

COLLECTIVE HAMILTONIAN FOR CHIRAL AND WOBBLING MODES: FROM ONE- TO TWO-DIMENSIONAL*

Q.B. CHEN

State Key Laboratory of Nuclear Physics and Technology, School of Physics
Peking University, Beijing 100871, China

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The recent progresses of the two-dimensional collective Hamiltonian and its applications for chiral and wobbling modes are reviewed. In particular, the comparisons between the results by one- and two-dimensional collective Hamiltonian are introduced.

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1. Introduction

Triaxial shape has been a long-standing subject in nuclear physics. The observation of chiral doublet or wobbling bands provides unambiguous experimental evidence of stable triaxiality. Nuclear chirality was first predicted in 1997 by Frauendorf and Meng [1]. Experimentally, more than 30 candidate chiral nuclei have been reported in the $A \sim 80, 100, 130$, and 190 mass regions; see, *e.g.*, Refs. [2, 3]. The wobbling motion was originally suggested by Bohr and Mottelson [4], and has been found mainly in the $A \sim 160$ mass region; see, *e.g.*, Ref. [5] and, very recently, in the $A \sim 130$ region [6].

Many theoretical models have been used to investigate the chirality and wobbling motion, such as the triaxial particle rotor model (PRM) [7–14], the tilted axis cranking (TAC) model [15, 16], and the random phase approximation (RPA) [17–20]. Based on the TAC model a one-dimensional collective Hamiltonian (1DCH) was proposed [5, 21]. It goes beyond the mean-field approximation and could microscopically describe not only the yrast sequence but also the highly excited bands. Using this model, the chiral vibration and rotational modes have been successfully described [21]. The simple, longitudinal and transverse wobblers [22] are systematically studied, and

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the variation trends of wobbling frequency of these three types of wobblers are confirmed [5]. This model is also adopted to describe the wobbling bands in ^{135}Pr [23]. Some of results were presented at the XXII Nuclear Physics Workshop “Marie and Pierre Curie” [24]. With the successes of 1DCH, we recently extend the collective Hamiltonian to two dimensions (2DCH) to explore the related new physics [25].

In this proceeding, the recent progresses of 2DCH and its comparison with 1DCH will be briefly reviewed.

2. Theoretical framework

Collective Hamiltonian, in terms of collective degree of freedom, is an effective tool for investigating various collective processes which involve small velocities [4]. In the chiral and wobbling modes collective Hamiltonian, the orientation of the nucleus in the rotating mean field is taken as the collective degree of freedom. The detailed theoretical framework of the collective Hamiltonian is presented in Refs. [5, 21, 23, 25].

In the 1DCH, the azimuth angle φ of nuclear orientation is taken as the collective coordinate. In the 2DCH, not only the φ but also the polar angle θ are considered as the collective variables. The quantized form of the 2DCH is written as

$$\begin{aligned} \hat{\mathcal{H}}(\theta, \varphi) = & -\frac{\hbar^2}{2\sqrt{w}} \left[\frac{\partial}{\partial \varphi} \frac{B_{\theta\theta}}{\sqrt{w}} \frac{\partial}{\partial \varphi} - \frac{\partial}{\partial \varphi} \frac{B_{\varphi\theta}}{\sqrt{w}} \frac{\partial}{\partial \theta} \right. \\ & \left. - \frac{\partial}{\partial \theta} \frac{B_{\theta\varphi}}{\sqrt{w}} \frac{\partial}{\partial \varphi} + \frac{\partial}{\partial \theta} \frac{B_{\varphi\varphi}}{\sqrt{w}} \frac{\partial}{\partial \theta} \right] + V(\theta, \varphi), \end{aligned} \quad (1)$$

in which the collective potential V and mass parameters $B_{\theta\theta}$, $B_{\theta\varphi}$, $B_{\varphi\theta}$, and $B_{\varphi\varphi}$ are all functions of (θ, φ) and calculated by the TAC model. The w is the determinant of the mass parameter tensor

$$w = \det B = \begin{vmatrix} B_{\theta\theta} & B_{\theta\varphi} \\ B_{\varphi\theta} & B_{\varphi\varphi} \end{vmatrix} = B_{\theta\theta}B_{\varphi\varphi} - B_{\theta\varphi}B_{\varphi\theta}. \quad (2)$$

Solving the collective Hamiltonian (1), the collective levels and the corresponding wave functions can be obtained.

3. Results and discussion

In the calculations, a system with one $h_{11/2}$ proton particle and one $h_{11/2}$ neutron hole coupled to a triaxial rigid rotor with $\gamma = -30^\circ$ is considered. The moment of inertia is chosen as $\mathcal{J}_0 = 40 \hbar^2/\text{MeV}$. The collective Hamiltonian is constructed by Eq. (1) with the collective potential and the

mass parameter obtained from TAC calculations. The diagonalization of the collective Hamiltonian yields the energy levels and wave functions for each cranking frequency [25]. In the following, taking $\hbar\omega = 0.50$ MeV as an example, the collective energy levels and the wave functions obtained by the 2DCH will be compared with those obtained by the 1DCH [21].

The comparisons for the collective energy levels are shown in Fig. 1. Since the 2DCH is invariant with the transformations $\varphi \rightarrow -\varphi$ ($\hat{P}_\varphi$) and $\theta \rightarrow \pi - \theta$ ($\hat{P}_\theta$), one could group the eigenenergies and eigenstates into four categories with different combinations of the symmetries P_θ and P_φ , *i.e.*, $(P_\theta P_\varphi) = (++)$, $(+-)$, $(-+)$, and $(--)$. For the 1DCH, it is invariant with respect to $\varphi \rightarrow -\varphi$ and thus, its solutions can be grouped as $(P_\varphi) = (+)$ and $(-)$. In Fig. 1, the collective energy levels of the two- and one-dimensional collective Hamiltonians are normalized to the corresponding lowest energy levels $(++)$ and $(+)$, respectively.

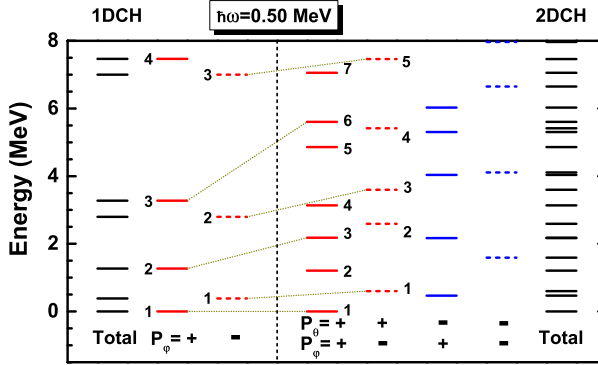


Fig. 1. (Color online) Collective energy levels obtained from the two-dimensional collective Hamiltonian in comparison with those from the one-dimensional collective Hamiltonian for $\hbar\omega = 0.50$ MeV. Taken from Ref. [25].

Apparently, one obtains more energy levels by solving the 2DCH than solving the 1DCH, since one more degree of freedom θ has been taken into account. Of course, one cannot find the corresponding energy levels in the groups of $(-+)$ and $(--)$ in the 1DCH results. Only the 2DCH energy levels in the groups of $(++)$ and $(+-)$ with zero-phonon excitation along the θ direction have their counterparts in the 1DCH. By analyzing the behaviors of the wave functions, one can easily build the connections between the solutions of the one- and two-dimensional Hamiltonians, as shown in Fig. 1 with the dotted lines.

The comparison of the wave functions can be found in Fig. 2. In order to compare 2DCH wave functions with the 1DCH ones, we chose $\theta = 78^\circ$ for all the wave functions, and this θ value is also the position of the minimum

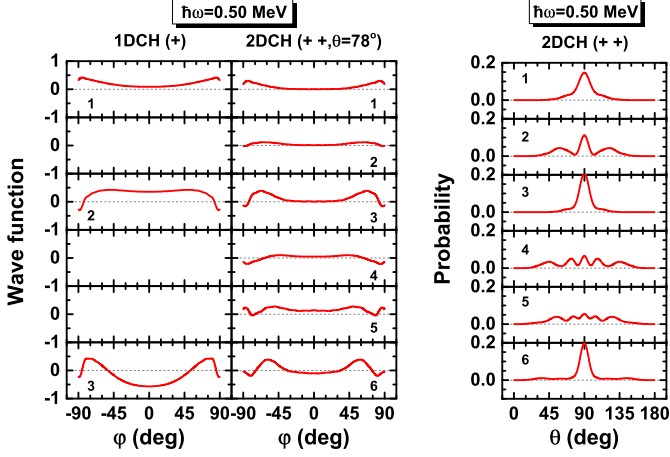


Fig. 2. (Color online) Left: The wave functions along the φ direction obtained by the 2DCH in comparison with those by the 1DCH. Right: The probability distributions $P(\theta)$ along the θ direction obtained by the 2DCH. Taken from Ref. [25].

in the collective potential. In Fig. 2, we present the wave functions of the six lowest energy states in the (+,+) group of the 2DCH, as well as the corresponding wave functions in the 1DCH. In Fig. 3, we also show the wave functions along the φ direction obtained by the 2DCH for $\theta = 0^\circ$. It can be

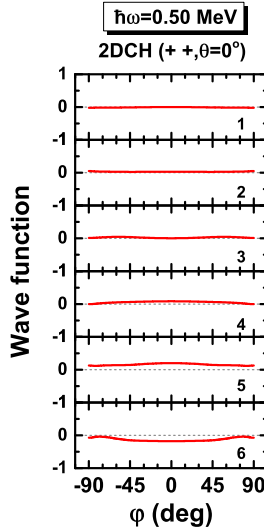


Fig. 3. (Color online) The wave functions along the φ direction obtained by the 2DCH for $\theta = 0^\circ$.

seen from Fig. 3 that the wave functions almost vanish, which is consistent with the fact that the minimum of the collective potential is far from $\theta = 0^\circ$. So in the following, the discussions are focused on the case for $\theta = 78^\circ$.

It can be seen from Fig. 2 that the behaviors of the wave functions for the 2DCH levels 1, 3, and 6 are similar to those for the one-dimensional levels 1, 2, and 3, respectively. This is consistent with the fact that the zero phonon excitation modes along the θ direction are weakly coupled with the excitations along the φ direction.

However, in Fig. 2, no one-dimensional counterpart has been found for the two-dimensional levels 2, 4, and 5. To further examine this point, we also plot the probability distributions of the 2DCH along the θ direction as shown in Fig. 2. Apparently, for each 2DCH level which has a one-dimensional correspondence, *i.e.*, 1, 3, 6, the corresponding probability distribution along the θ direction has only one peak, and this is associated with a zero-phonon vibration mode along the θ direction. In contrast, the probability distributions of the other levels have more than one peak, and they correspond to nonzero-phonon vibration mode along the θ direction. As a result, they have no counterpart in the solutions of the 1DCH. Note that we have also checked the wave functions for other rotational frequencies, and a similar conclusion can be obtained. To avoid repetition, the results are not shown here.

In Fig. 1, we label the counterparts between the 2DCH levels and the 1DCH ones with dotted lines. One can see that, with increasing energy, the energy differences between the 2DCH and 1DCH solutions become larger. This indicates that the one-dimensional approximation is reasonable at low excitation energy.

In addition, in Ref. [25], the angular momenta and energy spectra calculated by the 2DCH are compared with those by the TAC approach and the exact solutions of PRM. It is demonstrated that the 2DCH can well reproduce the PRM results by taking the fluctuations along the θ and φ directions into account.

4. Summary

The recent progresses of the 2DCH and its comparison with 1DCH are reviewed. More excitation modes have been obtained in the 2DCH calculations in comparison with the 1DCH ones. The 2DCH levels, which have the zero-phonon excitation along the θ direction, have their counterparts in the 1DCH solutions.

In addition, it is demonstrated that the 2DCH can well reproduce the PRM results by taking the fluctuations along the θ and φ directions into account. The success of the collective Hamiltonian has open the door to develop the collective Hamiltonian based on TAC density functional, *e.g.*, the TAC covariant density functional theory (TAC-CDFT) [26].

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