

RECENT RESULTS IN EUCLIDEAN DYNAMICAL TRIANGULATIONS*

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We study a formulation of lattice gravity defined via Euclidean dynamical triangulations (EDT). After fine-tuning a non-trivial local measure term, we find evidence that four-dimensional, semi-classical geometries are recovered at long distance scales in the continuum limit. Furthermore, we find that the spectral dimension at short distance scales is consistent with $3/2$, a value that is also observed in the causal dynamical triangulation (CDT) approach to quantum gravity.

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1. Introduction

EDT defines a spacetime of locally flat d -dimensional triangles, each with a fixed edge length. The model described in this work uses the partition function

$$Z_E = \sum_T \frac{1}{C_T} \left[\prod_{j=1}^{N_2} \mathcal{O}(t_j)^\beta \right] e^{-S_E}, \quad (1)$$

where the product is over all triangles, and $\mathcal{O}(t_j)$ is of the order of the triangle j , *i.e.* the number of 4-simplices to which the triangle belongs. The term in square brackets defines our non-trivial measure term, where β is a free parameter. The Einstein–Regge action is given by

$$S_E = -\kappa_2 N_2 + \kappa_4 N_4, \quad (2)$$

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where κ_2 and κ_4 are related to the bare Newton's constant and the cosmological constant, respectively. The parameter space of our model is depicted schematically in Fig. 1.

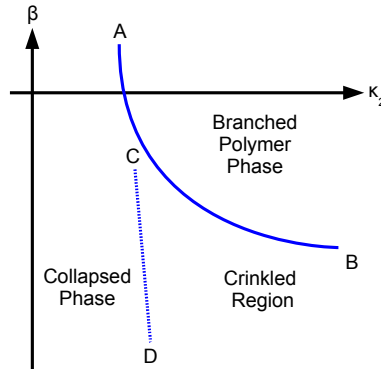


Fig. 1. A schematic of the EDT phase diagram as a function of κ_2 and β .

2. Global Hausdorff dimension

We determine the Hausdorff dimension of our ensembles by studying the finite-size scaling of the three-volume correlator $C_{N_4}(\delta)$ introduced in Ref. [1]. Figure 2 shows the rescaled correlator $c_{N_4}(x)$ for lattice volumes of 4 K, 8 K and 16 K four-simplices on the transition line AB for $\beta = 0$. We find that the overlap between the curves is maximised for $D_H = 4.1 \pm 0.3$.

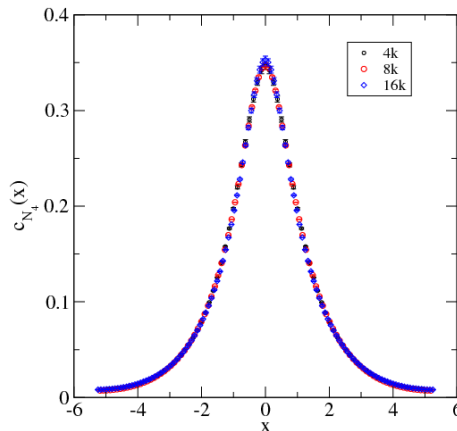


Fig. 2. Scaling of the volume-volume distribution as a function of the rescaled variable $x = \delta/N_4^{1/D_H}$ using lattice volumes of 4 K, 8 K and 16 K four-simplices.

3. Spectral dimension

The spectral dimension D_S is related to the probability of return P_r for a random walk over an ensemble of triangulations after σ diffusion steps, and is defined via

$$D_S(\sigma) = -2 \frac{\text{dlog}\langle P_r(\sigma) \rangle}{\text{dlog}\sigma}. \quad (3)$$

Assuming the fit function $D_S(\sigma) = a - \frac{b}{c+\sigma}$, we obtain a large distance spectral dimension in the range of $D_S = 2.7\text{--}3.3$ [2], which is inconsistent with 4-dimensional semi-classical general relativity. However, this discrepancy may be due to finite volume or discretisation effects associated with the lattice simulations. In order to investigate whether this is the case, we consider an additional extrapolation of $D_S(\infty)$ of the form of

$$D_S(\infty) = c_0 + c_1 \frac{1}{V} + c_2 a^2, \quad (4)$$

where c_i is a fit parameter, V is the volume and a the lattice spacing. This particular Ansatz is motivated by the fact that the data points are linear in $1/V$ and a^2 . Extrapolation to the continuum and infinite volume limit gives $D_S(\infty) = 3.94 \pm 0.16$ and $D_S(0) = 1.44 \pm 0.19$, as shown in Fig. 3 (a) and Fig. 3 (b), respectively. A value of $D_S(0)$ consistent with $3/2$ may have important implications for the asymptotic safety scenario [3].

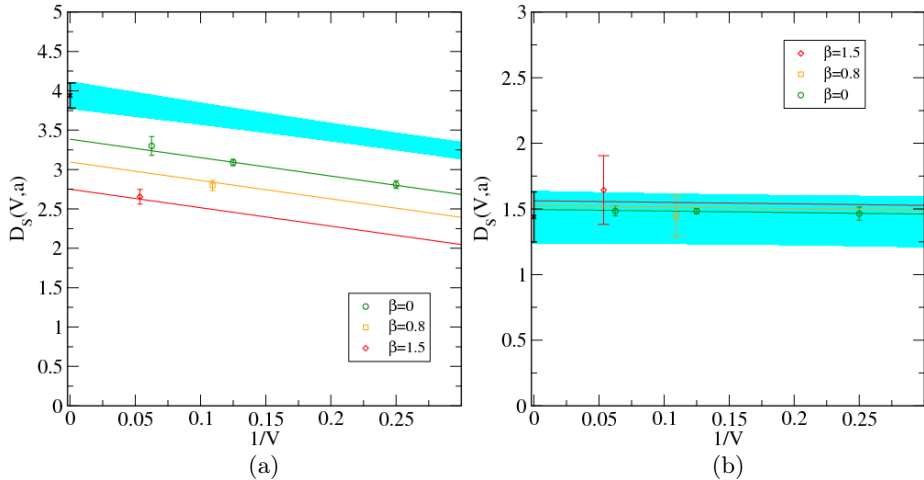


Fig. 3. The large (a) and small (b) distance scale spectral dimension D_S as a function of inverse lattice volume for 3 different β values, including an extrapolation to the infinite volume and continuum limit.

4. Discussion and conclusions

In this work, we determine the Hausdorff and spectral dimension for a specific fine-tuning of the bare coupling constants in Euclidean dynamical triangulations (EDT). Using a finite-size scaling analysis, we determine the Hausdorff dimension to be $D_H = 4.1 \pm 0.3$ on the transition line AB for $\beta = 0$, which is consistent with 4-dimensional general relativity and CDT results [4]. Furthermore, by applying an additional extrapolation to the continuum and infinite volume limits, we find a large scale spectral dimension of $D_S(\sigma) = 3.94 \pm 0.16$ and a small distance value of $D_S(\sigma) = 1.44 \pm 0.19$, results that are also similar to those reported in CDT [5,6].

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