

FROM COLD FERMI FLUIDS TO THE HOT QGP*

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Strongly coupled quantum fluids are found in different forms, including ultracold Fermi gases or tiny droplets of extremely hot Quark–Gluon Plasma. Although the systems differ in temperature by many orders of magnitude, they exhibit a similar almost inviscid fluid dynamical behavior. In this work, we summarize some of the recent theoretical developments toward better understanding of this property in cold Fermi gases at and near unitarity.

DOI:10.5506/APhysPolBSupp.10.481

1. Introduction

Strongly coupled quantum fluids are studied with high interest in recent years [1, 2]. Despite apparent differences in temperature T , density n , or underlying microscopic theory, various quantum fluids share interesting features. For example, similar nearly inviscid flow was observed in both cold Fermi gases [3] and the Quark–Gluon Plasma (QGP) [4]. Based on data analyses, the shear viscosity η to entropy density s ratios were concluded to be as small as a few times the conjectured holographic KSS-limit [5].

The QGP as color-deconfined state of hot and dense QCD matter is transiently formed in the expansion stage of high-energy heavy-ion collisions. Its properties can only indirectly be inferred from experimental data as only color-confined hadrons are measured in the detectors. In early attempts of describing the expanding matter with relativistic fluid dynamics [4], evolution-averaged values for the QGP shear viscosity were extracted. Recent efforts [6, 7] aim instead at determining η locally in dependence of T and n .

* Presented by M. Bluhm at the “Critical Point and Onset of Deconfinement” Conference, Wrocław, Poland, May 30–June 4, 2016.

In cold Fermi gases, one can control the dominant s -wave interaction among atoms by altering an external magnetic field. When the interaction range becomes large compared to the interparticle spacing, the system is tuned into a Feshbach resonant state and the scattering cross section is bound only by unitarity. In this limit, the system is scale and conformally invariant and the matter properties are universal functions of T and n [8]. At low T , this matter forms a superfluid. On different sides of the resonance, a Bose–Einstein condensate (BEC) of strongly bound molecules or a BCS-type atomic superfluid is realized with a smooth crossover in between [1]. With increasing T , the matter undergoes a phase transition to a dense normal fluid whose collective behavior is correctly described by non-relativistic fluid dynamics [9]. In the dilute gas limit, kinetic theory becomes applicable even in the unitarity limit.

One way to extract η in cold Fermi gases is to analyze expansion measurements of gas clouds [10–12]. In the experiments, the gas is cooled and trapped in a harmonic optical potential, which determines the initial properties of the cloud, and atomic interactions are controlled magnetically. After release from the trapping potential, the gas expands and its time-evolution can directly be observed. However, most of the previous analyses concentrated on deducing trap-averaged values for η and only recently a local determination became possible.

2. Anisotropic fluid dynamics for cold Fermi gases

The natural approach for studying the expansion dynamics of the dense gas cloud is the Navier–Stokes fluid dynamics. However in the dilute corona, where scatterings are rare and the system expands ballistically, the theory becomes inapplicable and leads to unphysical results. To overcome this situation, one needs an approach which is capable of describing locally the transition from fluid dynamical to free streaming behavior. Considering the full Boltzmann equation [13] is one possibility, as free streaming is contained in the theory and the fluid dynamical limit is reproduced in the dense regime. Determining locally the temperature and density dependence of the shear viscosity within this approach is, nonetheless, a difficult task.

Anisotropic fluid dynamics represents an alternative method. In contrast to the Navier–Stokes fluid dynamics, this theory can be derived from moments of the Boltzmann equation connected to an anisotropic distribution function. The form of this function is motivated by exact solutions of the kinetic theory in the free streaming limit for a harmonic trap potential [14–16]. Assuming that the symmetry axes of the potential are aligned with the Cartesian coordinates a simplifies the set of evolution equations, see [17] for details. In Lagrangian form, the continuity equation reads

$$\left(\partial_t + \vec{u} \cdot \vec{\nabla}\right) \rho = -\rho \vec{\nabla} \cdot \vec{u} \quad (1)$$

and the conservation equations for momentum and energy read

$$\left(\partial_t + \vec{u} \cdot \vec{\nabla}\right) u_i = -\frac{1}{\rho} \left(\nabla_i P + \nabla^j \delta \Pi_{ij}\right), \quad (2)$$

$$\left(\partial_t + \vec{u} \cdot \vec{\nabla}\right) \epsilon = -\frac{1}{\rho} \nabla^i \left(u_i P + u^j \delta \Pi_{ij}\right). \quad (3)$$

Here, $\rho = mn$ is the mass density, \vec{u} the fluid velocity, P the pressure and $\epsilon = \mathcal{E}/\rho$ the energy per mass. For a scale invariant Fermi gas at unitarity, pressure and energy density in the fluid rest frame are related via $P = \frac{2}{3}\mathcal{E}_0$ with $\mathcal{E}_0 = \mathcal{E} - \frac{1}{2}\rho\vec{u}^2$. The dissipative stress tensor $\delta \Pi_{ij}$ has a simple diagonal form, $\delta \Pi_{ij} = \sum_a \delta_{ia} \delta_{ja} \Delta P_a$, where $\Delta P_a = P_a - P$ for the anisotropic pressure components P_a . The latter are related to the anisotropic energy density components \mathcal{E}_a via $P_a = 2\mathcal{E}_a^0$, where $\mathcal{E}_a^0 = \mathcal{E}_a - \frac{1}{2}\rho u_a^2$ and $\mathcal{E} = \sum_a \mathcal{E}_a$.

In the anisotropic fluid dynamics, one treats these components as additional fluid dynamical variables obeying for given a the evolution equation

$$\left(\partial_t + \vec{u} \cdot \vec{\nabla}\right) \epsilon_a = -\frac{1}{\rho} \nabla_a [u_a P + u_a \Delta P_a] - \frac{P}{2\eta\rho} \Delta P_a. \quad (4)$$

By summing over a , the equation of energy conservation, Eq. (3), is recovered, *i.e.* only two of the three components $\mathcal{E}_a = \rho\epsilon_a$ are independent. Moreover, Eq. (4) represents a relaxation equation for the dissipative stresses ΔP_a : for small η/P , one finds $\Delta P_a = -\eta\sigma_{aa} + \mathcal{O}((\eta/P)^2)$ and anisotropic fluid dynamics reduces to the Navier–Stokes theory. For large η/P , instead, Eq. (4) becomes a conservation equation for the individual components \mathcal{E}_a . Anisotropic fluid dynamics provides, thus, a theoretical framework which, irrespective of the functional form of η , smoothly combines viscous fluid dynamical behavior in dense regions with ballistic expansion in the corona.

Relativistic anisotropic fluid dynamics was developed to achieve a more reliable description of the dynamics in a heavy-ion collision [18–20]. At early times, during the pre-equilibrium evolution, large momentum-space anisotropies build up. These manifest in sizeable differences between the longitudinal and transverse pressure components and in large viscous corrections which become even larger with increasing η/s and in the low- T regions of the fireball. Anisotropic fluid dynamics is able to handle such corrections, thus, extending the regime of applicability of fluid dynamics to situations far from isotropic thermal equilibrium.

3. Determination of η from high- T expansion data

The evolution equations of anisotropic fluid dynamics can be solved with the standard numerical techniques. Based on the PPMLR scheme of Colella and Woodward [21] implemented in [22, 23], we developed a code [17] that solves Eqs. (1)–(4) in the unitarity limit for a given initial state and functional dependence of η . The initial state is isothermal. The density profile is sensitive to the exact dependence of P on T and n , and determined by the hydrostatic equilibrium equation $\vec{\nabla}P = -n\vec{\nabla}V$ for trap potential V . In the high- T limit, we have $P = nT$ and the initial density has a simple Gaussian shape. After release from the trap, the system is governed by the evolution equations which are sensitive to $P(\mathcal{E}^0)$ only. To ensure stability of the relaxation equation (4), the simulation time-step has to be smaller than the minimal η/P in the gas cloud. In [24], we showed that the code is capable of perfectly reproducing the numerical results [13] for solving the Boltzmann equation in the dilute gas limit.

Figure 1 shows, as an example, the simulation result for the time-evolution of the two-dimensional column density $n_{2d}(x, z)$ in the case of a purely temperature-dependent shear viscosity. Initially, the gas cloud is an ellipsoid elongated in the z -direction. Pressure gradients accelerate the system preferably in the transverse plane (x – y -direction), but viscosity counteracts this acceleration and slows it down considerably for large η . The transition region from fluid dynamical behavior to free streaming is density-dependent and shifts during the evolution. In contrast to the Navier–Stokes theory, however, anisotropic fluid dynamics does not break down in the dilute regions for a density-independent η .

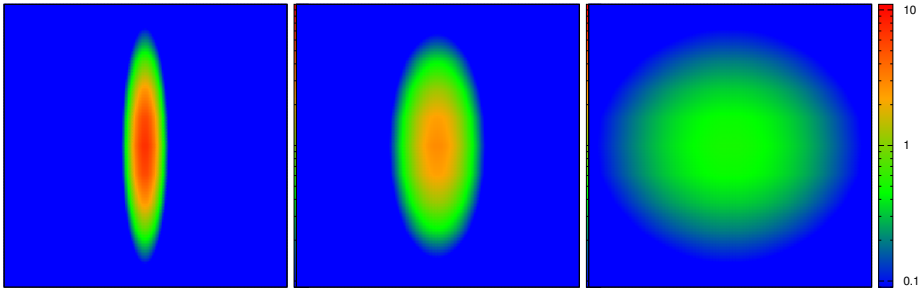


Fig. 1. Time-evolution of the column density $n_{2d}(x, z) = \int n(x, y, z) dy$ in the x – z -plane. The coordinates and n_{2d} are shown in arbitrary units, cf. [17, 23] for details. Snapshots are taken at times $t/t_0 = 0, 2, 8$ in units of the inverse geometric mean of the trap frequencies t_0 .

The framework can be confronted with the measured expansion data to deduce the dependence of η on T and n . This was done in [24]. In [24], we analyzed the expansion data of Cao *et al.* [10] for temperatures in the range of $0.79 \leq T/T_F \leq 1.54$ with Fermi temperature T_F . This is far above the superfluid transition, $T_c \simeq 0.23 T_F$, and the initial density profile can be approximated by a Gaussian. Scale invariance in the Fermi gas at unitarity implies that η is given by a universal function of T and n , $\eta = (mT)^{3/2} f(n/(mT)^{3/2})$. In [24], we verified this form with the Ansatz $\eta = \eta_0 (mT)^{3/2} (mT/n^{2/3})^c$ and found that the expansion data is best described with an exponent c consistent with zero. This implies that in the high- T limit, η depends only on temperature as predicted by the kinetic theory, where the best fit was obtained for $\eta = 0.282(mT)^{3/2}$ which agrees impressively with the kinetic theory expectation $\eta_{CE} \simeq 0.269(mT)^{3/2}$ [25–27].

4. Shear viscosity away from the unitarity limit

In the unitarity limit, the bulk viscosity ζ vanishes. For finite s-wave scattering length a , however, scale invariance is lost and $\zeta \neq 0$. In [28], this behavior was studied within kinetic theory for the dilute Fermi gas. It was found that ζ depends quadratically on the conformal symmetry breaking parameter, $z\lambda/a$, where z is the fugacity and λ the thermal wavelength of the fermions. In the high- T limit, where fermionic quasiparticles are well-defined, this can be understood as a medium effect due to a density-dependent fermion self-energy. To leading order in a , the ratio of bulk to shear viscosity reads $\zeta/\eta \sim (z\lambda/a)^2$. For the hot QGP, a comparable dependence of the viscosity ratio on the QCD measure for conformal symmetry breaking is found. In QCD, a non-zero bulk viscosity similarly arises from the scale breaking part in effective quark and gluon masses [29].

The scattering length dependence of the shear viscosity near unitarity was recently investigated in the expansion measurements by Elliott *et al.* [11]. For low T , an anomalous shift of the minimal shear viscosity away from unitarity toward positive $1/a$ is observed, which disappears with increasing T . Qualitatively, this behavior can again be understood as a medium effect due to modifications of the fermionic dispersion relation and the collision integral in the Boltzmann equation. As the dominant effect, Pauli-blocking in the scattering amplitude can be identified. A systematic fugacity expansion in the dilute regime yields near unitarity [30]

$$\eta = \eta_\infty \left(1 + c_0 \left(\frac{\lambda}{a} \right)^2 + c_1 \left(\frac{z\lambda}{a} \right) + \dots \right) \quad (5)$$

with $c_0 = 1/(4\pi)$, $c_1 \simeq -0.03325$ and η_∞ the shear viscosity at unitarity. At leading order, η is independent of z and even in a , and the minimum

is reached at unitarity, while the next-to-leading order correction becomes more important with decreasing T , shifting the minimum toward $a > 0$, *i.e.* to the BEC-side of the Feshbach resonance. This implies that Pauli-blocking is more efficient on the BCS-superfluid side. Away from the unitarity limit, the ratio ζ/η shows an interesting dependence on the conformal symmetry breaking parameter which can be compared to the behavior found for the QGP near the confinement transition [31].

5. Conclusions

Cold Fermi gases can form strongly coupled quantum fluids under experimentally controllable conditions. This provides a unique opportunity to study certain properties of other strongly coupled quantum systems like the QGP. Almost perfect fluidity, for example, is also seen in expansion measurements of cold Fermi gas clouds at and near unitarity. Anisotropic fluid dynamics provides a realistic framework that allows us to reliably extract from these data the temperature and density dependence of the shear viscosity. Advances in kinetic theory showed, furthermore, that the observed anomalous shift of the minimal shear viscosity near unitarity can be understood qualitatively from microscopic calculations for the dilute gas limit.

This work is supported in part by the U.S. Department of Energy under grant number DE-FG02-03ER41260. The work of M. Bluhm is funded by the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 665778 via the National Science Centre, Poland (NCN) under grant Polonez UMO-2016/21/P/ST2/04035.

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