VARIOUS APPROACHES TO ANISOTROPIC HYDRODYNAMICS*

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Various approaches to anisotropic hydrodynamics are shortly reviewed.

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1. Introduction

The standard viscous hydrodynamics is most often based on the expansion of the underlying microscopic kinetic theory in the Knudsen and inverse Reynolds numbers around the local equilibrium state [1]. This type of expansion may be questioned in the situation where space-time gradients and/or deviations from the local equilibrium are large. The goal of the anisotropic hydrodynamics program is to create a dissipative hydrodynamics framework that is better suited to deal with such cases and accurately describes several features such as the early time dynamics of the quark–gluon plasma (QGP) created in heavy-ion collisions, dynamics near the transverse edges of the nuclear overlap region, and temperature-dependent and possibly large shear viscosity-to-entropy density ratio: $\bar{\eta} = \eta/s$ [2]. In this conference proceedings, several aspects of different formulations of anisotropic hydrodynamics, which have been recently proposed in the literature, are shortly reviewed and compared.

2. Phenomenological vs. kinetic-theory formulation

The original concepts of anisotropic hydrodynamics were presented in Refs. [3, 4], see also [5, 6]. The approach of [3] was based on the energy-momentum conservation law and used an Ansatz for the entropy source

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term that defined the off-equilibrium dynamics. On the other hand, the approach of [4] was based on the kinetic theory, and employed the zeroth and first moments of the Boltzmann kinetic equation in the relaxation time approximation (RTA) [8]. It has been soon demonstrated in [7] that these two approaches are equivalent as the first moment of the Boltzmann equation yields the energy-momentum conservation law, while the zeroth moment can be interpreted as a special form of the entropy source. The two approaches referred also to the (quasi)particle picture, where the phase-space distribution function is given by the Romatschke–Strickland form [9]. In the covariant formulation, the latter takes the form of

$$f_{\rm RS} = \exp\left(-\frac{1}{\Lambda}\sqrt{(p\cdot U)^2 + \xi\ (p\cdot Z)^2}\right), \tag{1}$$

where Λ is the transverse-momentum scale, ξ is the anisotropy parameter, while U and Z are the two four-vectors that define a simple boost-invariant geometry, $U = (t/\tau, 0, 0, z/\tau)$ and $Z = (z/\tau, 0, 0, t/\tau)$ with $\tau = \sqrt{t^2 - z^2}$ being the longitudinal proper time. The distribution function (1) leads to the following form of the energy-momentum tensor:

$$T^{\mu\nu} = (\varepsilon + P_\perp)\, U^\mu U^\nu - P_\perp\, g^{\mu\nu} - \left(P_\perp - P_\parallel\right) Z^\mu Z^\nu\,, \tag{2}$$

where $P_\perp$ and $P_\parallel$ are the transverse and longitudinal pressures, respectively.

3. Two expansion methods

Further developments of anisotropic hydrodynamics were based exclusively on the kinetic theory and they may be classified as perturbative and non-perturbative schemes.

In the perturbative approach [10–12], one assumes that the distribution function has the form of $f = f_{\rm RS} + \delta f$, where $f_{\rm RS}$ is the leading order described by the Romatschke–Strickland form, accounting for the difference between the longitudinal and transverse pressures, while δf represents a correction. In this case, advanced methods of traditional viscous hydrodynamics are used to restrict the form of δf and to derive hydrodynamic equations. In this way, non-trivial dynamics may be included in the transverse plane and, more generally, in the full (3+1)D case.

In the non-perturbative approach, one starts with the decomposition $f = f_{\rm aniso} + \delta f$, where $f_{\rm aniso}$ is the leading order distribution function given by the generalised RS form. In this case, all effects due to anisotropy are included in the leading order, while the correction term δf is typically neglected. The generalised RS form includes more parameters than the original RS Ansatz, namely one uses the expression

$$f_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu} p^\nu}}{\lambda}\right), \tag{3}$$

where λ is the transverse momentum scale, $\Xi_{\mu\nu}$ is the anisotropy tensor and $f_{\rm iso}$ denotes the isotropic/equilibrium distribution (Boltzmann, Bose–Einstein or Fermi–Dirac).

The structure of the generalised RS distribution as well as the corresponding hydrodynamic equations have been gradually worked out for: (1+1)D conformal case [13] (with two independent anisotropy parameters), (1+1)D non-conformal case [14] (with two anisotropy parameters and one bulk parameter), and full (3+1)D case [15, 16] (five anisotropy parameters included in the tensor $\xi^{\mu\nu}$ and one bulk parameter Φ). In the latter case, one uses the parameterisation

$$\begin{aligned} \Xi^{\mu\nu} &= u^\mu u^\nu + \xi^{\mu\nu} - \Delta^{\mu\nu}\Phi\,, \\ u_\mu \xi^{\mu\nu} &= 0\,, \qquad \xi^{\mu}_{\ \mu} = 0\,. \end{aligned} \tag{4}$$

Here, $\Delta^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$ is the operator projecting on the space orthogonal to the flow vector U^μ. The second line in (4) indicates that the symmetric tensor $\xi^{\mu\nu}$ is orthogonal to U^μ and traceless, thus has indeed five independent parameters. These properties are similar to those characterising the shear stress tensor $\pi^{\mu\nu}$ commonly used in the formalism of the standard dissipative hydrodynamics. As a matter of fact, $\pi^{\mu\nu}$ becomes proportional to $\xi^{\mu\nu}$ for systems approaching local equilibrium. Similarly, in this case, the parameter Φ becomes proportional to the bulk viscous pressure Π.

To derive the hydrodynamic equations obeyed by the parameters $\xi^{\mu\nu}$ and Φ, one most often uses the moments of the RTA Boltzmann equation. The number of included moments corresponds to the number of unknown parameters. An alternative for this approach is the procedure where one first derives, directly from the RTA Boltzmann equation, the equations for the pressure corrections $\pi^{\mu\nu}$ and Π, and expresses them by the function $f_{\rm aniso}$. This is the latest development for the leading order, known as the anisotropic matching principle [17], that may be supplemented by next-to-leading terms following the perturbative approach [10, 11].

4. Anisotropic vs. standard dissipative hydrodynamics

4.1. Comparisons with the exact solutions of the kinetic equation

Anisotropic and viscous hydrodynamics predictions have been checked against exact solutions available for the Boltzmann kinetic equation in the relaxation time approximation [18, 19]. Such studies have been done for the one-dimensional Bjorken geometry [20–22] and for the Gubser flow that includes transverse expansion [23, 24]. The results of those studies showed that anisotropic hydrodynamics better reproduces the results of the underlying kinetic theory than the standard viscous hydrodynamics. In addition, important constraints on the structure of the hydrodynamic equations and the form of the kinetic coefficients have been obtained within such studies.

4.2. Gradient expansion

Another way of comparing between different theories, in particular, between different formulations of dissipative hydrodynamics is the study of their formal gradient expansion. The latter is an effective way to quantify the system's approach to equilibrium and to collect information about the non-hydrodynamic sector. This is attainable by considering perturbations around the perfect-fluid hydrodynamic solution.

Recently, the gradient expansion has been studied for anisotropic hydrodynamics, its underlying kinetic theory in the relaxation time approximation, and for different formulation of standard viscous hydrodynamics [25]. It has been found that the formulation of anisotropic hydrodynamics based on the anisotropic matching principle [17] yields the first three terms of the gradient expansion in agreement with those obtained for the kinetic theory [26]. This finding gives further support for this particular hydrodynamic model as a good approximation of the kinetic theory framework. It has been further found that the gradient expansion of anisotropic hydrodynamics is an asymptotic series, and the singularities of the analytic continuation of its Borel transform indicate the presence of non-hydrodynamic modes.

4.3. Far off-equilibrium behaviour

In one of the recent works [27], a comparison of the far off-equilibrium behaviour described by different hydrodynamics models have been analysed in the context of non-boost invariant expansion. It has been found that the results of anisotropic hydrodynamics and viscous hydrodynamics agree for the central hot part of the system, however, they differ at the edges where the approach of anisotropic hydrodynamics helps to control the undesirable growth of viscous corrections observed in standard frameworks.

5. Summary

In this work, we have shortly discussed several approaches to anisotropic hydrodynamics. We summarise with the statement that it offers a plausible alternative to more standard viscous hydrodynamics approaches. In many cases, anisotropic hydrodynamics describes more accurately the results obtained with underlying kinetic theory. It also eliminates several deficiencies of the standard approaches such as, for example, negative pressures or uncontrolled entropy production growth with increasing shear viscosity.

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