# QCD THERMODYNAMICS IN THE CROSSOVER/FREEZE-OUT REGION*

Frithjof Karsch

Fakultät für Physik, Universität Bielefeld, 33615 Bielefeld, Germany
and
Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

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We use results from a $6^{\mathrm{th}}$ order Taylor expansion of the QCD equation of state to construct expansions for cumulants of conserved charge fluctuations and their correlations. We show that these cumulants strongly constrain the range of applicability of hadron resonance gas model calculations. We point out that the latter is inappropriate to describe equilibrium properties of QCD at zero and non-zero values of the baryon chemical potential already at $T \sim 155$ MeV.

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## 1. Introduction

The existence of a critical point in the phase diagram of strong interaction matter will be clearly visible in the (singular) structure of higher order cumulants of net baryon number, strangeness and electric charge fluctuations, and their cross correlations. A prerequisite for utilizing these theoretically well-founded observables in experimental searches for the possible existence of a critical point is that experimentally observed charge fluctuations are indeed generated close to the pseudo-critical line that characterizes the chiral crossover transition and eventually ends in the critical point.

Establishing the relation between freeze-out conditions at different beam energies in heavy-ion experiments and the crossover line in QCD thus is of paramount importance. Although strategies have been developed to extract freeze-out conditions from observables that are directly accessible to experiment as well as calculations within QCD [1–3], applying these in practice is still hampered by large statistical errors and our poor control over systematic

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effects entering the measurement of higher order cumulants of charge fluctuations. The still preferred approach to extract freeze-out conditions from experimental data thus proceeds through a comparison of experimental data for particle yields with model calculations based on the thermodynamics of a hadron resonance gas (HRG) [4, 5].

We will confront here some QCD calculations of higher order cumulants of net-charge fluctuations with the corresponding HRG model calculations and discuss the range of applicability of the latter.

## 2. QCD thermodynamics at non-zero net-baryon-number density

The equation of state (EoS) of QCD with physical light and strange quark masses has been analyzed at non-zero values of the baryon number ($\mu_{\rm B}$), strangeness ($\mu_{\rm S}$) and electric charge ($\mu_Q$) chemical potentials. For the case of systems with vanishing net strangeness, $n_{\rm S} = 0$, and a fixed ratio on net-electric charge to baryon number densities, $n_Q/n_{\rm B} = 0$, the EoS has been calculated in a 6$^{\rm th}$ order Taylor expansion [6] as well as in simulations with an imaginary chemical potential [7]. These results agree well among each other and are considered to be reliable up to $\mu_{\rm B}/T \simeq 2$. Results from these calculations are shown in Fig. 1 (left).

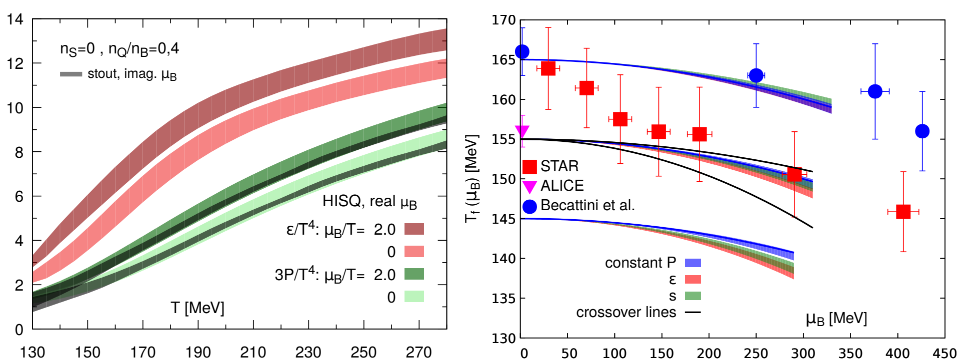


Fig. 1. Left: The pressure and energy density *versus* temperature for two values of the baryon chemical potential for strangeness neutral systems ($n_{\rm S} = 0$), and a fixed ratio of net-electric charge and net baryon number ($n_Q/n_{\rm B} = 0.4$). Shown are results from a 6$^{\rm th}$ order Taylor expansion [6] and from simulations with an imaginary chemical potential [7]. Right: Lines of constant pressure, energy density and entropy density *versus* baryon chemical potential in (2+1)-flavor QCD for three different initial sets of values fixed at $\mu_{\rm B} = 0$ and $T_0 = 145$ MeV, 155 MeV and 165 MeV, respectively.

Using the Taylor series for pressure ($P$), energy ($\epsilon$) and entropy ($s$) density, lines of constant physics (LCPs) can be determined

$$T_f(\mu_{\mathrm{B}}) = T_0 \left(1 - \kappa_2^f \left(\frac{\mu_{\mathrm{B}}}{T_0}\right)^2 - \kappa_4^f \left(\frac{\mu_{\mathrm{B}}}{T_0}\right)^4\right) + \mathcal{O}\left(\mu_{\mathrm{B}}^6\right) , \tag{1}$$

with $f$ labeling the observable that is kept constant, $f = P$, $\epsilon$ or $s$. It turns out that up to $\mu_{\mathrm{B}} \simeq 2T$, the correction arising from the quartic term is small. In the crossover region, 145 MeV $\leq T \leq$ 165 MeV, the quadratic expansion coefficients, $\kappa_2^f$, vary in a range of $0.006 \leq \kappa_2^f \leq 0.012$ [6]. These LCPs can be compared with results for the pseudo-critical temperature of the chiral transition, $T_{\mathrm{c}}(\mu_{\mathrm{B}})$, which can be parametrized as in Eq. (1). The spread of curvature coefficients determined for the chiral crossover line[1], $T_{\mathrm{c}}(\mu_{\mathrm{B}})$, agrees well with those obtained for the LCPs.

Results for lines of constant physics and the crossover line in the $T$–$\mu_{\mathrm{B}}$ phase diagram of QCD are shown in Fig. 1 (right). Also shown in this figure are experimental results for sets of freeze-out parameters, $(T_f, \mu_{\mathrm{B}}^f)$, determined by the ALICE Collaboration [5] at the LHC and the STAR Collaboration [4] at different beam energies at RHIC. These parameter sets have been obtained by comparing experimentally determined hadron yields with those calculated in HRG models. Unfortunately, at present, results from both collaborations differ significantly at large collision energies. Also shown in Fig. 1 (right) are results for the hadronization temperature extracted by Becattini *et al.* [8]. The latter analysis, unlike the STAR data, leads to hadronization parameters that nicely follow LCPs and have a curvature consistent with the QCD crossover line. However, the resulting hadronization temperatures are large and difficult to reconcile with the chiral transition temperature, $T_{\mathrm{c}} = 154(9)$ MeV. They seem to suggest that hadronization takes place at $T$-values, where QCD thermodynamics already shows many features of a partonic medium, albeit not a free gas of quarks and gluon, and is no longer compatible with HRG thermodynamics. This is apparent from the temperature dependence of $2^{\mathrm{nd}}$ and $4^{\mathrm{th}}$ order cumulants shown in Fig. 2. Obviously the agreement between cumulants that only differ by two derivatives with respect to $\mu_{\mathrm{B}}$ breaks down for $T \gtrsim 155$ MeV. As suggested in [9], this may hint at the appearance of quasi-particles with baryon number $B \neq \pm 1$, or at least the importance of interactions, which also may be interpreted as the appearance of clusters with $B \neq \pm 1$.

The $2^{\mathrm{nd}}$ and $4^{\mathrm{th}}$ order cumulants shown in Fig. 2 are also the first terms in Taylor expansions of net conserved charges[2], *e.g.*

[1] For a list of recent references see, for instance, [6].

[2] For simplicity, we set $\mu_Q = \mu_{\mathrm{S}} = 0$.

$$\frac{n_B}{T^3} = \chi_2^B \frac{\mu_B}{T} + \frac{1}{6}\chi_4^B \left(\frac{\mu_B}{T}\right)^3 + \mathcal{O}\left(\mu_B^5\right) ,$$
$$\frac{n_X}{T^3} = \chi_{11}^{BX} \frac{\mu_B}{T} + \frac{1}{6}\chi_{31}^{BX} \left(\frac{\mu_B}{T}\right)^3 + \mathcal{O}\left(\mu_B^5\right) , \qquad X = S,\ Q . \tag{2}$$

This suggests that even on the level of particle yields systematic differences between QCD and HRG, model calculations will show up at temperatures larger than $T \simeq 155$ MeV and will become more pronounced for $\mu_B \neq 0$.

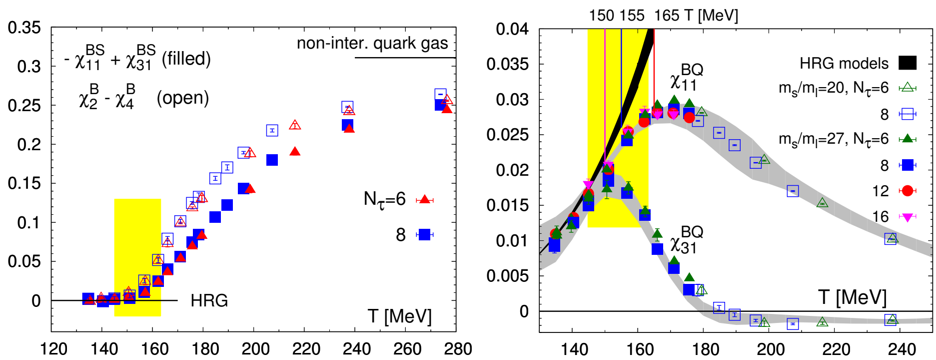


Fig. 2. Left: The difference of second and fourth order cumulants of net-baryon-number fluctuations and their correlation with net-strangeness fluctuations. Right: Second and fourth order cumulants of correlations between moments of net-baryon-number and electric charge fluctuations.

Moreover, even at temperatures where hadrons are the dominant degrees of freedom and HRG models may be appropriate to describe the thermodynamics of strong interaction matter, such a description is sensitive to the particle content in the HRG. In particular, it has been noted that systematic differences show up in the strangeness sector [3]. The correlation between net baryon number and strangeness fluctuations, $\chi_{11}^{BS}$, is systematically larger in QCD than in HRG model calculations that are based only on experimentally known hadron resonances (PDG-HRG). In fact, at temperatures $T \lesssim 160$ MeV, $\chi_{11}^{BS}$ is in a good agreement with HRG model calculations that also include resonances predicted to exist in quark models (QM-HRG). This is shown in Fig. 3 (left). A consequence of this difference is that values of $\mu_B$ and/or $T$ that describe identical thermal conditions, *e.g.* identical values of the charge fluctuations, do not match one-to-one between HRG and QCD calculations. Figure 3 (right) shows the values for the ratio $\mu_S/\mu_B$ needed to ensure strangeness neutrality, $n_S = 0$, in strong interaction matter. It is evident that a certain ratio $\mu_S/\mu_B$ corresponds to temperatures that differ by about 10 MeV in QCD and HRG models. Similar information can be deduced from Fig. 3 (left) when considering a fixed ratio $\chi_{11}^{BS}/\chi_2^S$.

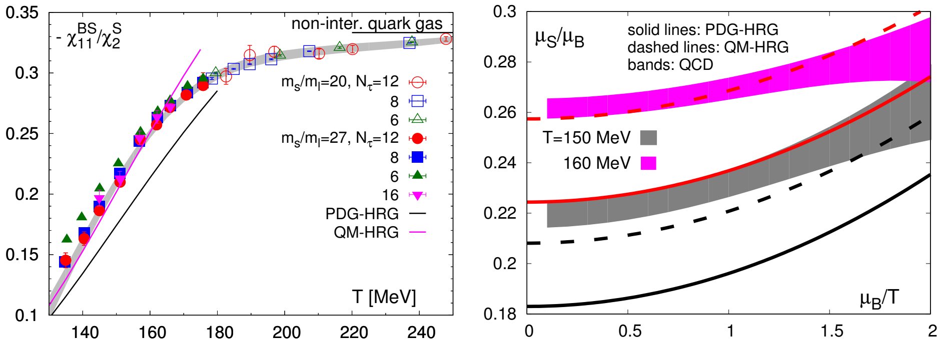


Fig. 3. Left: The correlation between net baryon number and strangeness fluctuations normalized to the variance of strangeness fluctuations (see the text for details). Right: Ratio $\mu_S/\mu_B$ needed to adjust $n_S = 0$ and $n_Q/n_B = 0.4$ in QCD. Lines show corresponding results for two different versions of HRG models (see the text).

## 3. Conclusions

Differences between the modeling of strong interaction matter in terms of HRG model thermodynamics and a QCD calculations rapidly become large for $T \gtrsim 155$ MeV. This can easily lead to a $\sim 10\%$ mismatch between temperature and chemical potential values determined as freeze-out parameters in QCD and model calculations. The width of the singular region around a critical point may be of a similar size. A 10 MeV accuracy on *e.g.* the freeze-out temperature $T^f(\mu_B)$ may thus decide whether or not critical behavior is at all detectable through measurements of cumulants of conserved charge fluctuations.

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