tag{2}$$

where $\mathcal{P}(f_{\mathrm{CP}})$ ($\bar{\mathcal{P}}(f_{\mathrm{CP}})$) is the probability that a particle identified as a B_d ($\bar{B}_d$) meson at $t = 0$ decay at a later time Δt to a CP-eigenstate, denoted as f_{CP}. In practice, in most of the cases, the extraction of the angles from the measured asymmetries is not straightforward due to difficulties in computing the contributions from strong interactions.

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[3] in B-factories, due to coherent production of the two B mesons, t must be replaced by the difference of their proper decay times Δt.

[4] I use here the results of the Bayesian analysis of [12], which do not differ substantially from the frequentist analysis of [13].

Measurements of β [15]. The measurement of sin 2β from the "golden modes" $B_d \to (c\bar{c})K_{\mathrm{S/L}}$, where $(c\bar{c})$ is a bound charmonium state, like J/Ψ, $\Psi(2s)$, *etc.*, is the most noticeable exception to the statement above. The dominating diagrams for these transitions are all determined by one single weak phase (β), so that, up to corrections of $o(1\%)$, theory predicts: $C_{(c\bar{c})K} = 0$, and $S_{(c\bar{c})K} = \mp \sin(2\beta)$ (the $-$ sign applies to the K_{S} final state). B-factories measure $\mathcal{A}_{(c\bar{c})K}(\Delta t)$ by comparing the rates for B-tagged and anti-B tagged $(c\bar{c})K_{\mathrm{S/L}}$ decays as a function of Δt. B-tagging is based on the same inclusive algorithms exploited for mixing measurements. Consistent results are obtained by BaBar and Belle (see second line on Table I). confirm that direct CP violation is negligible in this channel, by measuring $C_{(c\bar{c})K} = 0.002 \pm 0.021$. Even if many other CP eigenstates could be used to determine β in this same way (*e.g.* $\Phi K_{\mathrm{S,L}}, \eta K_{\mathrm{S,L}}, D^{(*)}\bar{D}^{(*)}$) in practice all of them are affected by larger theoretical uncertainties. They are just used as a test of the SM, by comparing their results with the charmonium measurement. The only inconsistency reported so far, the observation of a large direct CP asymmetry in $B^0 \to D^+D^-$ decays from Belle, $C_{(DD)} = -0.91 \pm 0.23 \pm 0.06$, is not confirmed by the BaBar result $C_{(DD)} = 0.11 \pm 0.22 \pm 0.07$.

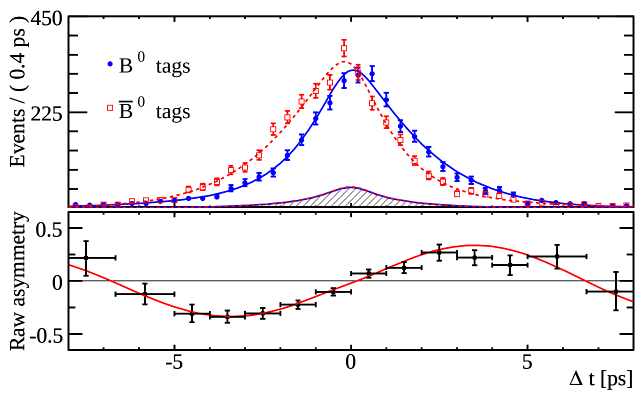

Fig. 1. Top: number of $(c\bar{c})K_S$ with a B_d (full circies and continues line) and a $\bar{B}_d$ (open circies and dashed line) tag as a function of Δt. Bottom: raw asymmetry.

Measurement of $\boldsymbol{\alpha}$ [16]. The tree level Feynman graph for the transition $B_d \to \pi^+\pi^-$, $B_d \to \rho^+\rho^-$ would imply $S_{\pi\pi,\rho\rho} = \sin 2\alpha$. Interference from penguin diagrams introduces a new phase, so that experiments measure an effective parameter $\alpha_{\mathrm{eff}} = \alpha + \Delta\alpha$. An Isospin analysis of $B \to h\bar{h}$ ($h = \pi, \rho$) provides the bound: $\sin^2(\Delta\alpha) \le \frac{\mathcal{B}(B_d \to h^0h^0) - \mathcal{B}(\bar{B}_d \to h^0h^0)}{\mathcal{B}(B^+ \to h^+h^0) + \mathcal{B}(B^- \to h^-h^0)}$ [17]. In practice, only weak bounds are obtained from the $\pi\pi$ final state. Tighter constraints come from the decays $B \to \rho\rho$, where the untagged Branching Ratio $\mathcal{B}(B_d \to \rho^0\rho^0) = (1.07 \pm 0.38) \times 10^{-6}$ is much smaller than $\mathcal{B}(B^+ \to \rho^+\rho^0) = \mathcal{B}(B^- \to \rho^-\rho^0) = (18.2 \pm 3.0) \times 10^{-6}$. Moreover, a transversity analysis proves that $\rho^+\rho^-$ form a 97% CP-even state. An analysis on the Dalitz plot of $B \to \rho\pi$ decays provides additional bounds on α. Results from the Bayesian analysis are reported in Table I. For sake of comparison, the results of the frequentist analysis is $\alpha = (115^{+9}_{-35})^\circ$.

Measurements of γ [18]. This angle is measured from the interference of the color allowed decay $B^- \to K^-D^0$ ($b \to c\bar{u}s$, amplitude $\propto V_{us}V^*_{cb} \propto \lambda^3$) and the color suppressed process $B^- \to K^-\bar{D}^0$ ($b \to u\bar{c}s$, amplitude $\propto V_{cs}V^*_{ub} \propto \lambda^3 e^{i\gamma}$). Interference, which takes place when the $\bar{D}^0$ and the D^0 are reconstructed in a common final state, leads to different B^+ and B^- decay rates (direct CP violation), according to the relation: $\Gamma(B^\mp \to D^0K^\mp) \propto |A_\mp + re^{i(\delta\mp\gamma)}A_\pm|^2$, where $A_-(A_+)$ is the amplitude for the $D^0(\bar{D}^0)$ decay, δ is the relative strong phase between the two amplitudes, and $r \simeq |V_{cs}V^*_{ub}|/|V_{us}V^*_{cb}|c_F \simeq 0.1$ accounts for the ratio of the CKM factors and for the color suppression $c_F \simeq 0.2$.

The analysis of the partial widths across the Dalitz plot [19] for the $K_s\pi^+\pi^-$ final state allows the simultaneous determination of δ, r, and γ, providing the tighter bounds on γ to date. The dependence of the amplitudes $A_\mp$ on the squared invariant masses of the $K_s\pi^+$ and $K_s\pi^-$ combinations are obtained by fitting a pure sample of about 10^5 $D^{*+} \to \pi^+D^0$ decays (and c.c.) from $e^+e^- \to c\bar{c}$, where the charge of the pion tags the flavor of the D meson. The fit model includes four Cabibbo allowed, three doubly Cabibbo suppressed, and three CP eigenstates; among these ρ^0K_s provides the only significant contribution, accounting for about 20% of the total width. Systematic uncertainties, mostly due to the Dalitz model, are still much smaller than the statistical uncertainties. This measurement includes also $B^\mp \to D^{*0}K^\mp$ decays, with $D^{*0} \to D^0\pi^0, D^0\gamma$. Other constraints on γ are obtained from decays to CP eigenstates (*e.g.* KK, $\pi\pi$, $\pi\pi\pi$, KKK, *etc.*) [20], and also to $K^+\pi^-$ states [21] , which are Cabibbo favored for the $\bar{D}^0$ and Cabibbo suppressed for the D^0. The present world average value of γ is reported in Table I.

TABLE I

Values for some parameters of the UT, as determined from measurements of its sides (first line), its angles (second line), and from their average.

	$\bar{\rho}$	$\bar{\eta}$	$\sin(2\beta)$	(α) $(^\circ)$	γ $(^\circ)$
sides	0.19 ± 0.04	0.37 ± 0.03	0.76 ± 0.04	91 ± 5	65 ± 5
angles	0.14 ± 0.04	0.33 ± 0.02	0.68 ± 0.03	91 ± 8	83 ± 19
average	0.16 ± 0.03	0.34 ± 0.02	0.69 ± 0.02	91 ± 5	67 ± 6

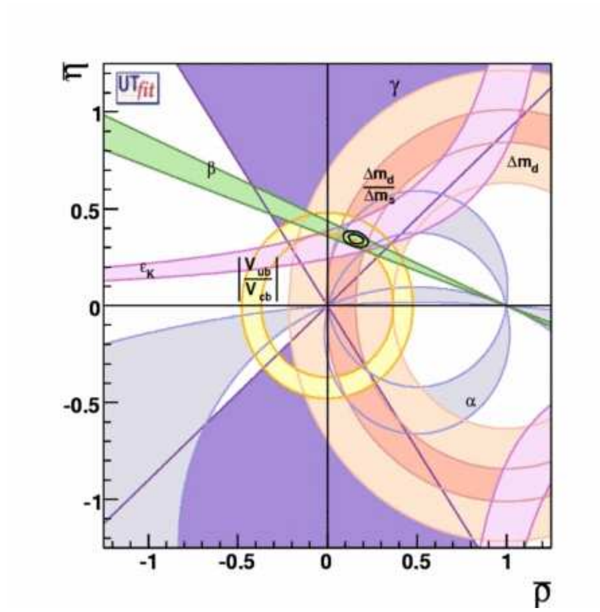

Fig. 2. Constraints to the UT from the measurements of B and K^0 mesons properties.

4. Conclusions

In the last years the B-physics program has scored a spectacular success, improving our understanding of the fundamental processes governing evolution and decay of B-hadrons. The SM description of CP-violating phenomena is confirmed by the existing results, as shown in Table I. The detailed analysis described in [22] shows that there is a slight inconsistency between the value of $|V_{ub}|$ measured from inclusive B semileptonic decays and the result of the CKM fits. However, as the value based on exclusive semileptonic decays is consistent with the fits, this cannot be considered as a crack in the SM.

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