ACROSS THE DECONFINEMENT*

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The deconfinement transition at vanishing chemical potential can be reliably studied by lattice simulations and its general features are by now well-known. On the contrary, what happens at finite density is still largely unknown and we will review the results obtained in the last year regarding the dependence, for small density, of the (pseudo)critical temperature on the baryonic chemical potential.

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1. Introduction

One of the goals of Lattice Quantum ChromoDynamics (LQCD) is to provide a first principles description of the QCD phase diagram and of the nonperturbative behaviour of the strongly interacting thermal medium. Beyond their purely theoretical interest, such informations are valuable for a better understanding of several phenomenologically relevant processes, among which heavy-ions ultrarelativistic collisions play a dominant role.

The study of the thermodynamical properties of QCD at the vanishing baryon density by means of LQCD numerical simulations is a standard (although not computationally easy) task. When a nonzero baryon density is present, this is no more true: the usual importance sampling Monte Carlo methods used in LQCD simulations cannot be used any more, due to the well-known sign problem.

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The only physically relevant case in which the sign problem can be circumvented is the case of small baryon density or, more precisely, small baryon chemical potential $\mu_{\rm B}$: if we are interested in the observable $A(\mu_{\rm B})$, we can expand it as $A(\mu_{\rm B}) \simeq A_0 + A_1\mu_{\rm B} + A_2\mu_{\rm B}^2 + \ldots$ and try to estimate the coefficients of the series. Which values of $\mu_{\rm B}$ are "small enough" for this procedure to give reliable results is something that can only be determined *a posteriori*, given the accuracy that one can obtain in the evaluation of the expansion coefficients.

This general idea can be implemented in two different ways: in the approach known as the Taylor expansion method, the coefficient A_n is obtained by evaluating $\partial^n_{\mu_{\rm B}} A(\mu_{\rm B})|_{\mu_{\rm B}=0}$ using standard $\mu_{\rm B} = 0$ simulations [1]. In the analytic continuation approach, simulations performed at imaginary values of the chemical potential $\mu_{\rm B} = i\mu_{B,I}$ (at which no sign problem is present) are performed, and the coefficients A_n are then extracted by fitting the results [2, 3]. The two approaches have complementary advantages and drawbacks: in the Taylor expansion method, the coefficients A_n are directly obtained, but their estimators become more and more noisy as n is increased; in the analytic continuation method, the A_n values have to be fitted, but one can use observables with good signal-to-noise ratios.

A property of the QCD phase diagram that can be investigated by using LQCD simulations is the dependence, at least for small values of $\mu_{\rm B}$, of the (pseudo)critical temperature on the baryon chemical potential: $T_{\rm c}(\mu_{\rm B})$ can be developed in even powers of $\mu_{\rm B}$, and the curvature κ of the critical line is defined by

$$T_{\rm c}(\mu_{\rm B})/T_{\rm c} = 1 - \kappa\,(\mu_{\rm B}/T_{\rm c})^2 + O\left(\mu_{\rm B}^4/T_{\rm c}^4\right)\,, \tag{1}$$

where $T_{\rm c}$ denotes the (pseudo)critical temperature at $\mu_{\rm B} = 0$. It is interesting to compare the curvature of the QCD (pseudo)critical line with the curvature of the freeze-out curve extracted from heavy-ion collisions, see *e.g.* [4, 5]. While there is no compelling theoretical reason for the critical and the freeze-out lines to coincide (in fact, the first is an equilibrium property of QCD, while the second depends also on out-of-equilibrium properties of the strongly interacting medium), a precise quantitative comparison of these observables could help in better understanding the physical processes involved in the cooling of the quark–gluon plasma.

2. Numerical results

At vanishing baryon chemical potential, a real phase transition is not present, but just an analytic crossover; as a consequence, it is important to specify the observable used in the determination of $T_{\rm c}(\mu_{\rm B})$, since different observables can, in principle, lead to different results. All LQCD results that

will be discussed in the following refer to observables related to the restoration of the chiral symmetry (chiral condensate or chiral susceptibility), see Fig. 1 for a graphical summary of the results.

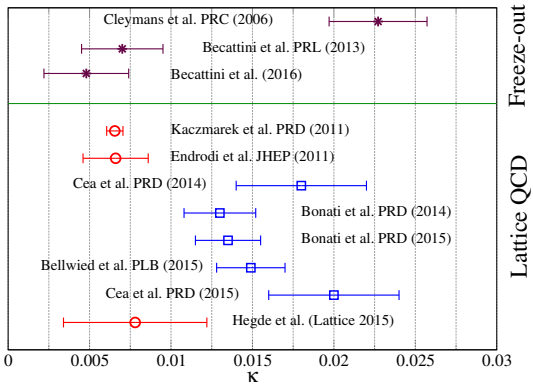

Fig. 1. (Colour on-line) Recent determinations of the curvature of the freeze-out curve (from [4, 5]) and of the QCD (pseudo)critical line (from [6–11]). Circles (red) denote data obtained by the Taylor expansion method, while squares (blue) correspond to data obtained by the analytic continuation method.

Given the great phenomenological significance of the curvature of the QCD (pseudo)critical line, after the seminal works [6, 7] several LQCD groups recently got involved in new computations of κ, with the principal aim of improving the control of the systematics. The analytic continuation method is attractive in this respect, since it enables to compute $T_{\rm c}(\mu_{\rm B})$ using the same standard procedures that are used in the $\mu_{\rm B} = 0$ case: looking for the inflection point of the chiral condensate or the maximum of the chiral susceptibility $\chi_{\bar{\psi}\psi}$ (see [9] for a discussion of the effect on κ of different renormalization procedures). A new source of systematics is in this case the fitting procedure, that however can be kept well under control by checking, *e.g.*, the dependence of the results on the fit range used, see Fig. 2.

Another improvement with respect to the first determinations is a better understanding of the role played by the quark chemical potentials: for the results to be of direct physical relevance in heavy-ions collisions, the chemical potentials have to be tuned in such a way that the total strangeness vanishes and the electric charge is related to the baryon number by $\langle Q \rangle = r \langle B \rangle$, with $r \simeq 0.4$. The first studies [6, 7] adopted the chemical potential setup $\mu_u = \mu_d = \mu_{\rm B}/3$, $\mu_s = 0$ and in [8], the setup $\mu_u = \mu_d = \mu_s = \mu_{\rm B}/3$ was used, while strangeness neutrality (near the transition) implies $\mu_s \approx \mu_u/4$ (see [12]). In [9], it was shown that the value of κ is insensitive (within the numerical accuracy) to the value of μ_s, while higher orders in the development of $T_{\rm c}(\mu_{\rm B})$ depends on μ_s (see Fig. 2), a fact that can explain the

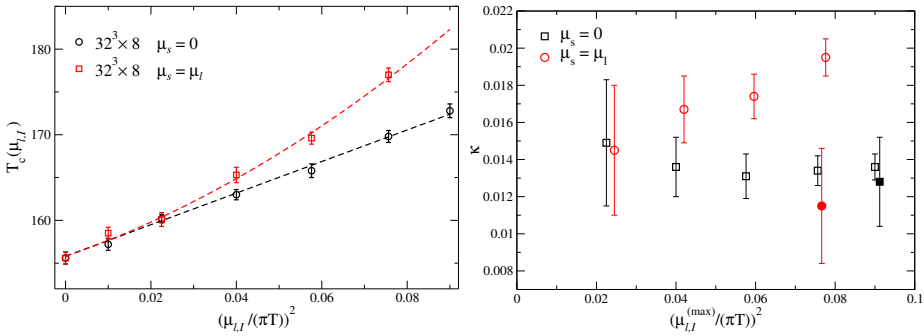

Fig. 2. Left: Dependence of the critical temperature on the (imaginary) light chemical potential for the two setups $\mu_s = \mu_l$ and $\mu_s = 0$. Right: Results of the fit to extract κ for different fit ranges; empty symbols denote the purely quadratic fit, while also the quartic correction is used for the filled symbols (from [9]).

slightly larger values of κ obtained in [8] and is likely related to the so-called Roberge–Weiss transition, taking place at $\mu_u = \mu_d = \mu_s = i\pi T/3$ [13]. A further confirmation of the μ_s-independence of κ is given by the results of [10]: a value of κ in a very good agreement with the ones of [9] is obtained by performing simulations directly at the strangeness neutrality. The effect on κ of the isospin breaking constraint $\langle Q\rangle = 0.4\langle B\rangle$ was also shown in [10] to be negligible with the present accuracy.

As a byproduct of the determination of the curvature κ, the continuum extrapolated chiral susceptibility was obtained in [9] for several values of the imaginary chemical potential, see Fig. 3 (left). The dependence on the chemical potential of $\chi_{\bar\psi\psi}$ can give some hints on the location of a critical

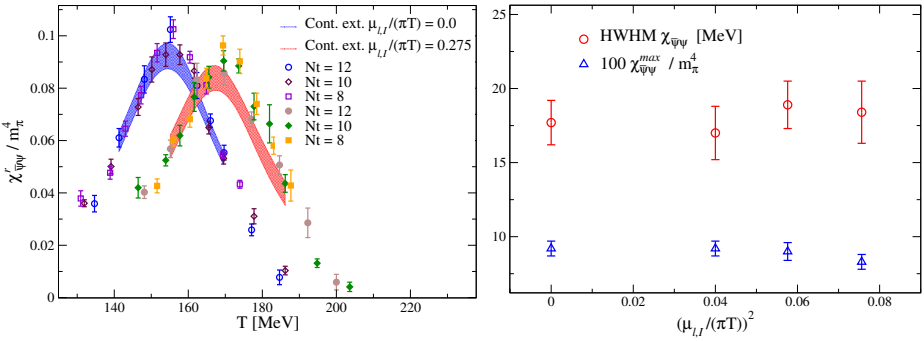

Fig. 3. Left: Continuum extrapolated chiral susceptibility for two values of the light chemical potential. Right: Dependence of the half-width at half-maximum (HWHM) of $\chi_{\bar\psi\psi}$ and of the maximum value of $\chi_{\bar\psi\psi}$ on the light chemical potential.

endpoint: a critical endpoint for $\mu_B > 0$ would suggest the maximum of $\chi_{\bar{\psi}\psi}$ to decrease and its half-width at half-maximum (HWHM) to increase as the imaginary chemical potential is increased. None of these behaviours was however observed in the numerical data, see Fig. 3 (right). While this is obviously not incompatible with the existence of a critical endpoint at $\mu_B > 0$, it is an indication that (if it exists) it cannot be too close to the real axis.

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