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FROISSART BOUND IN STRONG COUPLING LIMIT*

Chung-I. Tan

Physics Department, Brown University
Providence, RI 02912, USA

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We discuss the beginning of the unitarization program for high energy scattering based on String/Gauge duality.

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1. Introduction

We report on progress toward a fuller understanding on high energy scattering for hadrons, based on Maldacena's weak/strong duality relating Yang–Mills theories to string theories in (deformed) Anti-de Sitter space [1–4]. In this brief review, we focus on clarifying the properties of the Pomeron kernel, the structure of eikonal sum, and consequences of confinement.

There is in principle a clean definition for the notion of a Pomeron in QCD. Expand the 2-to-2 SU(N_c) QCD scattering amplitude in $g_0^2 \sim 1/N_c^2$:

$$A(s,t) = g_0^2 A_1(s,t,\lambda) + g_0^4 A_2(s,t,\lambda) + \ldots$$

at fixed 't Hooft coupling $\lambda = g^2_{\rm YM} N_c$: *Pomeron $\equiv$ leading contribution at large N_c to the vacuum exchange at large s and fixed t.*

In the Regge limit for an n-particle amplitude: $A(p_1, p_2, \ldots p_n)$, the rapidity gaps, $\ln(p_r^+ p_\ell^-)$, between any right- and left-moving particles are all $O(\log s)$, *i.e.*, can be specified by a large Lorentz boost, $\exp[yM_{+-}]$, with $y \sim \log s$. The J-plane is conjugate to rapidity, and is to be identified with the eigenvalue of the Lorentz boost generator M_{+-}. The boost operator can be approximated by $M_{+-} \simeq 2 - H_{+-}/(2\sqrt{\lambda})$ with H_{+-} expressible in terms of Casimirs of its commuting subgroup, SL$(2,C)$, of the full $O(4,2)$ conformal group. Indeed, the strong coupling conformal Pomeron

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kernel, $\mathcal{K}$, can be directly written in terms of the AdS_3 Green's function $G_3(j,v)$, *i.e.*, $\mathcal{K}(j,x^\perp - x'^\perp, z, z') = (zz'/R^4)G_3(j,v)$. As as a consequence of $\mathrm{SL}(2,C)$ invariance $G_3(j,v)$ depends only on the AdS_3 chordal distance, $v = ((x_\perp - x'_\perp)^2 + (z-z')^2)/2zz'$,

$$G_3(j,v) = \frac{1}{4\pi}\frac{e^{(2-\Delta_+(j))\xi}}{\sinh\xi}\,, \tag{1}$$

where $\cosh\xi = 1+v$, and the AdS_3 conformal dimension[1], $\Delta_+(j)-1$, $\Delta_+(j) \equiv 2 + \widetilde{\Delta}_+(j) = 2 + \sqrt{4 + 2\sqrt{\lambda}(j-2)} = 2 + \sqrt{2\sqrt{\lambda}(j-j_0)}$.

In both strong coupling and pQCD, the Pomeron contribution grows as $s^{1+\epsilon}$, $\epsilon > 0$, faster than the Froissart bound, and higher order corrections must be taken into account. It has been shown in [4] that the standard eikonal representation generalizes for the case of large 't Hooft coupling to

$$A(s,t) = -2is\int dz dz' P_{13}(z) P_{24}\left(z'\right)\int d^2b\; e^{-ib^\perp q_\perp}\left[e^{i\chi\left(s,b^\perp,z,z'\right)} - 1\right]\,. \tag{2}$$

To first order in g_0^2, the eikonal can be expressed in terms of an inverse Mellin transform, $\chi \sim -(2(zz')^2 s)^{-1}\int\frac{dj}{2\pi i}\left(\frac{\hat{s}^j + (-\hat{s})^j}{\sin\pi j}\right)\mathcal{K}(j,b^\perp,z,z')$. Here the scattering is of initial states $1,2$ to final states $3,4$, in the near-forward limit. The products of wave functions are for left-moving $(1\to 3)$ and right-moving $(2\to 4)$ states, respectively. When confinement is implemented, wave functions can be normalized so that $\int dz P_{ij}(z) = \delta_{ij}$.

2. Absorptive versus diffractive radii in the bulk

Consider a bulk cross-section, $\sigma(s,z,z') = 2\mathrm{Re}\int d^2b[1 - e^{i\chi}]$, where the physical total cross-section is obtained from this bulk cross-section by a convolution. At a given z and z', $\sigma(s,z,z')$ approaches its unitarity bound when $|\chi| \sim 1$. Since interactions become weaker at larger b, this leads to an effective "disk picture", $\sigma(s,z,z') \sim b_{\max}^2$.

If $\mathrm{Im}[\chi] > \mathrm{Re}[\chi]$, as is the case for the weak-coupling Pomeron, the point $b_{\max}$ is where absorption becomes of the order of one, one speaks of a *"black disk"* of radius b_{black}, set by $\mathrm{Im}[\chi] \sim 1$. If the reverse is true, then outside the black disk, there exists a *"diffractive disk"*. The radius of this disk, b_{diff}, is set roughly by the condition $\mathrm{Re}[\chi] \sim 1$.

The separation into dominant diffusive and diffractive regions, with $b_{\text{black}} \ll b_{\text{diff}}$ is a uniquely strong coupling feature [4–6]. This can already be illustrated by working with the example of an even-signatured Regge

[1] The demonstration of the DGLAP to BFKL relationship is given in [1].

exchange in 4-dim, with $A^{(1)}(s,t) \simeq \beta \left[\frac{2m_0^2}{(\pi\alpha')(m_0^2-t)} + i\right] (s/s_0)^{\alpha(t)}$, where $\alpha(t) \simeq 2 + \alpha'(t/m_0^2 - 1)$, and $\alpha(m_0^2) = 2$. To mimic the strong coupling limit, we treat $\alpha' \sim \lambda^{-1/2} \ll 1$.

The integral for $\mathrm{Im}[\chi]$ is gaussian, leading to the usual "Regge diffusion" in b. From the condition $\mathrm{Im}\chi = 0(1)$, one has

$$b_{\mathrm{black}} \sim \lambda^{-1/4} m_0^{-1} \log(\beta s/s_0)\,. \tag{3}$$

The real part can be found easily from $\mathrm{Re}\chi \simeq (2/\pi)e^{\tau}\int_\tau^\infty d\tau' e^{-\tau'}\mathrm{Im}[\chi(\tau', b)]$. When $b \ll b_{\mathrm{cross}} = (2\alpha'/m_0)\tau$, $\mathrm{Re}\chi \simeq (2/\pi\alpha')\mathrm{Im}\chi$, consistent with the expected Regge phase. However, for $b \gg b_{\mathrm{cross}}$, $\mathrm{Re}[\chi(\tau, b)] \sim s\, \frac{e^{-m_0 b}}{\sqrt{m_0 b}}$, *i.e.*, given by that for a $J = 2$ glueball exchange. From $\mathrm{Re}\chi = 0(1)$, one finds that the diffraction radius is

$$b_{\mathrm{diff}} \simeq m_0^{-1} \log(\beta s/s_0)\,, \tag{4}$$

and, with $\alpha' \sim \lambda^{-1/2} \ll 1$, $b_{\mathrm{diff}} >> b_{\mathrm{black}}$.

3. Conformal limit, confinement, and Froissart bound

We have shown in strong coupling, both in the conformal limit and the case with confinement, $\mathrm{Re}[\chi] > \mathrm{Im}[\chi]$ always holds at b sufficiently large with s fixed, and, in this region, $\mathrm{Re}[\chi]$ is given by spin-2 exchange. It follows that

$$b_{\mathrm{diff}} \gg b_{\mathrm{black}} \tag{5}$$

and the scale for the total cross-section is always set by ${b_{\mathrm{diff}}}^2$. For scattering in conformal limit, one has $b_{\mathrm{diff}} \sim \sqrt{zz'}\,(zz's/N^2)^{1/6}$, which leads to a total cross-section which grows as $\sigma_{\mathrm{tot}} \sim s^{1/3}$.

With confinement, the spectrum has a mass gap, which leads to a logarithmic growth. To be more precise, since the effects of the Pomeron cut are short-range, the spin-2 poles dominate the physics at very large b for fixed s and $z, z' \sim z_{\mathrm{max}}$ (where the hadron wave functions are largest), with the corrections from higher-spin states only becoming important at shorter range. Thus to understand the behavior of the cross-section, we may focus on the spin-two glueball states. From the lightest glueball of mass m_0, we find $|\chi| \sim 1$ inside a radius

$$b_{\mathrm{diff}} \simeq \frac{1}{m_0} \log(s/N^2 m_0^2) + \dots\,. \tag{6}$$

[2] We are speaking of disks in the bulk, for fixed z, z'; the corresponding disks in the gauge theory can be found only be integrating over z and z'.

It is important to check whether the eikonal approximation is self-consistent in the regimes we are discussing. A weak but necessary condition is that the scattering causes deflections at small angle, which requires b be larger than $b_{\theta\ll 1} \sim \frac{1}{2m_0}\log(s/N^4\Lambda^2) + \dots$. In other words, the area in which the scattering amplitude is reaching its unitarity bound, and in which the eikonal scattering is minimally self-consistent, is of order $(\log s)^2$. This provides strong evidence that, in the strong coupling limit, the Froissart bound on the total cross-section is not only satisfied, it is saturated.

4. Future directions

We have taken a step toward unitarization of high energy scattering using string/gauge duality. In future, it will be important to compute a variety of scattering amplitudes and interpret the results; [6] has recently begun this program in the context of deep-inelastic scattering. Eventually one would hope to extract appropriate lessons for QCD, though this will be a challenge, given the intricate dependence of the physics on s, b, λ and N. In particular, the approach to the region $\lambda \to 1$ holds some subtleties that are yet to be explored. One future goals is to show that the linearity approximation for the eikonal sum holds for a sufficiently large region in impact parameter space to prove saturation of the Froissart bound in strong coupling confining gauge theories. By isolating the leading contribution in the gravity limit first, we can proceed systematically to introduce the $1/\sqrt{\lambda}$ contributions to guide the development of a dual (Gribov) Reggeon effective field theory. These are ambitious goals but ones that have real promise to bring new clarity to high energy hadronic physics.

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