RELAXATION RATES AND PHASE TRANSITIONS AT STRONG COUPLING*

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In the present work, we compute relaxation rates of strongly coupled field theories exhibiting non-trivial phase structures. Our method of choice is a bottom-up gauge/gravity construction. Two different scenarios for a holographic first order phase transition are examined, and in both cases we establish the existence of a spinodal region. In addition, for a model with linear confinement in the meson spectrum, we find a region of temperatures with unstable non-hydrodynamic modes within a branch of black hole solutions.

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1. Introduction

It is a well-known fact that different strongly coupled systems exhibit different phase structures. For example, using lattice formulation of quantum field theory, it has been shown that a pure gluon system undergoes a first order phase transition [1], while inclusion of dynamical quarks changes this to a crossover behaviour [2]. While lattice techniques capture the static properties quite well, they do not reach real time dynamics easily, and one has to resort to other methods. One possibility of formulating the problem is the gauge/gravity duality [3], which has been proven useful in the heavy-ion collision phenomenology [4]. Following this line of reasoning, recently, an analysis has been performed aiming at quantifying the linearized excitations of strongly coupled systems with various phase structures [5,6]. The focus of this note is on the cases of a first order phase transition and different instabilities that appear along with it. One of them is the generically expected spinodal region [8], while the other is a brand new *dynamical* instability appearing in a confining model.

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2. Equations of state

The bottom-up constructions assume that holographic dictionary holds, and then model physics of interest by tuning the gravity-matter Lagrangian. In our case, we turn on a source for a relevant scalar operator in the boundary theory

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + \Lambda^{4-\Delta} O_\phi \,, \tag{1}$$

where Λ is some arbitrary energy scale and Δ is the conformal dimension of O_ϕ. The dual gravitational description of this deformation is given by the standard Einstein-scalar action [9, 10]

$$S = \frac{1}{2\kappa_5^2} \int\limits_{\mathcal{M}} \mathrm{d}^5 x \sqrt{-g} \left[R - \frac{1}{2} \left(\partial\phi\right)^2 - V(\phi) \right] . \tag{2}$$

The equations of state of the system are determined from the properties of the dual black hole solution in terms of the Hawking temperature and the Bekenstein–Hawking entropy [9, 10]. It is convenient to use the following parametric form of the scalar field potential:

$$V(\phi) = -12 \left(1 + a\,\phi^2\right)^{1/4} \cosh(\gamma\,\phi) + b_2\,\phi^2 + b_4\,\phi^4 + b_6\,\phi^6 \,, \tag{3}$$

where different choices of parameters corresponding to different phase structures were discussed in [5, 6]. In the rest of this note, we focus on two cases which give rise to two different types of a first order phase transition. The detailed parameter choices were presented in [5]. The main difference is in the parameter a, which is non-zero for the confining model.

In the first case (left panel of Fig. 1), the transition happens between two different stable black hole solutions (solid/green and dashed/blue lines). The dotted/red line in this case represents the spinodal instability region. The

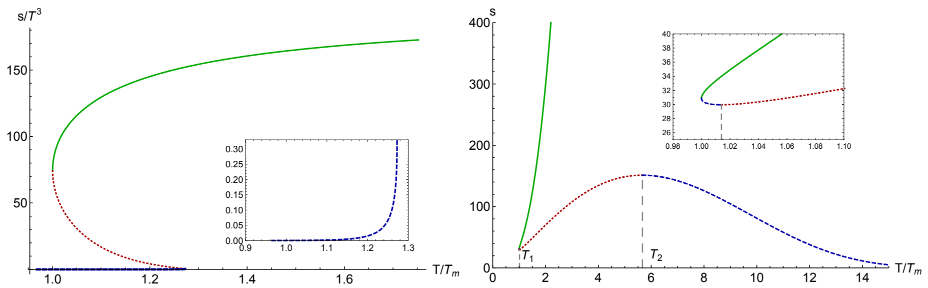

Fig. 1. (Colour on-line) Equations of state for two different choices of model parameters. Left panel: entropy density as a function of temperature for a first order phaste transition between two black hole solutions. Right panel: entropy density (in units of AdS radius) as a function of temperature for the IHQCD potential. Various lines represent different phases (see the text).

second case is a version of Improved Holographic QCD potential (IHQCD) [10] (right panel of Fig. 1), where the transition happens between a horizonless vacuum geometry and a black hole, which is similar in spirit to the original proposal [11]. Here, the dashed/blue line is the spinodal region, while the dotted/red line represents a novel phase which possesses unstable, non-hydrodynamical degrees of freedom.

3. Instabilities

In holography the linearized, collective excitations of a uniform system in a thermal equilibrium are quantified by quasinormal modes. According to the bulk/boundary dictionary these correspond to the poles of retarded Green's function of the properly chosen local operators [7]. In theories having classical dual description in terms of the General Relativity, there is a discrete spectrum of mode frequencies $\omega_n(k) = \Omega_n(k) - i\Gamma_n(k)$, with $n = 0, 1, 2, \ldots$ The real part, $\Omega_n(k)$, determines oscillatory behaviour, while the imaginary part $\Gamma_n(k)$, determines the decay rate. Whenever $\Gamma_n(k) > 0$, the mode is stable, while in the opposite case, the mode signals an instability of the system. In the spectrum, one always finds a hydrodynamic mode, defined by the relation $\lim_{k\to 0} \omega_{\mathrm{H}}(k) = 0$ with the usual dispersion relation

$$\omega(k) = \pm\, c_{\mathrm{s}}\, k - \frac{i}{2\,T}\left(\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s}\right) k^2 + O\left(k^3\right) \tag{4}$$

determined by the speed of sound and transport coefficients [7, 8]. Apart from that, there is an infinite set of higher, non-hydrodynamic frequencies.

It is generically expected that a system with a first order phase transition of any kind will exhibit a spinodal instability region and the related bubble formation effect. A classical example known from elementary textbooks is superheated water. A more sophisticated example is the spinodal region related to the nuclear matter liquid–gas transition [8]. Confirmation of this general prediction in the context of holographic models was put forward for the first time in [6]. In the following research, another, still more intriguing type of instability has been found [5,12]. We briefly review both phenomena, referring the interested reader to the original references.

3.1. Spinodal instability

In Fig. 2, we show quasinormal modes in the spinodal region of the first order phase transition without confining interactions. In this phase, the hydrodynamic sound mode has positive imaginary part for a range of momenta. This implies that the mode is growing in time and signals an unstable phase. There is a characteristic size for the instability formation given by the maximum of the imaginary part of the mode dispersion relation (called the growth rate [8]). As momentum increases, the viscus corrections stabilize the perturbation. This behaviour is exactly what we expect from the equations of state whenever $c_s^2 < 0$.

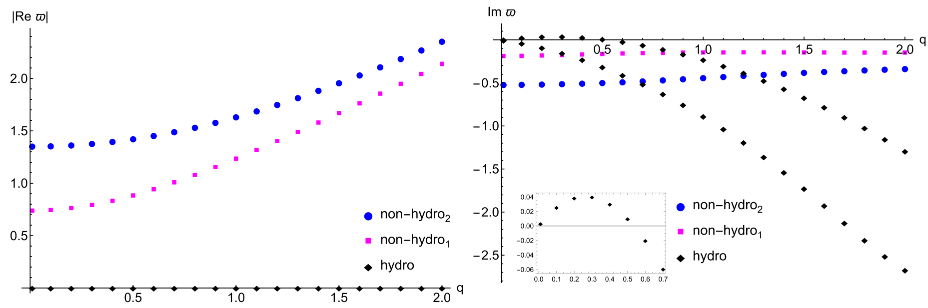

Fig. 2. The sound channel quasinormal modes for the first potential at $T \simeq 1.06T_m$. An instability of the spinodal region is shown. The speed of sound at that temperature is $c_s^2 \simeq -0.1$.

3.2. Dynamical instability

The second kind of instabilities was found in a confining model and has no known field theory counterpart. An example of this behaviour is shown in Fig. 3.

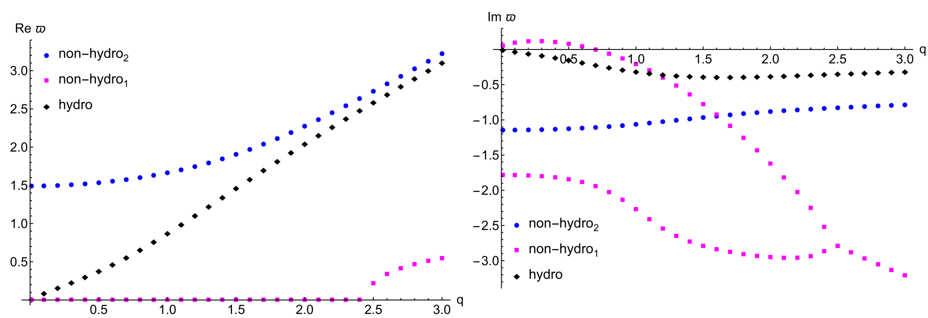

Fig. 3. Sound channel quasinormal modes for the IHQCD potential at $T = 1.027T_m$. System displays dynamical instability in spite of thermodynamical stability.

Here, the hydrodynamic mode is well-behaved, with the instability present in the lowest lying, non-hydrodynamic degrees of freedom. It goes all the way to zero momentum, which implies that the system is unstable against uniform perturbations. This is in a direct contrast to the spinodal instability, which was inherently related to a non-zero length scale. At the moment, there is no good understanding of this phenomena, as well as the fact that this effect does not explicitly appear on the level of equations of state.

4. Summary

The novel form of the unstable behaviour found in a class of holographic models is definitely an interesting new direction of study. A possible direct experimental connection has been anticipated in [12], given that in a full time evolution of a homogeneous system, one could fine-tune the initial conditions so that during the evolution of the system, the unstable phase would be attainable for an arbitrary long time. This would cause the termalization time to be much larger than the scale set by the inverse effective temperature of the system. However, it might turn out that this is in some form an artifact of the system, and that configurations suffering such instabilities are not well-defined physical states. This question remains an open one.

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