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MODELS OF A DIFFUSIVE DM/DE INTERACTION*

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We discuss relativistic particle and field theoretic models of an interaction with an environment. We show that in a Markovian approximation such models lead to a diffusion. We interpret dark energy as an environment for the dark matter.

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1. Introduction

The successful ΛCDM model determines the global evolution of the Universe. In order to go into its more detailed structure, we must construct particular models of the energy-momentum (cosmological fluids, inflaton fields). Simple models involve non-interacting components of the energy-momentum. If we wish to explain a relation between the components and their evolution in time, we need models with a non-trivial dynamics. We do not know details of the interaction but in a system of an infinite number of particles, a Markovian approximation may be sufficient. We discuss here a model where the unknown components of the cosmological fluid (dark matter and dark energy) are treated in the scheme of a system and an environment. We think that such a scheme is in a sense unique and has some testable consequences: a relation between the temperature and the interaction coupling constant.

2. Particle environment

In the non-relativistic mechanics (classical as well as quantum), the interaction with an environment consisting of an infinite system of particles and its Markovian approximation have been well-studied. The effect of decoherence of such an interaction shows that even a weak interaction with an infinite system may result in a profound change of the behavior of complex systems. An interaction of a relativistic system with an environment has not

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been so well-studied yet. There is the well-known problem of a relativistic description of a many-particle system. However, instead of an introduction of an environment of many particles, we may consider an electromagnetic field F produced by an environment of such (charged) particles. Then, the equations of motion read

$$\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = p^\mu \,, \tag{2.1}$$

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} + \Gamma^\mu_{\nu\alpha} p^\nu p^\alpha = F^{\mu\nu} p_\nu \,, \tag{2.2}$$

where τ is the proper time and $\Gamma^\mu_{\nu\alpha}$ are the Christoffel symbols. In the Liouville description, we write an equation for the probability distribution Ω in the phase space (we eliminate the proper time τ in favor of the coordinate time t)

$$\partial_t \Omega = -\left(p^0\right)^{-1}\left(p^k \partial^x_k \Omega - \Gamma^k_{\nu\alpha} p^\nu p^\alpha \partial_k \Omega + F^{k\nu} p_\nu \partial_k\right)\Omega \equiv (X+Y)\Omega \,, \tag{2.3}$$

where ∂_k is a derivative over momenta, ∂^x_k a derivative over spatial coordinates and

$$Y = \left(p^0\right)^{-1} F^{k\nu} p_\nu \partial_k \,. \tag{2.4}$$

If $F = 0$, then the solution of the Liouville equation can be expressed as

$$\Omega_t(\boldsymbol{x}, \boldsymbol{p}) = \left(\exp\left(\int^t \mathrm{d}s X_s\right)\Omega\right)(\boldsymbol{x}, \boldsymbol{p}) = \Omega(x_t, p_t) \,, \tag{2.5}$$

where (x_t, p_t) is the geodesic and the exp can be treated as a time ordered exponential of the vector field X. In the presence of the electromagnetic field, we may write

$$\Omega_t(\boldsymbol{x}, \boldsymbol{p}) = \left(\exp\left(\int^t \mathrm{d}s X_s\right)\exp\left(\int^t \mathrm{d}s Y^X_s\right)\Omega\right)(\boldsymbol{x}, \boldsymbol{p}) \,, \tag{2.6}$$

where

$$Y^X_t = \exp\left(-\int^t \mathrm{d}s X_s\right) Y_t \exp\left(\int^t \mathrm{d}s X_s\right) . \tag{2.7}$$

We assume that the electromagnetic field F is produced by an infinite system of randomly moving relativistic particles. In such a case, the field F is also random. If the randomness of F is the result of a sum of small random perturbations resulting from a large number of particles, then we may assume on the basis of the central limit theorem that the random variables are Gaussian. For a Gaussian F and any operator $\mathcal{M}$ linear in F, we may write an expansion in powers of $\mathcal{M}$ (we assumed that $\langle F\rangle = 0$) $\langle \exp \mathcal{M}\rangle = \exp(\frac{1}{2}\langle \mathcal{M}^2\rangle + \ldots)$. Under general assumptions, we have calculated [1,2]

$$\left\langle \left(\int_0^t Y_s^X \mathrm{d}s \right)^2 \right\rangle \simeq \left(p^0\right)^{-1} \Delta_m^H \,, \tag{2.8}$$

where Δ_m^H is the Laplace–Beltrami operator defined as follows: in the space-time metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - g_{jk}\mathrm{d}x^j\mathrm{d}x^k \equiv \mathrm{d}t^2 - a^2 h_{jk}\mathrm{d}x^j\mathrm{d}x^k \tag{2.9}$$

(where for later purposes, the time dependence is contained in the scale factor a), we define the metric on the mass-shell

$$G^{jk} = g^{jk} + m^{-2}p^j p^k \,. \tag{2.10}$$

Then

$$\Delta_H^m = \frac{1}{\sqrt{G}}\partial_j G^{jk}\sqrt{G}\partial_k \,. \tag{2.11}$$

We assume that the time evolution consists of "strokes" at subsequent time intervals Δt by the random electromagnetic field (Markovian approximation). As a result, the random Liouville equation is approximated by the diffusion equation generated by Δ_H^m. By means of the phase-space distribution Ω, we can define the energy-momentum tensor $\tilde{T}^{\mu\nu}$ which is not conserved. It must be supplemented either by the energy-momentum of the random electromagnetic field F or by the energy-momentum $T_\Lambda^{\mu\nu}$ of the particles which are the source of the electromagnetic field.

3. An environment of quantum fields

In this section, we consider an environment of an infinite number of scalar fields χ^b. Let the scalar field ϕ with a potential V be linearly coupled to χ and satisfy equations of motion

$$g^{-\frac{1}{2}}\partial_\mu g^{\frac{1}{2}}\partial^\mu \phi + m^2\phi + V'(\phi) \;=\; -\sum_b \lambda_b \chi^b \,, \tag{3.1}$$

$$g^{-\frac{1}{2}}\partial_\mu g^{\frac{1}{2}}\partial^\mu \chi^b + m_b^2\chi^b \;=\; -\lambda_b\phi \,, \tag{3.2}$$

where $g_{\mu\nu}$ is the metric tensor and $g = |\det[g_{\mu\nu}]|$. In general, we need Green's function of the operator on the l.h.s. of Eq. (3.2) in order to express χ by ϕ. We restrict ourselves to a solution for a fixed ("frozen") a of Eq. (2.9). Then, the metric g can be treated as time-independent. We solve Eq. (3.2) for a time-independent metric and insert the solution in the r.h.s. of Eq. (3.1) obtaining

$$\sum_b \lambda_b \left(\cos(K_b t)\chi_{b0} + K_b^{-1}\sin(K_b t)\pi_{b0}\right) - \sum_b \lambda_b^2 \int_0^t K_b^{-1}\sin(K_b(t-s))\phi\, \mathrm{d}s\,, \tag{3.3}$$

where $K_b = -g^{-\frac{1}{2}}\partial_j g^{jk} g^{\frac{1}{2}}\partial_k + m_b^2$. In Eq. (3.3), the friction $\mathcal{R}$ is defined as the operator in the term $\mathcal{R}\phi$ linear in ϕ and the rest is interpreted as the noise W. The noise depends on the probability distribution of the initial values χ^b and $\partial_t\chi^b = \pi^b$ (which can be chosen as quantum fields at finite temperature T). Then, (in the classical limit) $\langle\chi_{b0}\chi_{b0}\rangle = TK_b^{-2}$ and $\langle\pi_{b0}\pi_{b0}\rangle = T$. These correlations determine correlations (3.5) for the noise W. The Markovian approximation results from a neglect of spatial dependence of $K_b^{-2}(\boldsymbol{x},\boldsymbol{x}') \simeq m_b^{-2}\delta(\boldsymbol{x}-\boldsymbol{x}')$ and the assumption $\lambda_b^2 \simeq \gamma m_b^2$. The $g^{-\frac{1}{2}} = a^{-3}$ term (where a is the scale expansion factor of Eq. (2.9)) follows from the dependence of the temperature $T = a^{-3w}T_0$ on the scale factor a and on the equation of state parameter w which for a kinetic energy dominance gives $w = 1$ [3]. We obtain from Eqs. (3.1)–(3.2) [3]

$$g^{-\frac{1}{2}}\partial_\mu g^{\mu\nu} g^{\frac{1}{2}}\partial_\nu\phi + m^2\phi + V'(\phi) = \gamma W\,, \tag{3.4}$$

where

$$\left\langle W(x) W\left(x'\right)\right\rangle = g^{-\frac{1}{2}}\delta\left(x,x'\right) \tag{3.5}$$

(with a four-dimensional δ-function on the r.h.s. of Eq. (3.5)). The diffusing particles of Sec. 2 and diffusing quantum fields of this section (which can create particles) are considered as models of the dark matter. The energy-momentum $\tilde{T}^{\mu\nu}$ of the dark matter is not conserved. We must supplement it by an energy-momentum tensor $T_\Lambda^{\mu\nu}$ of the environment so that the total energy momentum $\tilde{T}^{\mu\nu} + T_\Lambda^{\mu\nu}$ is conserved. The conservation law can determine $T_\Lambda^{\mu\nu}$ and the dynamical relation between the two components. In a general time-dependent metric, there will be no static (equilibrium) distribution of observables. We have shown [4] that there are probability distributions of the particle model of Sec. 2 and the field theoretic model of this section which can be interpreted as equilibrium distributions with a time-dependent temperature. The dependence of the temperature on the diffusion constant results from a fluctuation–dissipation theorem [3, 5]. This relation could, in principle, be checked in astronomical observations as the temperature and the rate of the energy dissipation are measurable.

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