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HEAVY QUARK PROPAGATION IN AN AdS/CFT PLASMA*

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We compute the momentum broadening of a heavy probe in $\mathcal{N} = 4$ super-symmetric Yang–Mills in the large number of colors limit and strong coupling. The mean momentum transferred squared per unit length, κ, is expressed in terms of derivatives of a Wilson line. This definition is used to compute κ via the AdS/CFT correspondence.

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1. Introduction

Two of the major experimental findings of the RHIC program are the strong collective effects and the high p_t particle suppression. Hydrodynamic models, in which the viscosity of matter is assumed to vanish, have been successful in describing the observed radial and elliptic flow. The tomographic analysis of the particle suppression (jet quenching) shows that the opacity of the matter produced at RHIC is large. These findings indicate that the physics of strong coupling are important in the region of the QCD phase diagram accessible to RHIC.

The need for strong coupling techniques has led to a burst of interest in the AdS/CFT correspondence. This correspondence establishes a duality between the $\mathcal{N} = 4$ super-symmetric Yang–Mills in the strong coupling limit ($\lambda = g^2 N_c \to \infty$) and Type II B Super Gravity, a gravity theory in 10 dimension. While the theory described is not QCD, the possibility of extracting dynamical quantities in a gauge theory in a regime not accessible to lattice techniques makes the AdS/CFT correspondence a very powerful tool in the understanding of heavy ion data. In this work we discuss the propagation of a heavy probe via this correspondence.

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2. Momentum broadening in gauge theories

The dynamics of a heavy probe in the plasma can be described by a classical equation of motion under a random force, $\mathcal{F}$, due to the thermal fluctuations of the (chromo) electric fields in the bath [1]

$$\mathcal{F} \equiv \int d^3x\, Q^\dagger(t,\boldsymbol{x}) T^a Q(t,\boldsymbol{x})\, E_a(t,x)\,, \tag{1}$$

with $Q(\boldsymbol{x},t)$ the heavy quark field.

The force distribution can be characterized by its second moment,

$$\left\langle \mathcal{F}_i(t)\mathcal{F}_j\left(t'\right)\right\rangle = \kappa\delta_{ij}\delta\left(t-t'\right)\,, \tag{2}$$

where the constant κ is the mean squared momentum transferred to the probe. In this expression, the average is taken with the partition function of the gauge theory plus the heavy quark. Since the correlator in Eq. (2) is not time ordered it is convenient to express κ in terms of the retarded correlator. By using the Kubo–Martin–Schwinger relations we obtain

$$\kappa = \lim_{\omega\to 0} -\frac{2T}{\omega}\mathrm{Im}G_{\mathrm{R}}(\omega)\,, \qquad iG_{\mathrm{R}}(t) = \theta(t)\left\langle [\mathcal{F}(t),\mathcal{F}(0)]\right\rangle_{HQ}\,. \tag{3}$$

The expression above can be obtained from a Wilson line along the time contour with a deformation in the $\hat{y}$ direction δy

$$W_C[\delta y] = T_C \exp\left\{-i\int\limits_C dt\,(A_0 + \delta\dot{y}A_y)\right\}, \tag{4}$$

by taking (functional) derivatives on the path.

$$\langle T_C[\mathcal{F}(t_C)\mathcal{F}(0)]\rangle_{HQ} = \frac{1}{\langle W_C[0]\rangle}\left.\left\langle \frac{\delta^2 W_C[\delta y]}{\delta y(t_C)\,\delta y(0)}\right\rangle\right|_{\delta y=0}\,. \tag{5}$$

Thus, the broadening is obtained by studying small fluctuations of the Wilson line along the heavy quark path.

3. AdS/CFT computation of the broadening

According to the AdS/CFT correspondence, the gravity theory describing a thermal state of $\mathcal{N}=4$ SYM is characterized by the metric

$$ds^2 = \frac{r^2}{R^2}\Big(-f dt^2 + dx_1^2 + dx_2^2 + dx_3^2\Big) + \frac{R^2}{r^2}\frac{1}{f}dr^2 + R^2 d\Omega_5\,, \tag{6}$$

with R the AdS radius, $f = 1 - r_0^4/r^4$ and $r_0 = \pi T R^2$ the horizon.

A static Wilson line in the gauge theory is described in the gravity theory by a single string pending down from the boundary of AdS. The momentum broadening is computed by studying the propagation of small fluctuations in the boundary along a path δy into the AdS bulk, Fig. 1.

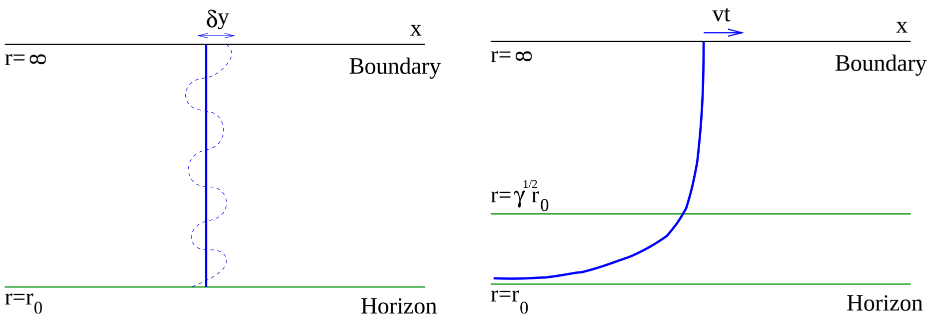

Fig. 1. Left: Static string solution. The dashed line represents the fluctuations in the transverse coordinates. Right: Trailing string corresponding to a prove moving at finite velocity [2, 3]. The string approaches logarithmically to the horizon.

The string solution is a surface parameterized by

$$X^M_{\rm cl} = (t, r, 0, y(t, r), 0) \ , \tag{7}$$

such that, at the boundary $y(t, r \to \infty) = \delta y(t)$ and is a minimum of the Nambu–Goto action. For small fluctuations, the action reads

$$S_{\rm NG} = \frac{R^2}{2\pi\alpha'} \int \frac{d\bar{t}\, du}{2u^{3/2}} \left[1 - \frac{1}{2} \left(\frac{\dot{\bar{y}}_{\|}^2}{f} - 4fu \left(\bar{y}'_{\|} \right)^2 \right) \right] \ , \tag{8}$$

where $u = r_0^2/r^2$, $\bar{t} = \pi T t$, $\bar{y} = \pi T y$. Performing a Fourier transform

$$\bar{y}(t, u) = \int e^{-i\mathfrak{w}\bar{t}}\, \bar{y}(\mathfrak{w})\, Y_{\mathfrak{w}}(u)\, \frac{d\mathfrak{w}}{2\pi} \, , \tag{9}$$

where $Y_{\mathfrak{w}}(u = 0) = 1$ and, thus, $\bar{y}(\mathfrak{w})$ is the boundary value.

Imposing infalling boundary conditions at the horizon, the solution to the equations of motion inferred from Eq. (8) is

$$Y_{\mathfrak{w}} = (1 - u)^{-i\mathfrak{w}/4}\, F(u) \, , \tag{10}$$

with $F(u)$ a regular function which at lowest order in frequency is

$$F(u) = 1 + \frac{i\mathfrak{w}}{8} \left\{ \pi - 4 \tan^{-1}\left(\sqrt{u}\right) - 6 \log 2 + 4 \log\left(1 + \sqrt{u}\right) + 2 \log(1 + u) \right\} . \tag{11}$$

Following the AdS/CFT correspondence, the retarded correlator is

$$G_{\rm R}(\omega) = -A(u)Y_{-\omega}(u)\partial_u Y_\omega(u)|_{u\to 0}\,, \qquad A(u) = \frac{R^2}{\pi\alpha'}\frac{(\pi T)^2}{}\frac{f}{u^{1/2}}\,. \quad (12)$$

Inserting Eq. (11) into Eq. (12) and Eq. (3) we have

$$\kappa \;=\; \lim_{\omega\to 0}\frac{-2T}{\omega}\,{\rm Im}\,G_{\rm R}(\omega) \;=\; \sqrt{\lambda}\,T^3\,\pi\,, \qquad (13)$$

where $R^2/\alpha' = \sqrt{\lambda}$.

This computation was extended to a probe moving at a velocity v in [4]. In this case the string solution trails back the boundary endpoint, as illustrated in Fig. 1 [2,3]. The transverse fluctuations yields

$$\kappa \;=\; \sqrt{\gamma\lambda}\,T^3\,\pi\,, \qquad (14)$$

with $\gamma = 1/\sqrt{1-v^2}$ the usual Lorentz factor.

The obtained κ has several remarkable features. First of all it depends linearly on the coupling constant g instead of quadratically, as expected from perturbation theory. Second, it also depends explicitly on the number of colors. Thus, the result is not universal, since it depends on the number of degrees of freedom of the theory. Finally, it depends explicitly on the velocity of the probe. Let us note that the above computation cannot be extended to the ultra-relativistic limit, since the stability of the brane solution that supports the string demands [4]

$$\gamma < \left(\frac{M}{\sqrt{\lambda}T}\right)^2\,. \qquad (15)$$

In a Langevin description, the momentum broadening leads to the heavy quark diffusion constant. Using the Einstein relations we obtain [1]

$$D = \frac{2T^2}{\kappa} \approx \frac{0.9}{2\pi T}\left(\frac{1.5}{\alpha_{\rm s}N_c}\right)^{1/2}\,. \qquad (16)$$

For the typical values of the coupling constant expected at RHIC the computed diffusion constant is comparable to the values extracted from the data. This is remarkable for a theory which is not QCD, which also indicates that the physics of strong coupling are important for RHIC.

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