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DECOHERENCE OF QUARK COLOUR STATES IN QCD VACUUM*

Viacheslav Kuvshinov, Pavel Buividovich

JIPNR, National Academy of Science
Acad. Krasin str. 99, 220109 Minsk, Belarus

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Strong interactions which lead to quark confinement are analyzed from the point of view of quantum information theory. It is shown how states with nontrivial colour charge can be transformed into white mixed states described by density matrices and how the information about colour states of quarks is lost due to interactions with non-Abelian quantum gauge fields. For mixed state of a single quark the entropy growth rate is found to be proportional to the tension of QCD string.

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The most interesting feature of non-Abelian gauge theories is the permanent confinement of colour charges, which explains why the most fundamental constituents of matter — quarks — have never been observed experimentally. Only white, colourless bound states of quarks — hadrons — can be directly detected. It is commonly believed that quarks can effectively behave as individual particles only under extremal physical conditions. Such conditions, however, can only be created for extremely short periods of time, after which all quarks and gluons recombine back into hadrons. It is important to know the details of such hadronization processes in order to describe correctly the experimental data, however, hadronization and confinement are essentially nonperturbative phenomena and up to date can only be described by various phenomenological models [1]. From numerical simulations on the lattice it is known that confinement is governed by strong interactions of quarks with gluonic degrees of freedom, and the role of sea quarks is negligible [3]. For instance, hadron masses are mainly saturated by gluons, not quarks. Thus the information about the states with nontrivial colour charge is completely lost due to strong interactions with gluonic degrees of freedom.

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Such situation is common in quantum physics — consider some system A which is strongly coupled to another, much larger, system B. This much larger system B is usually called the environment or the heat bath [2]. Due to interaction the states of A and B become entangled, and the information which characterized the individual state of A is redistributed between A and B. However, the state of the environment can not be determined in practice, therefore some information about the state of A is lost. In this case the state of A can only be described by the reduced density matrix $\hat{\rho}_A$:

$$\hat{\rho}_A = \mathrm{Tr}_{\,B} \left|AB\right\rangle \left\langle AB\right| , \tag{1}$$

where the trace is taken over all states of B. The loss of information about the initial state of A can be characterized by the von Neumann entropy $S = -\mathrm{Tr}\,(\hat{\rho}_A \ln \hat{\rho}_A)$, which is equal to zero for pure states with $\hat{\rho}_A = |A\rangle\langle A|$ and which takes its maximal value for the mixed state with $\hat{\rho}_A = \left(\mathrm{Tr}\,\hat{I}\right)^{-1} \hat{I}$. This is a mixed state where all pure states of A are mixed with equal probabilities, thus all information about the initial state of A is lost.

In practice the effect of the system A on the system B can often be neglected. For example, if the system A corresponds to quark degrees of freedom and the system B — to gluonic degrees of freedom, we obtain the quenched approximation which describes QCD with heavy quarks with a good precision [3]. In this approximation one can describe the evolution of the state vector of A by a Schrödinger equation with the hamiltonian which depends on the state of B and after that average the density matrix of A over the ground state of B:

$$\hat{\rho}_A = \langle\, |A\rangle\, \langle A|\, \rangle_B \, . \tag{2}$$

The aim of this paper is to demonstrate how white mixed states described by density matrices can emerge in QCD. For simplicity only the colour states of a single infinitely heavy colour charge which interacts with quantum gauge fields with $\mathrm{SU}(N_\mathrm{c})$ gauge group will be considered. The space-time is assumed to be Wick-rotated from Minkowski to Euclidean metric. The effect of colour charge on Yang–Mills vacuum will be neglected and the approximation (2) will be used. In contrast to [4], the analysis presented will not be based on any particular model of QCD vacuum.

An infinitely heavy colour charge in a fixed gauge field is characterized by its worldline γ and the state vector in colour space $|q\rangle$, which depends on the point on γ. The vector $|q\rangle$ is the unit vector in the complex N_c-dimensional space, whose evolution can be described as the parallel transport along γ. The final and the initial colour states of the charge can only be compared if γ is a closed loop. In Minkowski space-time this corresponds to the process of creation and annihilation of a quark–antiquark pair. It is convenient to consider the rectangular loop γ with the size $R \times T$, so that the pair is separated by the distance R for the time T.

In order to demonstrate how white mixed states can emerge due to interaction with non-Abelian quantum gauge fields, assume that in some point x_0 on the worldline γ the colour charge was prepared in some state $|q_0\rangle$, which is then parallel transported along γ. After that the colour state of the charge becomes:

$$|q\rangle = \mathcal{P} \exp\left(i \int\limits_{\gamma} dx^{\mu} \hat{A}_{\mu} \right) |q_0\rangle \, , \tag{3}$$

where $\hat{A}_{\mu}$ is the gauge field and $\mathcal{P}$ is the path-ordering operator. Correspondingly, the density matrix for the state $|q\rangle$ is [4]:

$$|q\rangle\langle q| = N_{\mathrm{c}}^{-1} \, \hat{I} + \hat{T}_a \langle q_0|\hat{T}_b|q_0\rangle \mathcal{P} \exp\left(\int\limits_{\gamma} dx^{\mu} \hat{A}^{\mu} \right)_{ab} , \tag{4}$$

where $\hat{T}_a$, $a = 1, \ldots, N_{\mathrm{c}}^2 - 1$ are the generators of the $\mathrm{SU}(N_{\mathrm{c}})$ gauge group normalized as $\mathrm{Tr}\left(\hat{T}_a \hat{T}_b\right) = \delta_{ab}$ and $\mathcal{P} \exp\left(\int\limits_{\gamma} dx^{\mu} \hat{A}^{\mu} \right)_{ab}$ is the holonomy of the gauge field over the loop γ calculated in the adjoint representation of the gauge group. According to (2), the expression (4) should be averaged over the ground state of non-Abelian quantum gauge fields. If the theory is regularized in such a way that there is no need to fix the gauge (an example of such a regularization is the lattice gauge theory), it is convenient to integrate the holonomy in (4) over gauge orbits first, which shows that its expectation value is equal to $\delta_{ab} W_{\mathrm{adj}}[\gamma]$ [4], where $W_{\mathrm{adj}}[\gamma]$ is the Wilson loop in the adjoint representation of the gauge group. Using (2) and (4), one can express the density matrix which describes the mixed state of the colour charge as:

$$\hat{\rho} = \langle \, |q\rangle \, \langle q| \, \rangle = \; N_c^{-1} \hat{I} + \left(|q_0\rangle\langle q_0| - N_c^{-1} \hat{I} \right) W_{\mathrm{adj}}[\gamma] \; . \tag{5}$$

As the loop γ becomes larger and the Wilson loop $W_{\mathrm{adj}}[\gamma]$ decays, this density matrix changes from the pure state to the white mixed state with $\hat{\rho} = N_{\mathrm{c}}^{-1} \hat{I}$, for which all colour states are mixed with equal probabilities and all information about the initial colour state $|q_0\rangle$ is lost. Thus interactions with non-Abelian quantum gauge fields indeed lead to the emergence of white mixed states.

Now the entropy which corresponds to the density matrix $\hat{\rho}$ can be calculated. When R or T are of order of 1 fm (for SU(3) theory), Wilson loops decay exponentially with the area of the surface spanned on γ: $W_{\rm adj}\left[\gamma\right] = \exp\left(-\sigma_{\rm adj} RT\right)$. At this scale the QCD string with the tension $\sigma_{\rm adj}$ is formed. Thus when T is not very large, the entropy can be expanded as follows:

$$S = \sigma_{\rm adj} RT \left(1 - N_{\rm c}^{-1}\right) \left(1 - \ln\left(\frac{\sigma_{\rm adj} RT}{N_{\rm c}}\right)\right) + O\left(\sigma RT\right) . \tag{6}$$

This expression implies that at distances of order of 1 fm the entropy growth rate is proportional to the adjoint string tension $\sigma_{\rm adj}$ and the length of QCD string, which supports the statement about the relation between the dissipation of information about states of colour charges and their permanent confinement. When the size of the loop γ becomes much larger compared to hadronic scale, $W_{\rm adj}\left[\gamma\right]$ decays exponentially with the length of γ. In this case $W_{\rm adj}\left[\gamma\right] \ll 1$ and the entropy can be approximated as $S = \ln N_{\rm c} - \frac{N_c - 1}{2} W^2_{\rm adj}\left[\gamma\right]$. Thus the maximal possible entropy $S_{\rm max} = \ln N_{\rm c}$ is slowly approached at a rate determined by quark self-energy. It is interesting to note that similar behavior is also common for decoherence processes, which can be usually separated into fast dephasing (decay of the off-diagonal components of the density matrix) and slow relaxation of the diagonal components [2].

It is interesting that in the above analysis the quantity which characterizes the loss of information about colour states is the adjoint string tension, while most physical properties of hadrons are related to the string tension in the fundamental representation. The reason is that the simplest white mixed state described by (5) does not correspond to any experimentally observable particle. In order to study mixed states which can describe real hadrons, one should consider the joint density matrix of several quarks, which will be done in subsequent publications.

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