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# ELASTIC $qq$ CROSS SECTIONS AT FINITE CHEMICAL POTENTIAL IN THE NAMBU–JONA-LASINIO LAGRANGIAN*

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We discuss the elastic $qq$ cross sections at finite chemical potential in the Nambu–Jona-Lasinio model. We comment the generic features of the cross sections as functions of the chemical potential, temperature and collision energy. Finally, we discuss their relevance in the construction of a relativistic transport model for heavy-ion collisions based on this effective Lagrangian.

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## 1. Introduction

One of the most widely used low-energy realizations of quantum chromodynamics (QCD) for the quark degrees of freedom is the Nambu–Jona-Lasinio (NJL) model [1–5]. In this effective model, one considers only dynamical quarks which exchange small momenta in their interaction, whereas gluons are integrated out. This model shares the global symmetries of QCD, and it is able to describe the phenomenon of spontaneous symmetry breaking and its high-temperature/density restoration. In particular, the local version of the model (where quarks interact locally in space through a four-point interaction) has the property of simplicity and transparency, as opposed

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to other more complicated approximations to QCD. In addition, the NJL model has a straightforward extension to finite temperatures and densities, allowing for the study of different areas of the QCD phase diagram.

We consider the Lagrangian of the NJL model with (color neutral) pseudoscalar and scalar interactions (neglecting the vector and axial-vector vertices for simplicity) [6]

$$\begin{aligned}\mathcal{L}_{\mathrm{NJL}} &= \sum_i \bar{\psi}_i (i\partial\!\!\!/ - m_{0i} + \mu_i \gamma_0)\psi_i \\ &+G\sum_a \sum_{ijkl} \left[\left(\bar{\psi}_i \; i\gamma_5 \tau^a_{ij}\psi_j\right) \; \left(\bar{\psi}_k \; i\gamma_5 \tau^a_{kl}\psi_l\right) + \left(\bar{\psi}_i \tau^a_{ij}\psi_j\right) \; \left(\bar{\psi}_k \tau^a_{kl}\psi_l\right)\right] \\ &-K \det_{ij}\left[\bar{\psi}_i \; (\mathbb{I}-\gamma_5)\psi_j\right] - K\det_{ij}\left[\bar{\psi}_i \; (\mathbb{I}+\gamma_5)\psi_j\right] , \end{aligned} \tag{1}$$

where the flavor indices $i, j, k, l = 1, 2, 3$ and $\tau^a$ $(a = 1, \ldots, 8)$ are the $N_{\mathrm{f}} = 3$ flavor generators with normalization

$$\mathrm{tr}_{\mathrm{f}} \; \left(\tau^a \tau^b\right) = 2\delta^{ab} , \tag{2}$$

with $\mathrm{tr}_{\mathrm{f}}$ denoting the trace in flavor space. In Lagrangian (1), the bare quark masses are represented by $m_{0i}$ and their chemical potential by $\mu_i$. The coupling constant for the scalar and pseudoscalar interaction $G$ is taken as a free parameter (fixed *e.g.* by the pion mass in vacuum). The third term of Eq. (1) is the so-called 't Hooft Lagrangian, which mimics the effect of the axial U(1) anomaly, accounting for the physical splitting between the $\eta$ and the $\eta'$ meson masses. $K$ is an unknown coupling constant (fixed by the value of $m_{\eta'} - m_\eta$) and $\mathbb{I}$ is the identity matrix in Dirac space. As the NJL model is non-renormalizable, it requires also an ultraviolet regulator, which we introduce in the form of a cutoff $\Lambda$.

This Lagrangian has been widely used to study strongly interacting systems in the vacuum and at finite temperature. In the present calculation, we employ the following parameters, determined by vacuum meson masses and decay constants, see Table I. For all details, we refer to the above reviews [1–5].

TABLE I

Parameters used in the calculation and the critical chemical potential $\mu_{\mathrm{crit}}$, where the chiral restoration occurs as a second-order phase transition.

| $\Lambda$ | $G$ | $K$ | $m_{0u}$ | $m_{0s}$ | $\mu_{\mathrm{crit}}$ |
|---|---|---|---|---|---|
| 569 MeV | $2.3/\Lambda^2$ | $11/\Lambda^5$ | 5.5 MeV | 134 MeV | 338 MeV |

## 2. A transport theory based on the NJL Lagrangian

After experiments have confirmed that in heavy-ion reactions at ultra-relativistic energies for a very short time a quark–gluon plasma (QGP) is created, the study of the properties of the QGP is presently the key challenge for all these experiments. These plasma properties show up only very indirectly in the experimental observables and, therefore, theoretical approaches are needed which show how the measured observables are related to the properties of the plasma. These theoretical transport approaches either assume that during the expansion the plasma is in local thermal equilibrium or that effective degrees of freedom and cross sections can be determined from the equation of state of the plasma which allows for describing the expansion by the Kadanoff–Baym equations [7, 8]. The equation of state of the QGP at vanishing baryochemical potential can be reliably calculated by lattice gauge calculations [9, 10]. Both approaches are confronted with the fact that at the end of its expansion the QGP hadronizes. For the hadronization, only phenomenological approaches are presently available and it is presently under discussion whether strange hadrons hadronize earlier.

In this situation, it is challenging to develop a transport theory based on the NJL Lagrangian. This approach has the advantage that it is based on a Lagrangian which shares the fundamental symmetries of QCD. In addition, it has the advantage that no parametrization of the equation of state, nor assumptions about cross sections or the hadronization have to be made. All follows directly from the Lagrangian whose few parameters are fixed by vacuum properties. Also the extension to finite baryon chemical potential, where lattice gauge calculation cannot presently provide guidelines, is straightforward what is of importance in view of the upcoming experiments at NICA and FAIR. The drawback is that the NJL Lagrangian contains no gluons. In an extended version, the PNJL approach [11, 12], they can be added as a mean field which assures the right thermal properties of the plasma at large temperatures. The PNJL approach describes in a reasonable way the lattice-QCD equation of state [13].

Such a transport theory based on the lattice equation of state has been developed in the last years [14] and has been compared to the Parton–Hadron-String Dynamics model for RHIC energies [15]. One of the main ingredients of these calculations are the elastic cross sections between the plasma constituents.

## 3. Quark–quark cross section

The $qq$ and $q\bar{q}$ cross sections have been calculated by Rehberg *et al.* [16]. Here, we report on the extension of these cross sections to finite chemical potential. Figure 1 shows the cross section $uu \to uu$ as a function of the

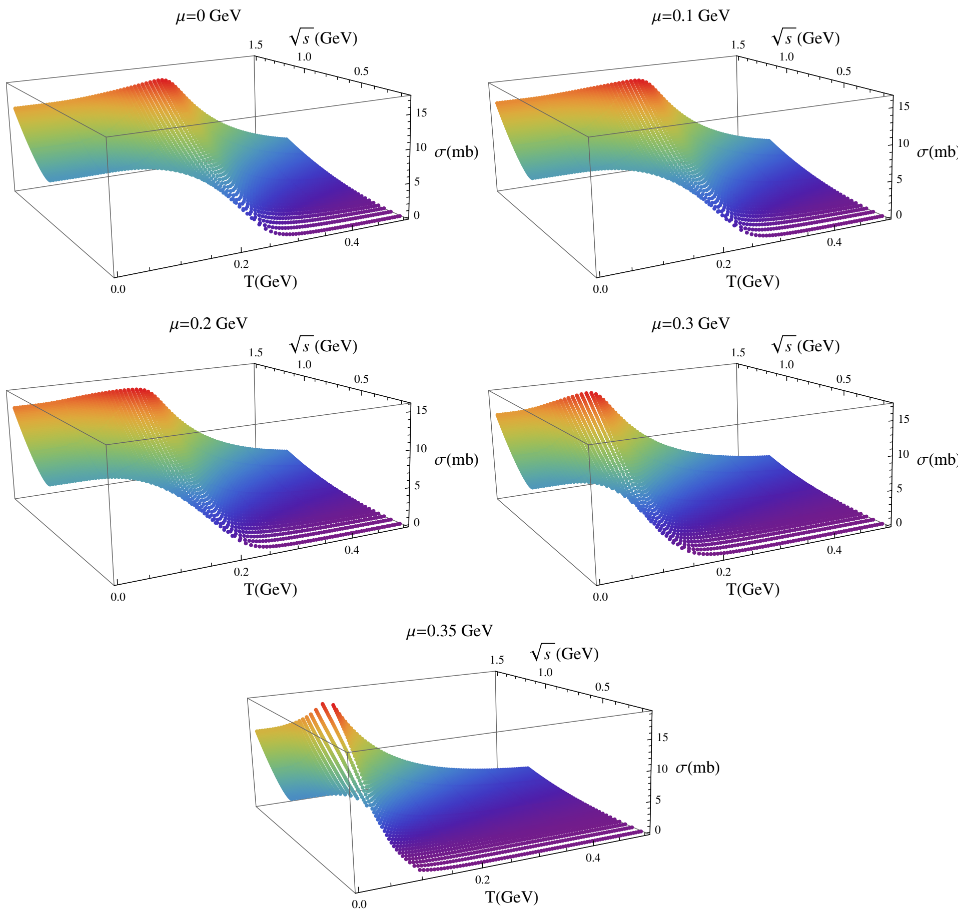


Fig. 1. $uu \to uu$ cross section as a function of the temperature, the center-of-mass energy for different chemical potentials $\mu$.

temperature, the chemical potential and the center-of-mass energy $\sqrt{s}$. In the region where the quarks are the degrees of freedom of the system, *i.e.* beyond the hadronization temperature $T_{\rm had}(\mu)$, the cross section is of the order of ten millibarn. With increasing $\mu$, the maximum moves to lower temperature following the line on which the sum of the masses of then two quarks equals that of the corresponding meson and the mesons become the theromodynamically dominating degrees of freedom. At $\mu_{\rm crit} = 338$ MeV, the transition between the QGP and the hadronic change from a cross over to a first order phase transition. Close to this value the cross section increases, especially close to the critical temperature.

## 4. Conclusions

We have presented our results for the $qq$ cross sections at finite chemical potential in the context of the NJL model. The quark–quark elastic scattering cross sections show no resonant behavior and they can take values around $\simeq 10$ mb. For increasing chemical potential, the maximum of the cross section moves to lower temperatures. The results presented here serve as a preliminary computation for an eventual implementation of the NJL/PNJL model in a transport simulation for relativistic heavy-ion collisions at low energies.

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