$\mathrm{SU}(2N_{\mathrm{F}})$ SYMMETRY OF QCD AT HIGH TEMPERATURE AND ITS IMPLICATIONS*

L.Ya. Glozman

Institute for Physics, University of Graz
Universitätsplatz 5, 8010 Graz, Austria

(Received February 16, 2017)

If above a critical temperature not only the $\mathrm{SU}(N_{\mathrm{F}})_{\mathrm{L}} \times \mathrm{SU}(N_{\mathrm{F}})_{\mathrm{R}}$ chiral symmetry of QCD but also the $\mathrm{U}(1)_{\mathrm{A}}$ symmetry is restored, then the actual symmetry of the QCD correlation functions and observables is $\mathrm{SU}(2N_{\mathrm{F}})$. Such a symmetry prohibits existence of deconfined quarks and gluons. Hence, QCD at high temperature is also in the confining regime and elementary objects are $\mathrm{SU}(2N_{\mathrm{F}})$ symmetric "hadrons" with not yet known properties.

DOI:10.5506/APhysPolBSupp.10.583

1. Introduction

Nonperturbatively QCD is defined in terms of its fundamental degrees of freedom, quarks and gluons in the Euclidean space-time. These fundamental degrees of freedom are never observed in the Minkowski space, a property of QCD which is called confinement. Only hadrons are observed. It is believed, however, that at high temperature, QCD is in a deconfinement regime and its fundamental degrees of freedom, quarks and gluons, are liberated. Is it true? Here, we present results of our recent findings [1] that suggest that this is actually not true.

In the Minkowski space-time, the QCD Lagrangian in the chiral limit is invariant under the chiral transformations

$$\mathrm{SU}(N_{\mathrm{F}})_{\mathrm{L}} \times \mathrm{SU}(N_{\mathrm{F}})_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{A}} \times \mathrm{U}(1)_{\mathrm{V}} \,. \tag{1}$$

The axial $\mathrm{U}(1)_{\mathrm{A}}$ symmetry is broken by anomaly [2]. The $\mathrm{SU}(N_{\mathrm{F}})_{\mathrm{L}} \times \mathrm{SU}(N_{\mathrm{F}})_{\mathrm{R}}$ symmetry is broken spontaneously by the quark condensate in

* Presented at the "Critical Point and Onset of Deconfinement" Conference, Wrocław, Poland, May 30–June 4, 2016.

the vacuum. According to the Banks–Casher relation [3], the quark condensate in the Minkowski space can be expressed through a density of the near-zero modes of the Euclidean Dirac operator

$$\lim_{m\to 0}\langle 0|\bar{\Psi}(x)\Psi(x)|0\rangle = -\pi\rho(0)\,. \tag{2}$$

Consequently, if we remove by hands the near-zero modes of the Dirac operator, we can expect a restoration of the chiral $\mathrm{SU}(N_\mathrm{F})_\mathrm{L}\times\mathrm{SU}(N_\mathrm{F})_\mathrm{R}$ symmetry in correlation functions. If hadrons survive this "surgery", then the chiral partners should become degenerate. The chiral partners of the $J=1$ mesons are shown in Fig. 1.

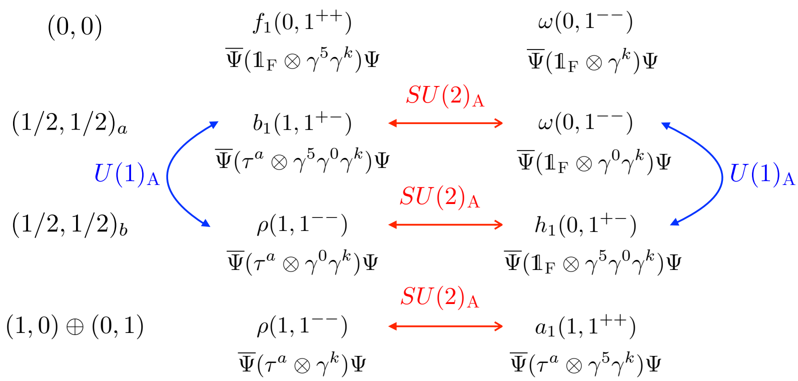

Fig. 1. $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}$ and $\mathrm{U}(1)_\mathrm{A}$ classification of the $J=1$ meson operators.

It was observed in $N_\mathrm{F}=2$ dynamical simulations with the overlap Dirac operator that, indeed, hadrons survive this truncation (except for the ground states of $J=0$ mesons) and the chiral partners get degenerate [4–7]. Not only the $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}$ restoration was observed. Mesons that are connected by the $\mathrm{U}(1)_\mathrm{A}$ transformation get also degenerate. We conclude that the same low-lying modes of the Dirac operator are responsible for both $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}$ and $\mathrm{U}(1)_\mathrm{A}$ breakings, which is consistent with the instanton-induced mechanism for both breakings [8].

Restoration of the full chiral symmetry $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}\times\mathrm{U}(1)_\mathrm{A}$ of the QCD Lagrangian assumes degeneracies marked by arrows in Fig. 1. However, a larger degeneracy that includes all possible chiral multiplets in Fig. 1 was detected, see Fig. 2.

This unexpected degeneracy implies a symmetry that is larger than the chiral symmetry of the QCD Lagrangian. This not yet known symmetry was reconstructed in Refs. [9, 10] and turned out to be

$$\mathrm{SU}(2N_\mathrm{F})\supset\mathrm{SU}(N_\mathrm{F})_\mathrm{L}\times\mathrm{SU}(N_\mathrm{F})_\mathrm{R}\times\mathrm{U}(1)_\mathrm{A}\,. \tag{3}$$

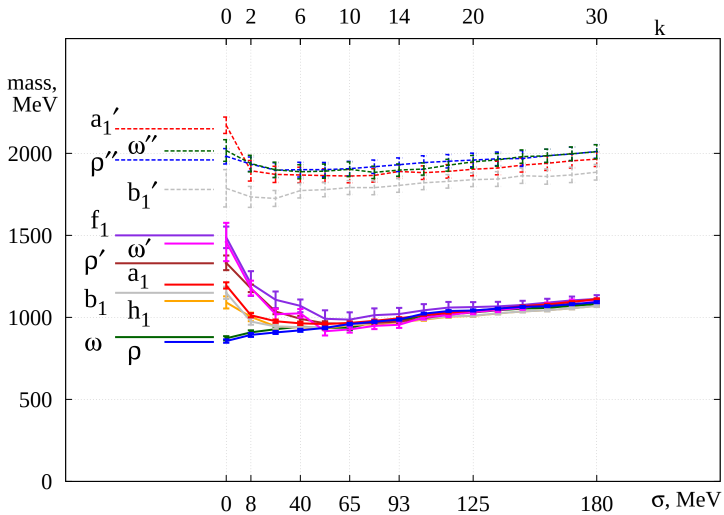

Fig. 2. $J = 1$ meson mass evolution as a function of the number k of truncated lowest-lying Dirac modes. σ shows energy gap in the Dirac spectrum.

This group includes as a subgroup the $\mathrm{SU}(2)_{\mathrm{CS}}$ (chiral spin) invariance. The $\mathrm{SU}(2)_{\mathrm{CS}}$ chiral spin generators are

$$\boldsymbol{\Sigma} = \left\{\gamma^0, i\gamma^5\gamma^0, -\gamma^5\right\}\,, \qquad \left[\Sigma^i, \Sigma^j\right] = 2i\epsilon^{ijk}\,\Sigma^k\,.$$

The Dirac spinor transforms under a global or local $\mathrm{SU}(2)_{\mathrm{CS}}$ transformation as

$$\Psi \rightarrow \Psi' = e^{i\boldsymbol{\varepsilon}\,\boldsymbol{\Sigma}/2}\Psi\,. \tag{4}$$

The γ^5 generates an $\mathrm{U}(1)_{\mathrm{A}}$ subgroup of $\mathrm{SU}(2)_{\mathrm{CS}}$. The γ^0 and $i\gamma^5\gamma^0$ mix the left- and right-handed components of the Dirac spinors. When we combine the $\mathrm{SU}(2)_{\mathrm{CS}}$ generators with the $\mathrm{SU}(N_{\mathrm{F}})$ generators, we arrive at the $\mathrm{SU}(2N_{\mathrm{F}})$ group.

2. SU(2N_{F}) as a hidden classical symmetry of QCD [11]

The SU(4) symmetry of $N_{\mathrm{F}} = 2$ Euclidean QCD was obtained in lattice simulations. This means that this symmetry must be encoded in the nonperturbative Euclidean formulation of QCD. Obviously, the Euclidean Lagrangian for N_{F} degenerate quarks in a given gauge background $A_\mu(x)$

$$\mathcal{L} = \Psi^\dagger(x)(\gamma_\mu D_\mu + m)\Psi(x) \tag{5}$$

is not $\mathrm{SU}(2)_{\mathrm{CS}}$ and $\mathrm{SU}(2N_{\mathrm{F}})$-symmetric, because the Dirac operator does not commute with the generators of $\mathrm{SU}(2)_{\mathrm{CS}}$. A fundamental dynamical reason for the absence of these symmetries are zero modes of the Dirac operator, $\gamma_\mu D_\mu \Psi_0(x) = 0$. The zero modes are chiral, L or R. With a gauge configuration of a nonzero global topological charge, the number of the left-handed and right-handed zero modes is, according to the Atiyah–Singer theorem, not equal. Consequently, there is no one-to-one correspondence of the left- and right-handed zero modes. The $SU(2)_{CS}$ chiral spin rotations mix the left- and right-handed Dirac spinors. Such a mixing can be defined only if there is a one-to-one mapping of the left- and right-handed spinors: The zero modes break the $SU(2)_{CS}$ invariance.

We can expand independent fields $\Psi(x)$ and $\Psi^\dagger(x)$ over a complete and orthonormal set $\Psi_n(x)$ of the eigenvalue problem

$$i\gamma_\mu D_\mu \Psi_n(x) = \lambda_n \Psi_n(x)\,, \tag{6}$$

$$\Psi(x) = \sum\nolimits_n c_n \Psi_n(x)\,, \qquad \Psi^\dagger(x) = \sum\nolimits_k \bar{c}_k \Psi_k^\dagger(x)\,, \tag{7}$$

where $\bar{c}_k, c_n$ are Grassmann numbers. The fermionic part of the QCD partition function takes the following form:

$$Z = \int \prod_{k,n} \mathrm{d}\bar{c}_k \mathrm{d}c_n e^{\sum_{k,n} \int \mathrm{d}^4 x \bar{c}_k c_n (\lambda_n + im) \Psi_k^\dagger(x) \Psi_n(x)}\,. \tag{8}$$

In a finite volume, the eigenmodes of the Dirac operator can be separated into two classes. The exact zero modes, $\lambda = 0$, and nonzero eigenmodes, $\lambda_n \neq 0$. It is well-understood that the exact zero modes are irrelevant since their contributions to the Green functions and observables vanish in the thermodynamic limit $V \to \infty$ as $1/V$ [12–14]. Consequently, in the finite-volume calculations, we can ignore the exact zero-modes.

Now, we can read off the symmetry properties of the partition function (8). For any $SU(2)_{CS}$ and $SU(2N_F)$ rotation, the Ψ_n and $\Psi_k^\dagger$ Dirac spinors transform as

$$\Psi_n \to U\Psi_n\,, \qquad \Psi_k^\dagger \to (U\Psi_k)^\dagger\,, \tag{9}$$

where U is any transformation from the groups $SU(2)_{CS}$ and $SU(2N_F)$, $U^\dagger = U^{-1}$. It is then clear that the exponential part of the partition function is invariant under global and local $SU(2)_{CS}$ and $SU(2N_F)$ transformations, because

$$(U\Psi_k(x))^\dagger U\Psi_n(x) = \Psi_k^\dagger(x)\Psi_n(x)\,. \tag{10}$$

The exact zero modes contributions

$$\Psi_0^\dagger(x)\Psi_n(x), \Psi_k^\dagger(x)\Psi_0(x), \Psi_0^\dagger(x)\Psi_0(x)\,,$$

for which equation (10) is not defined, are irrelevant in the thermodynamic limit and can be ignored. In other words, QCD classically without the irrelevant exact zero modes has in a finite volume V local $\mathrm{SU}(2)_{\rm CS}$ and $\mathrm{SU}(2N_{\rm F})$ symmetries. These are hidden classical symmetries of QCD.

The integration measure in the partition function is not invariant under a local $\mathrm{U}(1)_{\rm A}$ transformation [2], which is a source of the $\mathrm{U}(1)_{\rm A}$ anomaly. The $\mathrm{U}(1)_{\rm A}$ is a subgroup of $\mathrm{SU}(2)_{\rm CS}$. Hence, the axial anomaly breaks $\mathrm{SU}(2)_{\rm CS}$ and $\mathrm{SU}(2N_{\rm F}) \rightarrow \mathrm{SU}(N_{\rm F})_{\rm L} \times \mathrm{SU}(N_{\rm F})_{\rm R}$.

In the limit $V \rightarrow \infty$ the otherwise finite lowest eigenvalues λ condense around zero and provide according to the Banks–Casher relation at $m \rightarrow 0$ a nonvanishing quark condensate in the Minkowski space. The quark condensate in the Minkowski space-time breaks all $\mathrm{U}(1)_{\rm A}$, $\mathrm{SU}(N_{\rm F})_{\rm L} \times \mathrm{SU}(N_{\rm F})_{\rm R}$, $\mathrm{SU}(2)_{\rm CS}$ and $\mathrm{SU}(2N_{\rm F})$ symmetries to $\mathrm{SU}(N_{\rm F})_{\rm V}$. In other words, the hidden classical $\mathrm{SU}(2)_{\rm CS}$ and $\mathrm{SU}(2N_{\rm F})$ symmetries are broken both by the anomaly and spontaneously.

3. Restoration of $\mathbf{SU(2)_{CS}}$ and $\mathbf{SU(2\mathit{N}_F)}$ at high temperature [1]

Above the chiral restoration phase transition, the quark condensate vanishes. If, in addition, the $\mathrm{U}(1)_{\rm A}$ symmetry is also restored [15–17] and a gap opens in the Dirac spectrum, then above the critical temperature, the $\mathrm{SU}(2)_{\rm CS}$ and $\mathrm{SU}(2N_{\rm F})$ symmetries are manifest. The precise meaning of this statement is that the correlation functions and observables are $\mathrm{SU}(2)_{\rm CS}$ and $\mathrm{SU}(2N_{\rm F})$ symmetric.

These $\mathrm{SU}(2)_{\rm CS}$ and $\mathrm{SU}(2N_{\rm F})$ symmetries of QCD imply that there cannot be deconfined free quarks and gluons at any finite temperature in the Minkowski space-time. Indeed, the Green functions and observables calculated in terms of unconfined quarks and gluons in the Minkowski space (*i.e.* within the perturbation theory) cannot be $\mathrm{SU}(2)_{\rm CS}$ and $\mathrm{SU}(2N_{\rm F})$ symmetric, because the chromo-magnetic interaction necessarily breaks both symmetries. Then it follows that above $T_{\rm c}$, QCD is in a confining regime. In contrast, color-singlet $\mathrm{SU}(2N_{\rm F})$-symmetric "hadrons" (with not yet known properties) are not prohibited by the $\mathrm{SU}(2N_{\rm F})$ symmetry and can freely propagate. "Hadrons" with such a symmetry in the Minkowski space can be constructed [18].

4. Predictions

Restoration of the $\mathrm{SU}(2)_{\rm CS}$ and of $\mathrm{SU}(2N_{\rm F})$ symmetries at high temperatures can be tested on the lattice.

Transformation properties of hadron operators under $\mathrm{SU}(2)_{\rm CS}$ and $\mathrm{SU}(2N_{\rm F})$ groups are given in Refs. [7, 10]. For example, the isovector $J = 1$ operators $\bar{\Psi}\vec{\tau}\gamma^i\Psi, (1^{--})$; $\bar{\Psi}\vec{\tau}\gamma^0\gamma^i\Psi, (1^{--})$; $\bar{\Psi}\vec{\tau}\gamma^0\gamma^5\gamma^i\Psi, (1^{+-})$ form an irreducible representation of $SU(2)_{CS}$. One expects that below T_c, all three diagonal correlators will be different and the off-diagonal cross-correlator of (1^{--}) operators will not be zero. Above T_c, an $SU(2)_{CS}$ restoration requires that all diagonal correlators should become identical and the off-diagonal correlator of (1^{--}) currents should vanish. A restoration of $SU(2)_{CS}$ and of $SU(N_F)_L \times SU(N_F)_R$ implies a restoration of $SU(2N_F)$.

A similar prediction can be made with the baryon operators.

We acknowledge partial support from the Austrian Science Fund (FWF) through the grant P26627-N27.

REFERENCES

[1] L.Ya. Glozman, `arXiv:1512.06703 [hep-ph]`.
[2] K. Fujikawa, *Phys. Rev. D* **21**, 2848 (1980) [*Erratum ibid.* **22**, 1499 (1980)].
[3] T. Banks, A. Casher, *Nucl. Phys. B* **169**, 103 (1980).
[4] M. Denissenya, L.Ya. Glozman, C.B. Lang, *Phys. Rev. D* **89**, 077502 (2014).
[5] M. Denissenya, L.Ya. Glozman, C.B. Lang, *Phys. Rev. D* **91**, 034505 (2015).
[6] M. Denissenya, L.Ya. Glozman, M. Pak, *Phys. Rev. D* **91**, 114512 (2015).
[7] M. Denissenya, L.Ya. Glozman, M. Pak, *Phys. Rev. D* **92**, 074508 (2015).
[8] T. Schäfer, E.V. Shuryak, *Rev. Mod. Phys.* **70**, 323 (1998).
[9] L.Ya. Glozman, *Eur. Phys. J. A* **51**, 27 (2015).
[10] L.Ya. Glozman, M. Pak, *Phys. Rev. D* **92**, 016001 (2015).
[11] L.Ya. Glozman, `arXiv:1511.05857 [hep-ph]`.
[12] H. Leutwyler, A.V. Smilga, *Phys. Rev. D* **46**, 5607 (1992).
[13] R. Brower, S. Chandrasekharan, J.W. Negele, U.J. Wiese, *Phys. Lett. B* **560**, 64 (2003).
[14] S. Aoki, H. Fukaya, Y. Taniguchi, *Phys. Rev. D* **86**, 114512 (2012).
[15] G. Cossu *et al.*, *Phys. Rev. D* **87**, 114514 (2013); **88**, 019901 (2013).
[16] A. Tomiya *et al.*, *PoS* **LATTICE 2014**, 211 (2015) [`arXiv:1412.7306 [hep-lat]`]; G. Cossu *et al.* [JLQCD Collaboration], `arXiv:1511.05691 [hep-lat]`.
[17] S. Aoki [JLQCD Collaboration], *PoS* **(CD15)**, 045 (2015) [`arXiv:1603.00997 [hep-lat]`].
[18] M. Shifman, *Phys. Rev. D* **93**, 074035 (2016).