Vol. 1 (2008) *Acta Physica Polonica B Proceedings Supplement* No 3

RUNNING COUPLING IN SMALL x EVOLUTION, A SUMMARY*

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(Received October 16, 2008)

The Color Glass Condensate has become an important tool to understand saturation phenomena in high energy collisions involving large nuclei. Until recently, the underlying JIMWLK and BK evolution equations have been known only to one loop accuracy. Here I summarize results of the first calculations to step beyond leading order and provide running coupling corrections to both equations.

PACS numbers: 12.38.–t, 12.38.Cy

1. High energies and saturation in QCD

With the advent of modern colliders, from the Tevatron and HERA to RHIC and planned facilities like LHC and EIC, the high energy asymptotics of QCD has gained new prominence. In *all* these experiments soft gluon emission is strongly enhanced by phase space logarithms in the total energy s. In QCD, such multiple soft emission is intrinsically nonlinear since gluons carry color charge: Produced gluons act as sources for further emission: emission accelerates. Once densities have grown, further emission into an already dense environment will be modified by recombination and absorption. This slows growth and leads to saturation. Accelerated growth and saturation are both irrelevant in the electroweak sector, they require the gauge boson to be both massless and non-Abelian.

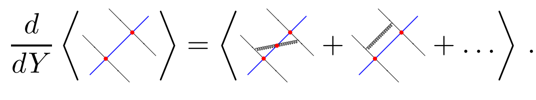
The key points can be most easily understood in the context of deep inelastic scattering (DIS) of leptons on proton or nuclear targets. The dynamics is governed by the deeply space-like momentum $q^2 = -Q^2 < 0$ imparted on the nuclear target and rapidity (the "energy logarithms") $Y = \ln(1/x) \approx \ln\left(s/Q^2\right)$ (with Bjorken $x := Q^2/(2p.q)$, p the target momentum). Q^2 defines the transverse resolution of the probe.

* Presented at the XXXVII International Symposium on Multiparticle Dynamics, Berkeley, USA, August 4–9, 2007.

At large Q^2 with fixed x such a system may be analyzed in terms of the twist expansion, a short-distance expansion in powers of $1/Q^2$. Since Q^2 controls the apparent size of the particles encountered, this may be viewed as a small density expansion.

Going to small x at fixed Q^2, no matter how large, one enters a domain of high densities since one keeps adding particles that are all of a fixed effective size of order $1/Q^2$. Density will grow to a point where particle correlations become essential and a description in terms of distribution functions alone becomes untenable. Appropriate degrees of freedom and more general evolution equations are needed to describe the system beyond this point. The most general of these existing to date are the JIMWLK equation, or, the completely equivalent Balitsky hierarchies, with their factorized truncation, the Balitsky–Kovchegov (BK) equation. In the low density limit they reduce to the Balitsky–Fadin–Lipatov–Kuraev (BFKL) equations. [1–3] may serve as a guide to the literature.

JIMWLK and BK equations are based on a no recoil approximation for the constituents of the projectile wave function (justified by the high collision energies). It allows a description of the interaction of each constituent of the projectile with the target in terms of path-ordered exponentials $U_{\boldsymbol{x}}$ aligned with the projectile trajectory. $\boldsymbol{x}$ represents the transverse coordinate of the constituent during the interaction. In leading order the $\gamma^* A$ cross-section in DIS turns then into a convolution of the absolute value squared of the projectile (photon) wave function and the dipole cross-section $\int d^2b\ \langle \hat{N}_{\boldsymbol{xy}} \rangle (Y)$. The former gives the probability to find a $q\bar{q}$ pair of a given size $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y}$ in the projectile at momentum transfer Q^2. The latter gives the cross-section of a $q\bar{q}$ pair at transverse positions $\boldsymbol{x}$ and $\boldsymbol{y}$ with the target. (The integral is over impact parameter $\boldsymbol{b} = (\boldsymbol{x} + \boldsymbol{y})/2$.) The T-matrix average $\langle \hat{N}_{\boldsymbol{xy}} \rangle := \langle \mathrm{Tr}\,(1 - U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)/N_{\mathrm{c}} \rangle (Y)$ encodes all details of the interaction with the target field via U-fields and the averaging procedure. Its energy dependence is accessible perturbatively and can be traced via terms enhanced by $\alpha_{\mathrm{s}} \ln(1/x)$. These corrections are resummed by the JIMWLK equation, an equation for the statistical weight functional for the average $\langle \ldots \rangle (Y)$. The functional equation is equivalent to infinite hierarchies of equations for multi-U-correlators, the Balitsky hierarchies. The evolution equation for the dipole operator at leading order has the form



(1)

The diagrams contain q and $\bar{q}$ lines (leaning left along the x^--direction) and interactions with the target gluons (U-fields) concentrated at $x^- = 0$ (the line leaning to the right), marked as dots. The diagrams on the right represent real corrections (1st) in which the interaction of the newly produced gluon with the target frees it to reach the final state, and virtual corrections (2nd) where the new gluon does not interact with the target. "..." represents diagrams with alternative locations of the $q\bar{q}g$ vertices. The left hand side and the virtual term on the right hand side only contain target interaction of a dipole via the operator $\mathrm{Tr}\,(U_{\boldsymbol{x}}U^{\dagger}_{\boldsymbol{y}})$. The real emission term on the other hand contains an additional gluonic Wilson line in the form $U^{ab}_{\boldsymbol{z}}\ \mathrm{Tr}\,(t^a U_{\boldsymbol{x}} t^b U^{\dagger}_{\boldsymbol{y}})$ and couples this equation to the other equations in its Balitsky hierarchy. At this order of accuracy the right hand side is proportional to the strong coupling at a fixed but undetermined scale μ^2.

2. Running coupling triumvirates

It has been noted early that running coupling effects play a role that is much more prominent than in the well known DGLAP evolution that would resum Q^2 corrections in a dilute regime. There the natural scale for the coupling is Q^2 and thus is one fixed number in each step of the evolution. For evolution towards small x, Eq. (1) on its r.h.s. contains integrals over all transverse space and samples the coupling on all scales in each individual step of the evolution. Non-perturbative influence near the Landau pole is largely shielded by saturation effects, but the perturbative variation has a large impact: it is solely responsible for a reduced gluon emission from small dipoles [4]. Such slowdown effects are essential for phenomenology. Yet, these corrections have only recently been *calculated* [5–7] using a resummation of quark loops to trace the N_f part of the β-function. The gluon contributions are restored by substituting $N_f \to -6\beta_0$. The problem is then reduced to a study of the quark bubble insertions shown in Fig. 1. The right-

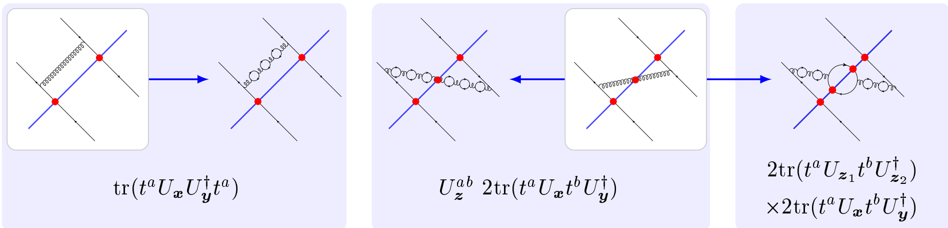
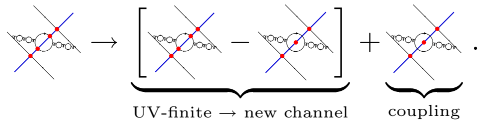


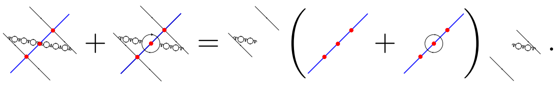
Fig. 1. Running coupling corrections to virtual (left) and real diagrams (right).

most diagram contains a new channel, where a $q\bar{q}$ pair instead of a gluon interacts with the target. This contribution overlaps with the running of the coupling in the UV, where the interacting $q\bar{q}$ pair coalesces and its interaction with the target reduces to that of a gluon: $\lim_{\boldsymbol{z}_1,\boldsymbol{z}_2\to\boldsymbol{z}} 2\mathrm{Tr}\,(t^a U_{\boldsymbol{z}_1} t^b U^{\dagger}_{\boldsymbol{z}_2}) = U^{ab}_{\boldsymbol{z}}$. To generate a renormalized coupling and to guarantee real virtual cancellation in the absence of target interaction (probability conservation), separately for the running coupling corrected leading order terms and the new channel, one has to separate this UV-divergent contribution from the new $q\bar{q}$ channel by adding and subtracting the contribution of this diagram with the quark interaction $2\mathrm{Tr}\,(t^a U_{\boldsymbol{z}_1} t^b U^\dagger_{\boldsymbol{z}_2})$ replaced by a gluon interaction $U^{ab}_{\boldsymbol{z}}$ placed at a suitable coordinate $\boldsymbol{z}$ according to



(2)

While this implies a scheme dependence for the separation of these contributions it does not affect the sum. The running coupling corrected real emission term now takes the form


(3)

This structure reveals that the running coupling factor in momentum space takes the form of a triumvirate $\frac{\alpha_s(\boldsymbol{q}^2)\,\alpha_s(\boldsymbol{q}'^2)}{\alpha_s(\tilde{Q}^2)}$ where the numerator factors arise from the bubble chains to the left and right of the interaction. They depend on the off-shell-ness of the gluon lines $\boldsymbol{q}^2$ and $\boldsymbol{q}'^2$ before and after the interaction. $\alpha_s(\tilde{Q}^2)$ obtains its logarithm from the UV part of the interacting quark loop. Its scale $\tilde{Q}$ is a function of $\boldsymbol{q}$ and $\boldsymbol{q}'$ whose details depend on the separation scheme for the new channels with explict expressions given in [6]. This contribution is $\mathcal{O}(\alpha_s)$ while the new channel in (2) is $\mathcal{O}(\alpha_s^2)$. Numerical work shows that these corrections go a long way to help match up with experimental data at HERA [8] and RHIC [9].

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