HOW TO USE LATTICE AND EXPERIMENTAL DATA FOR QCD CRITICAL POINT SEARCH*

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We show that the canonical approach is a promising tool to find the critical point of the QCD phase both in the experimental study and the lattice QCD simulations.

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1. Introduction — sign problem in the finite density QCD

One of the important objectives of the workshop "Critical Point and Onset of Deconfinement" is to clarify the QCD phase structure at finite temperature, T, and finite baryon number density, μ, from experiments and lattice simulations.

The lattice simulation at finite baryon density is a very tough issue because of the sign problem: The fermion determinant $\det \Delta(\mu)$ in the partition function, Z, is complex if the baryon chemical potential μ is real, where Z has the form of

$$Z(\mu, T) = \int \mathcal{D}\mathrm{U} \left(\det \Delta(\mu)\right)^{N_f} e^{-S_G} \,. \tag{1}$$

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In the Monte Carlo simulations, configurations are updated with the probability $P \propto (\det \Delta(\mu))^{N_f} e^{-S_G}$. Therefore, for the real chemical potential, the probability is a complex number and the simulation becomes impossible.

2. Canonical approach

A key formula of our study is

$$Z(\mu, T) = \sum_n Z_n(T)\xi^n \quad \text{with} \quad \xi \equiv \exp(\mu/T)\,. \tag{2}$$

The formula is neither an assumption nor a model. We can obtain Eq. (2) as follows:

$$Z(\xi, T) = \operatorname{Tr} e^{-\left(H-\mu\hat{N}\right)/T} = \sum_{n=-N_{\max}}^{+N_{\max}} \langle n|e^{-H/T}|n\rangle e^{\mu n/T} = \sum_{n=-N_{\max}}^{+N_{\max}} Z_n(T)\xi^n\,, \tag{3}$$

where

$$Z_n = \langle n|\exp(-H/T)|n\rangle\,. \tag{4}$$

Here, we assume that the number operator $\hat{N}$ commutes with H, that is, $\hat{N}$ is a conserved quantity. Z and Z_n are the grand canonical partition function and the canonical partition function, respectively. Both are a function of the system volume.

Note that Z_n do *not* depend on μ, therefore, formula (2) is valid for any μ, *i.e.*, not only the real number, but also the pure imaginary and the complex numbers.

3. Results

3.1. Experimental data

Suppose a system is described by the grand partition function, $Z(\mu, T)$, of the temperature T and chemical potential μ. Then the formula

$$Z(\mu, T) = \sum_n Z_n(T)\xi^n \tag{5}$$

means that the probability to find a net multiplicity n is proportional to $Z_n(T)\xi^n$, *i.e.*,

$$P_n = Z_n\xi^n/Z\,, \qquad P_{-n} = Z_{-n}\xi^{-n}/Z\,. \tag{6}$$

From the CP invariance, we can impose $Z_n = Z_{-n}$. Then

$$\sqrt{P_n P_{-n}} = Z_n/Z\,. \tag{7}$$

Therefore, from the experimental net-multiplicity data, we can construct Z_n.

Once we have Z_n from formula (5), we can calculate $Z(\mu, T)$ at *any* μ. Further, the n^{th} moments, $\lambda^n = (T\partial/\partial\mu)^n \log Z = (\xi\partial/\partial\xi)^n \log Z$ are easily estimated.

In references [1] and [2], the net-proton multiplicities were reported for searching the critical point. The proton number is not a conservative quantity, but we consider it as a proxy of the net-baryon multiplicity.

Once we have Z_n, it is straightforward to calculate the moments λ_k. In Fig. 1, we show the ratio of two moments, λ_4 and λ_2, which is related to kurtosis, for 19.6 and 200 GeV. Details of the analysis are described in Ref. [3] where the method to estimate error bars is explained.

Freeze-out points in Fig. 1 stand for a point where the fire-ball is created in (μ, T) plane. The canonical partition functions, $Z_n(T)$, are determined from the experimental data on this point and we can predict $Z(\mu/T, T)$ at different μ/T values with the same T.

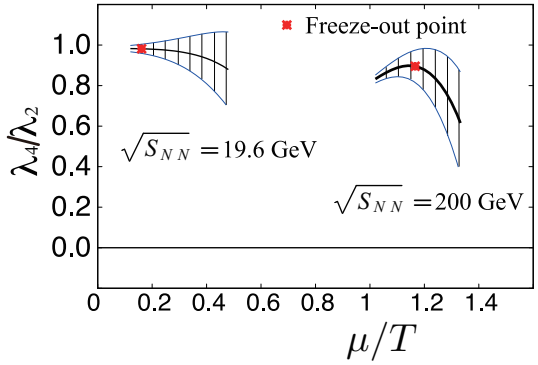

Fig. 1. The ratio of the moments λ_4 and λ_2, which corresponds to $\kappa\sigma^2/T^2$ as a function of μ/T for $\sqrt{s_{NN}} = 19.6$ GeV and 200 GeV. Here, κ and σ^2 are the kurtosis and the variance, respectively. They are calculated from Z_n which is constructed from the net-proton multiplicities at RHIC [1] and [2].

3.2. Lattice QCD analysis

In the lattice QCD simulation, there are three methods to calculate the canonical partition functions, Z_n:

— The fermion determinant can be expanded as the fugacity polynomial, $\det \Delta(\mu) = \sum_n c_n \xi^n$. We insert this into Eq. (1), and obtain formula (2). We can reduce the CPU time and the memory by using a so-called reduction formula [4].

— When the chemical potential is pure imaginary, $\mu = i\mu_I$, the fermion determinant is real, and the canonical partition functions are obtained by the following formula [5]:

$$Z_n = \int_{-\pi}^{+\pi} \frac{\mathrm{d}\theta}{2\pi} e^{in\theta} Z\left(\theta \equiv \frac{\mu_I}{T}\right) . \tag{8}$$

— Wilson fermion has the form of $\Delta = I - \kappa Q$ and allows the hopping parameter expansion

$$\det(I - \kappa Q(\mu)) = \exp \mathrm{Tr} \log(I - \kappa Q) = \exp\left(- \mathrm{Tr} \sum_n \frac{\kappa^n}{n} Q^n \right) .$$

From this expression, we can construct Eq. (2). See "Algorithm 1 Winding Numbers via Hopping Parameter Expansion" in Ref. [6].

Here, we use the first method to calculate Z_n shown in Fig. 2 for $T/T_\mathrm{c} = 1.04, 1.01$ and 0.99. The lattice size is $8^3 \times 4$, and we use the Iwasaki improved gauge action and the clover Wilson fermions. In Fig. 3, we show λ_4/λ_2, constructed from Z_n in Fig. 2.

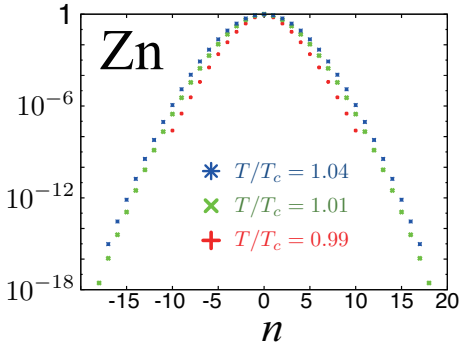

Fig. 2. Canonical partition functions, Z_n, obtained by the lattice QCD simulations for $T/T_\mathrm{c} = 1.04, 1.01$ and 0.99. ($\beta = 1.89, 1.87$ and 1.85, respectively.)

Above T_c, λ_4/λ_2 is almost monotonous as μ/T increases, since we do not hit the transition line there. On the contrary, below T_c it decreases, which may suggest we are reaching the transition line.

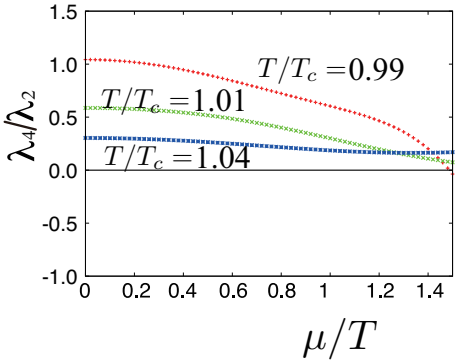

Fig. 3. The ratio of the moments λ_4 and λ_2 as a function of μ/T for the lattice QCD simulations for $T/T_c = 1.04, 1.01$ and 0.99.

4. Concluding remarks

In this report, we show that the canonical partition functions, Z_n, can be calculated both from experimental net multiplicity and by the lattice QCD simulations. This is, we hope, an important step towards searching the QCD phase diagram and the critical point, but we still have to complete several steps to go further:

— It is interesting to apply this method to the net-charge multiplicity. We can compare experimental results and the lattice calculation.

— We will study the second (pure imaginary chemical potential) and third (hopping parameter expansion) methods for calculating Z_n, and see if we can get more reliable Z_n for large n.

— The lattice calculation here is still on a small lattice with heavy quarks. More realistic simulations will provide us with data that can be compared with experiments.

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