

## HBT FROM PERFECT FLUID DYNAMICS\*

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In this paper we summarize the ellipsoidally symmetric Buda-Lund model's results on HBT radii. We calculate the Bose-Einstein correlation function from the model and derive formulas for the transverse momentum dependence of the correlation radii in the Bertsch-Pratt system of out, side and longitudinal directions. We show a comparison to  $\sqrt{s_{NN}} = 200$  GeV RHIC PHENIX two-pion correlation data and make prediction on the same observable for different particles.

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**1. Perfect fluid hydrodynamics**

Perfect fluid hydrodynamics is based on local conservation of entropy and four-momentum. The fluid is perfect if the four-momentum tensor is diagonal in the local rest frame. The conservation equations are closed by the equation of state, which gives the relationship between energy density  $\epsilon$ , pressure  $p$ . Typically  $\epsilon = \kappa p$  is assumed, where  $\kappa$  may be a constant, but can be an arbitrary temperature dependent function.

There are only a few exact solutions for these equations. One (and historically the first) is the famous Landau-Khalatnikov solution discovered more than 50 years ago [1–3]. This is a 1+1 dimensional solution, and has realistic properties: it describes a 1+1 dimensional expansion, does not lack acceleration and predicts a Gaussian longitudinal rapidity distribution.

Another renowned relativistic hydrodynamical solution is the Hwa-Bjorken solution [4–6], which is a simple, explicit and exact, but accelerationless solution. This solution is boost-invariant in its original form, but this

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approximation fails to describe the data [7, 8]. However, the solution allowed Bjorken to obtain a simple estimate of the initial energy density reached in high energy reactions from final state hadronic observables.

There are solutions which interpolate between the above two solutions [9, 10], are explicit and describe a relativistic acceleration.

## 2. The Buda–Lund model

We focus here on the analytic approach in exploring the consequences of the presence of such perfect fluids in high energy heavy ion experiments in Au+Au collisions at RHIC. Such exact analytic solutions were published recently in Refs. [9–13]. A tool, that is based on the above listed exact, dynamical hydro solutions, is the Buda–Lund hydro model of Refs. [14, 15].

The Buda–Lund hydro model successfully describes BRAHMS, PHENIX, PHOBOS and STAR data on identified single particle spectra and the transverse mass dependent Bose–Einstein or HBT radii as well as the pseudorapidity distribution of charged particles in central Au+Au collisions both at  $\sqrt{s_{NN}} = 130$  GeV [16] and at  $\sqrt{s_{NN}} = 200$  GeV [17] and in  $p+p$  collisions at  $\sqrt{s} = 200$  GeV [18], as well as data from Pb+Pb collisions at CERN SPS [19] and  $h+p$  reactions at CERN SPS [20, 21]. The model is defined with the help of its emission function; to take into account the effects of long-lived resonances, it utilizes the core-halo model [22]. It describes an expanding fireball of ellipsoidal symmetry (with the time-dependent principal axes of the ellipsoid being  $X$ ,  $Y$  and  $Z$ ).

## 3. HBT from the Buda–Lund model

Let us calculate the two-particle Bose–Einstein correlation function from the Buda–Lund source function of the Buda–Lund model as a function of  $q = p_1 - p_2$ , the four-momentum difference of the two particles. The result is

$$C(q) = 1 + \lambda e^{-q_0^2 \Delta\tau_*^2 - q_x^2 R_{*,x}^2 - q_y^2 R_{*,y}^2 - q_z^2 R_{*,z}^2} . \quad (1)$$

with  $\lambda$  being the intercept parameter (square of the ratio of particles emitted from the core *versus* from the halo [22]), and

$$\frac{1}{\Delta\tau_*^2} = \frac{1}{\Delta\tau^2} + \frac{m_t}{T_0} \frac{d^2}{\tau_0^2} , \quad (2)$$

$$R_{*,x}^2 = X^2 \left( 1 + m_t \left( a^2 + \dot{X}^2 \right) / T_0 \right)^{-1} , \quad (3)$$

$$R_{*,y}^2 = Y^2 \left( 1 + m_t \left( a^2 + \dot{Y}^2 \right) / T_0 \right)^{-1} , \quad (4)$$

$$R_{*,z}^2 = Z^2 \left( 1 + m_t \left( a^2 + \dot{Z}^2 \right) / T_0 \right)^{-1} , \quad (5)$$

with  $\dot{X}, \dot{Y}, \dot{Z}$  being the time-derivative of the principal axes,  $m_t$  the average transverse mass of the pair.  $T_0$  is the central temperature at the freeze-out,  $\Delta\tau$  is the mean emission duration and  $\tau_0$  is the freeze-out time. Furthermore,  $a$  and  $d$  are the spatial and time-like temperature gradients, defined as  $a^2 = \langle \frac{\Delta T}{T} \rangle_\perp$  and  $d^2 = \langle \frac{\Delta T}{T} \rangle_\tau$ . From the mass-shell constraint one finds  $q_0 = \beta_x q_x + \beta_y q_y + \beta_z q_z$ , if expressed by the average velocity  $\beta$ . Thus we can rewrite Eq. 1 to

$$C(q) = 1 + \lambda_* \exp \left( - \sum_{i,j=x,y,z} R_{i,j}^2 q_i q_j \right) \quad (6)$$

with the modified radii of

$$R_{i,i}^2 = R_{*,i}^2 + \beta_i^2 \Delta\tau_*^2, \quad \text{and} \quad R_{i,j}^2 = \beta_i \beta_j \Delta\tau_*^2. \quad (7)$$

From this, we can calculate the radii in the Bertsch–Pratt frame [23] of out (o, pointing towards the average momentum of the actual pair, rotated from  $x$  by an azimuthal angle  $\varphi$ ), longitudinal (l, pointing towards the beam direction) directions and side (s, perpendicular to both l and o) directions. If one averages on the azimuthal angle, and goes into the LCMS frame (where  $\beta_l = \beta_s = 0$ ), the Bertsch–Pratt radii are:

$$R_o^2 = (R_{*,x}^{-2} + R_{*,y}^{-2})^{-1} + \beta_o^2 \Delta\tau_*^2, \quad (8)$$

$$R_s^2 = (R_{*,x}^{-2} + R_{*,y}^{-2})^{-1}, \quad (9)$$

$$R_l^2 = R_{*,z}^2. \quad (10)$$

These can be fitted then to the data [24] as was done in Ref. [25], see Fig. 1.

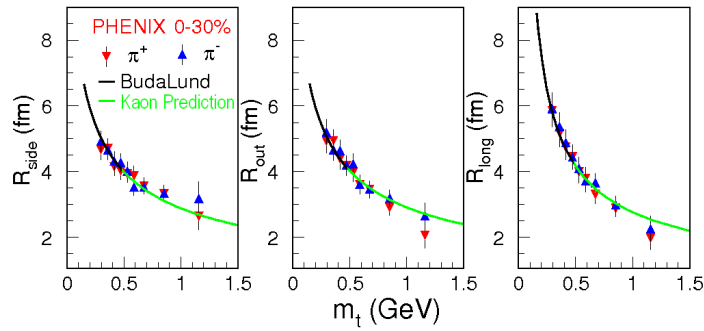


Fig. 1. HBT radii from the axially Buda–Lund model from Ref. [25], compared to data of Ref. [24]. We also show a prediction for kaon HBT radii on this plot: these overlap with that of pions if plotted *versus* transverse mass  $m_t$ .

This allows us to predict the transverse momentum dependence of the HBT radii of two-kaon correlations as well: if they are plotted *versus*  $m_t$ , the data of all particles fall on the same curve. This is also shown for kaons in Fig. 1.

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