PROBING THE NATURE OF PHASES ACROSS THE PHASE TRANSITION AT FINITE ISOSPIN CHEMICAL POTENTIAL*

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We compare the low-eigenvalue spectra of the Overlap Dirac operator on two sets of configurations at $\mu_I/\mu_I^{\rm c} = 0.5$ and 1.5 generated with dynamical staggered fermions at these isospin chemical potential on $24^3 \times 6$ lattices. We find very small changes in the number of zero modes and low-lying modes which is in stark contrast with those across the corresponding finite temperature phases where one sees a drop across the phase transition. Possible consequences are discussed.

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1. Introduction

The baryon density-temperature ($\mu_{\rm B}$–T) phase diagram of quantum chromodynamics (QCD) has received a lot of attention for the past few decades, starting from skeleton diagrams on the basis of simple hadronic models, which explain the hadron spectrum reasonably well, to the increasingly quantitative attempts to pin it down *ab initio* from QCD itself using the non-perturbative lattice approach. As is well-known, one has to face the famous fermion sign(phase) problem at nonzero baryon density or equivalently nonzero baryon chemical potential, $\mu_{\rm B}$, adding an extra layer of uncertainty to the results obtained. In addition to baryon number, the up and down quarks also carry isospin. Defining μ_I as the chemical potential for I_z, and μ_u, μ_d for the up and down quarks, one has $\mu_{\rm B} = 3(\mu_u + \mu_d)/2$ and

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$\mu_I = (\mu_u - \mu_d)/2$ or, alternatively, $\mu_u = \mu_B/3 + \mu_I$ and $\mu_d = \mu_B/3 - \mu_I$. The fermion determinant is real [1, 2] for $\mu_I \neq 0$ and $\mu_B = 0$, and one thus has no sign problem in that case. From a theoretical point of view, the ability to simulate the theory enables tests of many conceptual issues related to confinement and chiral symmetry breaking in the entire μ_I–T phase diagram, as we set out to show below.

Staggered fermions are often used for such investigations due to their remnant chiral symmetry. Kogut–Sinclair [2] introduced the corresponding fermion action to investigate also whether the isospin symmetry is spontaneously broken

$$S_F = \sum_{\text{sites}} \bar{\chi} \left[\not{D}(\tau_3 \mu_I) + m + i\lambda_I \epsilon \tau_2 \right] \chi \,. \tag{1}$$

Here $\chi, \bar{\chi}$ are two component flavour spinors, τ_i are the SU(2) flavour generators, $\epsilon = (-1)^{x+y+z+t}$ is the 'γ_5' for staggered fermions, μ_I and m are isospin chemical potential and quark mass, respectively, and λ_I is a pionic source that is sent to zero at the end of the analysis. Reference [2] worked out the symmetry-breaking patterns and the corresponding observables which signal them. Further, it was argued that the fermion determinant is positive definite, enabling simulations.

Employing staggered fermions on 8^4 lattices with $a = 0.299(2)$ fm at a lattice quark mass $ma = 0.025$, corresponding to $m_\pi \simeq 260$ MeV, Endrődi [3] recently investigated the phase structure. As can be seen from his results in Fig. 1 on the chiral condensate, the pion condensate, the isospin density

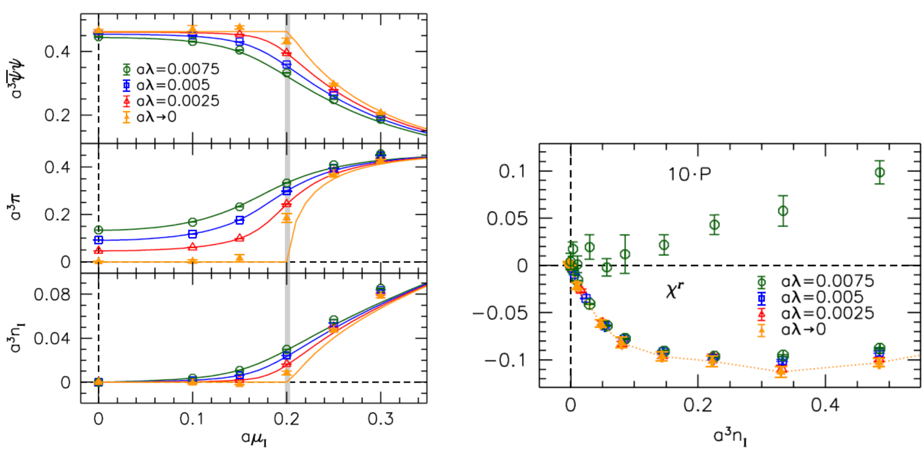

Fig. 1. Results for the chiral condensate, pion condensate, isospin density (left panel) and Polyakov line (upper right panel) on 8^4 lattice from Ref. [3] for $m_\pi \simeq 260$ MeV.

and the Polyakov line, obtained from the partition function Z defined by S_F above and the Wilson gluonic action by using $\langle\bar{\psi}\psi\rangle = \frac{T}{V}\frac{\partial \log Z}{\partial m}$, $\langle\pi\rangle = \langle\bar{\psi}_u\gamma_5\psi_d - \bar{\psi}_d\gamma_5\psi_u\rangle = T\partial\ \log Z/V\partial\lambda_I$, $\langle n_I\rangle = T\partial \log Z/V\partial\mu_I$ suggest an $a\mu_I^c \simeq 0.2$ in the $\lambda_I \to 0$ limit.

A linear $\lambda_I \to 0$ extrapolation of the data for the three λ_I values indicated is displayed by points whereas the corresponding line is a chiral theory fit. The grey vertical band denotes the value of $m_\pi/2$ in the lattice units. The chiral condensate drops rapidly around $\mu_I^c \simeq m_\pi/2$, where the pion condensate and isospin density also become nonzero as does the Polyakov loop displayed in the upper half of the right panel. These results show deconfinement to occur as well with the chiral symmetry restoration at the transition point, μ_I^c. The similarity of this phenomenon with the finite temperature transition, *i.e.* $\mu_{\mathrm{B}} = 0 = \mu_I$, prompts further investigation of the nature of this transition in terms of the established ideas, such as topological excitations, or phenomenological models, such as the well-known instanton-liquid model [4] built on instanton-fermion couplings.

Lattice QCD simulations support for the model was observed in the peak of the instanton-distribution at a radius $\rho = 0.3$ fm [5]. Note that Overlap Dirac operator, which has *exact* chiral symmetry on the lattice as well as an index theorem, was used for this analysis, by studying its low-eigenmodes spectrum. Such studies were also carried out for the high-temperature phase. Number of low eigenmodes were found to get depleted as T increased away from T_{c} [6,7]. Furthermore, a gap appeared to separate the low modes from the others. Localized zero modes were observed [7] for $1.25 \leq T/T_{\mathrm{c}} \leq 2$, suggesting the axial symmetry group $\mathrm{U_A}(1)$ to be restored only gradually up to $2T_{\mathrm{c}}$. Indeed, the scalar and pseudoscalar meson correlators were equal, as expected in a chiral symmetry restored phase, only after the contribution of these zero modes was subtracted out from the former. Clearly, a similar investigation will be interesting for the nonzero chemical potential case as well, in view of the both the naïve model expectations and the results in Fig. 1 for $\mu_I \neq 0$.

2. Our results

Employing dynamical configurations on $24^3 \times 6$ lattices, generated with a Symanzik improved action with 2 stout steps and for a quark mass tuned to have the physical pion mass, we investigated the eigenvalue spectra of the Overlap Dirac operator both below and above the isospin breaking phase transition at $a\mu_I^c = 0.1$, which again corresponds to μ_I^c being $m_\pi/2$. We employed the Arnoldi method to extract the eigenvalues of Overlap Dirac operator, demanding a residue $r = ||DX - \eta|| \leq 10^{-10}$. It may be noted that the dynamical configurations are with a nonzero $\mu_I = 0.05$ and 0.15, but there is no explicit μ_I in the operator itself, since our intention is to study the topological fluctuations. We extracted ~ 500 eigenvalues from each configuration. At both the μ_I values, computations were done for two different values of λ_I — the isospin breaking parameter in the quark matrix.

Since the eigenvalue λ is complex for $D_{\rm ov}$, we display in Fig. 2 the $|\lambda|$-distributions for $\lambda_I = 0.0006$ both below and above the transition. Fairly uniform distribution with some low modes are seen in both the cases. Surprisingly, the distributions are very similar as well, and by overlaying

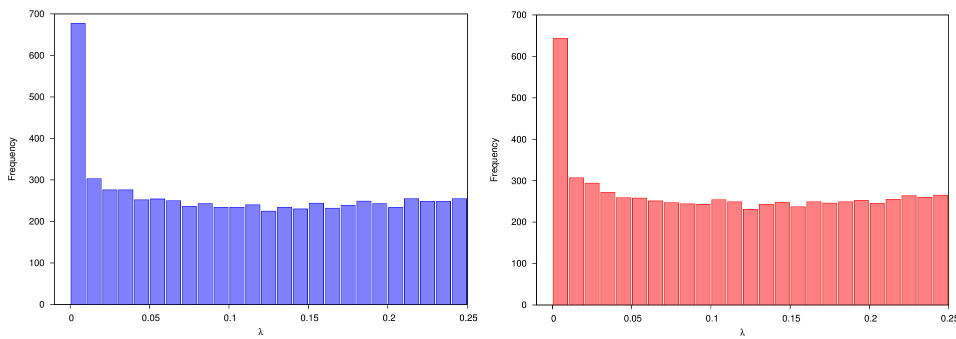

Fig. 2. Eigenvalue spectrum of the Overlap Dirac operator on $24^3 \times 6$ lattice for $\lambda_I = 0.0006$ and $\mu_I/\mu_I^c = 0.5$ (left panel) and 1.5 (right panel).

them, one finds them almost indistinguishable with minor quantitative differences. Re-plotting them on a log scale, one can easily identify the zero modes from the gap in the spectrum. Explicit chirality checks were made to confirm their nature. Zooming in on the eigenvalue distribution on the log scale, one can see if the near-zero modes have any visible differences. While a nice smooth rise is seen in Fig. 3, as one moves away from the zero eigenvalue, the similarity in the distributions for $\mu_I/\mu_I^c = 0.5$ and 1.5 per-

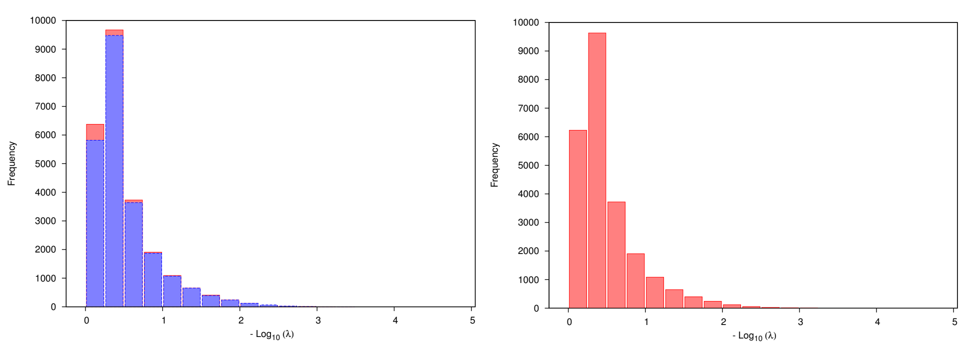

Fig. 3. Near-zero eigenvalue spectrum of the Overlap Dirac operator on $24^3 \times 6$ lattice for $\lambda_I = 0.0025$ for both the μ_I values (left panel) and for $\lambda_I = 0.0006$ with $\mu_I/\mu_I^c = 1.5$ (right panel).

sists for even higher $\lambda_I = 0.0025$, where one observes qualitatively the same picture as discussed above. The displayed overlay of near-zero modes for $\mu_I/\mu_I^{\rm c} = 0.5$ and 1.5 for $\lambda_I = 0.0025$ in the left-hand panel also compares well with that of $\mu_I/\mu_I^{\rm c} = 1.5$, $\lambda_I = 0.0006$ in the right-hand panel.

The exact chiral symmetry of the overlap fermions implies that nonzero modes are doubly degenerate with opposite chirality, while the zero modes possess only a specific chirality. The latter act as a measure of topology due to the index theorem the overlap fermions satisfy. Table I lists the number of zero modes we observed in a sample of 50 independent gauge configurations as a function of μ_I and λ_I. The last two columns list the corresponding results of Ref. [7] which are also on samples of 50 configurations but as a function of temperature in the vicinity of the finite temperature transition at $\mu = 0$. While a steep fall off is seen in the latter as a function of $T/T_{\rm c}$, almost no variation is observed across μ_I for $\lambda_I = 0.0006$ and a mild one for $\lambda_I = 0.0025$, ∼25% reduction.

TABLE I

Number of zero modes N^λ as a function of μ_I and λ_I along with corresponding results for finite temperature from Ref. [7].

μ_I/μ_I^c	$N_{\rm zero}^{0.0006}$	$N_{\rm zero}^{0.0025}$	$T/T_{\rm c}$	$N_{\rm zero}$
0.5	426	416	1.25	18
1.5	451	310	1.5	8
—	—	—	2.0	1

3. Summary

We investigated the eigenvalue distribution for chirally exact Overlap Dirac operator for $\mu_I/\mu_I^c = 0.5$ and 1.5, *i.e.*, below and above the isospin phase transition, which is indicated [3] to be similar to the finite temperature transition in having both chiral symmetry restoration and a rise of the Polyakov loop at the transition point. The distribution of zero and near-zero modes is nearly the same for both at $\lambda_I = 0.0006$, with a 25 % reduction in former at $\lambda_I = 0.0025$.

This should be contrasted with the earlier $T \neq 0$ results [6, 7], where these modes were also present above the transition but decreased sharply as one moved away from the transition. Further quantitative investigations in pinning down the changes in these modes may help in efforts to understand the difference in T and μ_I directions, if there are any.

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