KALUZA–KLEIN UNIVERSE FILLED WITH WET DARK FLUID IN $f(R,T)$ THEORY OF GRAVITY*

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Kaluza–Klein metric is considered with wet dark fluid (WDF) source in $f(R,T)$ gravity, where R is the Ricci scalar and T is the trace of the energy-momentum tensor proposed by Harko *et al.* (2011). The exact solutions of the field equations are derived from a time varying deceleration parameter.

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1. Introduction

The nature of the dark energy (DE), a component of the Universe [1–3], remains one of the greatest mysteries of cosmology. There are many candidates for DE such as: cosmological constant, quintessence [4], k-essence [5], phantom energy [6] *etc.* Modified or alternative theories of gravity are the second proposal to justify the current expansion of the Universe. The recently developed $f(R,T)$ theory of gravity is one such example.

In this work, we use WDF as a candidate for DE. This model is in the spirit of generalized Chaplygin gas (GCG), where a physically motivated EOS is offered with the properties relevant for DE problem. The EOS for WDF [7] is

$$p_{\rm WDF} = \omega(\rho_{\rm WDF} - \rho^*)\,. \tag{1}$$

This EOS is a good approximation for many fluids, including water, in which the internal attraction of molecules makes negative pressure. The parameters ω and ρ^* are taken to be positive and $0 \leq \omega \leq 1$. If $c_{\rm s}$ denote the adiabatic sound speed in WDF, then $\omega = c_{\rm s}^2$ [8]. The energy conservation equation for WDF is

$$\overset{*}{\rho}_{\rm WDF} + 4H(p_{\rm WDF} + \rho_{\rm WDF}) = 0\,. \tag{2}$$

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Using $4H = \frac{V'}{V}$ and EOS (1), the above equation can be written as

$$\rho_{\rm WDF} = \frac{\omega}{1+\omega}\rho^* + \frac{C}{V^{(1+\omega)}}\,, \tag{3}$$

where C is the constant of integration and V is the volume expansion. Wet dark fluid naturally includes two components: one of them behaves as a cosmological constant as well as a standard fluid with an equation of state $p = \omega\rho$. If we consider $C > 0$, this fluid will not violate the strong energy condition $p + \rho \geq 0$. Hence,

$$p_{\rm WDF} + \rho_{\rm WDF} = (1+\omega)\rho_{\rm WDF} - \omega\rho^* = (1+\omega)\frac{c}{V^{(1+\omega)}} \geq 0\,. \tag{4}$$

The action for $f(R,T)$ gravity is

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\,{\rm d}^4x + \int L_{\rm m}\sqrt{-g}\,{\rm d}^4x\,, \tag{5}$$

where $f(R,T)$ is an arbitrary function of Ricci scalar R, T be the trace of stress-energy tensor T_{ij} of the matter. $L_{\rm m}$ is the matter Lagrangian density. The energy momentum tensor T_{ij} is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\,\right)L_{\rm m}}{\delta g^{ij}}\,. \tag{6}$$

By the help of matter Lagrangian $L_{\rm m}$, the matter energy tensor is given by

$$T_{ij} = (p_{\rm WDF} + \rho_{\rm WDF})u_i u_j - p_{\rm WDF}g_{ij}\,, \tag{7}$$

where $u^i = (1,0,0,0,0)$ is the five velocity in comoving coordinates satisfying the condition $u^i u_i = 1$ and $u^i\nabla_j u_i = 0$. In equation (7), $\rho_{\rm WDF}$ is the energy density, $p_{\rm WDF}$ is pressure and the matter Lagrangian can be taken as $L_{\rm m} = -p_{\rm WDF}$.

Harko *et al.* [9] presented three classes of models. In this paper, we consider $f(R,T) = R + 2f(T)$, where $f(T)$ is an arbitrary function of energy tensor. The $f(R,T)$ gravity field equations are derived by varying the action S with respect to metric tensor g_{ij}. For a WDF matter source, (7) takes the form of

$$R_{ij} - \tfrac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + \left[2p_{\rm WDF}f'(T) + f(T)\right]g_{ij}\,. \tag{8}$$

2. Field equations and solution

The five-dimensional Kaluza–Klein metric is

$$ds^2 = dt^2 - A(t)^2\left(dx^2 + dy^2 + dz^2\right) - B(t)^2 d\psi^2\,, \tag{9}$$

where the fifth coordinate ψ is space-like.

Here, we consider $f(T) = \lambda T$, where λ is a constant.

The $f(R,T)$ gravity field equations (8) for the metric (9) can be written as

$$2H_x' + 3H_x^2 + 2H_xH_\psi + H_\psi' + H_\psi^2 = (8\pi + 4\lambda)p_{\mathrm{WDF}} - \lambda\rho_{\mathrm{WDF}}\,, \tag{10}$$

$$3\left(H_x' + 2H_x^2\right) = (8\pi + 4\lambda)p_{\mathrm{WDF}} - \lambda\rho_{\mathrm{WDF}}\,, \tag{11}$$

$$3\left(H_x^2 + H_xH_\psi\right) = -(8\pi + 3\lambda)\rho_{\mathrm{WDF}} + \lambda p_{\mathrm{WDF}}\,, \tag{12}$$

where prime denotes derivative with respect to time t. $H_x = \frac{A'}{A} = H_y = H_z$ and $H_\psi = \frac{B'}{B}$ are the directional Hubble parameters.

Here, we have three equations with four unknowns. To get a deterministic solution, we assume the deceleration parameter as

$$q = -\frac{aa''}{a'^2} = -1 + \frac{\beta}{1 + a^\beta}\,, \tag{13}$$

where $\beta > 0$ is a constant and a is the average scale factor.

The mean Hubble parameter $H = \frac{a'}{a}$ can be obtained from the above equation as

$$H = \frac{\dot{a}}{a} = A_1\left(1 + a^{-\beta}\right)\,, \tag{14}$$

where A_1 is an integration constant. Again, integrating (14), we get

$$a = \left(e^{A_1\beta t} - 1\right)^{\frac{1}{\beta}}\,. \tag{15}$$

Using the spatial volume $V = a^4 = A^3B$, we obtain the scale factors A, B as

$$A = \left(e^{A_1\beta t} - 1\right)^{\frac{1}{3\beta}}\,, \qquad B = \left(e^{A_1\beta t} - 1\right)^{\frac{3}{\beta}}\,. \tag{16}$$

The directional Hubble parameters are obtained as

$$H_x = \frac{A_1}{3\left(e^{A_1\beta t} - 1\right)} = H_y = H_z\,, \qquad H_\psi = \frac{3A_1}{e^{A_1\beta t} - 1}\,. \tag{17}$$

Using the above values, the energy density (ρ_{WDF}) and the pressure (p_{WDF}) are obtained as

$$\rho_{\mathrm{WDF}} = \frac{A_1^2}{(8\pi+3\lambda)(8\pi+4\lambda)-\lambda^2}\left[\frac{\beta\lambda e^{A_1\beta t}}{e^{A_1\beta t}-1} - \frac{(3\beta\lambda+80\pi+38\lambda)e^{2A_1\beta t}}{3(e^{A_1\beta t}-1)^2}\right], \tag{18}$$

$$p_{\mathrm{WDF}} = \frac{A_1^2}{(8\pi + 3\lambda)(8\pi + 4\lambda) - \lambda^2} \times \left[\frac{(8\pi + 3\lambda)\beta e^{A_1\beta t}}{e^{A_1\beta t} - 1} + \frac{\{16\pi - 4\lambda - 3\beta(8\pi + 3\lambda)\}e^{2A_1\beta t}}{3(e^{A_1\beta t} - 1)^2}\right]. \tag{19}$$

The anisotropy parameter of the expansion is $\varDelta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2 = \frac{4}{3}$. The scalar expansion ($\theta = 4H$) and the shear ($\sigma^2 = \frac{4}{2}\Delta H^2$) are

$$\theta = 4A_1 e^{A_1\beta t}\left(e^{A_1\beta t}-1\right)^{-1}, \qquad \sigma^2 = \frac{8}{3}A_1^2 e^{2A_1\beta t}\left(e^{A_1\beta t}-1\right)^{-2}. \tag{20}$$

3. Conclusion

The spatial volume and average scale factor of the model is zero at initial time $t \to 0$ indicating that the model starts at Big Bang and has a point type singularity too. The anisotropic parameter becomes constant and our model is expanding with time. From Figs. 1 (left) and (right), the energy density of WDF is a decreasing function of time and remains positive throughout the Universe whereas the pressure is the increasing function of time and becomes constant.

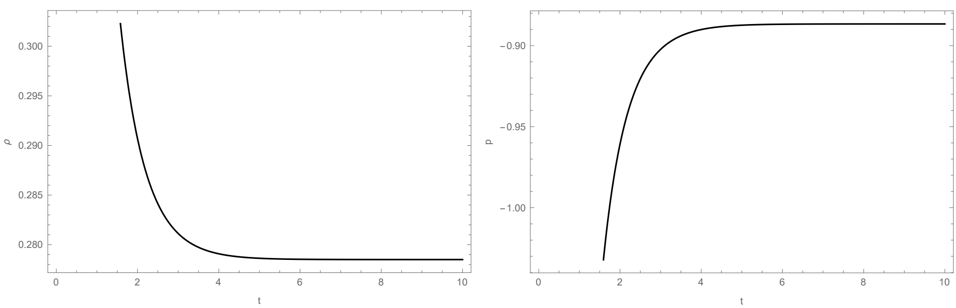

Fig. 1. Left: Plot of ρ_{WDF} *vs.* t with $A_1 = 1, \beta = 1.5, \lambda = -6$. Right: Plot of p_{WDF} *vs.* t with $A_1 = 1, \beta = 1.5, \lambda = -6$.

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