

NON-PERTURBATIVE PRODUCTION RATE OF PHOTONS WITH A LATTICE QUARK PROPAGATOR: EFFECT OF VERTEX CORRECTION*

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We analyze the production rate of photons from the thermal medium above the deconfinement temperature with a quark propagator obtained from a lattice QCD numerical simulation. The photon-quark vertex is determined gauge-invariantly, so as to satisfy the Ward-Takahashi identity. The obtained photon-production rate shows a suppression compared to perturbative results.

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1. Introduction

The photon-production yield is an important experimental observable in the relativistic heavy-ion collisions because it serves as a direct signal from a hot medium. Recently, interesting experimental results on the p_T spectra [1, 2] and its anisotropic flow [3] are measured. Theoretically, the production rate can be calculated perturbatively, and sophisticated analyses on the leading order [4] and next-to-leading order [5] have been performed based on the hard thermal loop (HTL) resummed perturbation theory. It is, however, known that the hot medium near the critical temperature, T_c , is a strongly coupled system. Therefore, non-perturbative analysis is more desirable to calculate the production rate relevant for the relativistic heavy-ion collisions.

In Ref. [6], the analysis of the production rate of virtual photons, observed as dileptons in experiments, at zero momentum has been performed using a quark propagator obtained on a lattice simulation. The vertex

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function is constructed so as to satisfy the Ward–Takahashi identity. In the present study, we apply this analysis to the study of the real photon-production rate.

2. Formalism of photon-production rate

The photon-production rate per unit time per unit volume is related to the retarded photon self energy $\Pi_{\mu\nu}^R(\omega, \mathbf{q})$ as

$$\omega \frac{dN_\gamma}{d^3q d^4x} = -\frac{2}{(2\pi)^3} \frac{1}{e^{\beta\omega} - 1} \text{Im} \Pi_\mu^{R,\mu}(\omega, \mathbf{q}), \quad (1)$$

with the inverse temperature $\beta = 1/T$ [7].

The full photon self energy in the Matsubara formalism is written as

$$\Pi_{\mu\nu}(i\omega_m, \mathbf{q}) = -\sum_f e_f^2 T \sum_n \int \frac{d^3p}{(2\pi)^3} \text{Tr}_C \text{Tr}_D [S(P) \gamma_\mu S(P+Q) \Gamma_\nu(P+Q, P)], \quad (2)$$

with the full quark propagator $S(P)$ and the full photon–quark vertex $\Gamma_\nu(P+Q, P)$. For notational simplicity, the color, flavor and Dirac indices of $S(P)$ are suppressed. $\omega_m = 2\pi T m$ and $\nu_n = (2n+1)\pi T$ with integers m and n represent the Matsubara frequencies for bosons and fermions, respectively. $Q_\mu = (i\omega_m, \mathbf{q})$ and $P_\mu = (i\nu_n, \mathbf{p})$ are four momenta of the photon and quarks, respectively. e_f is the electric charge of a quark and the index “f” represents the quark flavor. Tr_C and Tr_D are the traces over the color and Dirac indices, respectively.

In the present study, we use a quark propagator obtained on the lattice as the full quark propagator in Eq. (2). On the lattice with a gauge fixing, the imaginary-time correlator $S_{\mu\nu}(\tau, \mathbf{p})$ can be measured, which is related to the spectral function $\rho_{\mu\nu}(\nu, \mathbf{p})$ as

$$S_{\mu\nu}(\tau, \mathbf{p}) = \int_{-\infty}^{\infty} d\nu \frac{e^{(1/2 - \tau/\beta)\beta\nu}}{e^{\beta\nu/2} + e^{-\beta\nu/2}} \rho_{\mu\nu}(\nu, \mathbf{p}). \quad (3)$$

We take the Landau gauge and color indices are suppressed.

When the chiral symmetry is restored, the spectral function can be decomposed with the projection operators $\Lambda_\pm(\mathbf{p}) = (1 \pm \gamma_0 \hat{\mathbf{p}} \cdot \boldsymbol{\gamma})/2$ as

$$\rho(\nu, \mathbf{p}) = \rho_+(\nu, p) \Lambda_+(\mathbf{p}) \gamma_0 + \rho_-(\nu, p) \Lambda_-(\mathbf{p}) \gamma_0, \quad (4)$$

where $\rho_\pm(\nu, p) \equiv \text{Tr}_D[\rho(\nu, \mathbf{p}) \gamma_0 \Lambda_\pm(\mathbf{p})]/2$ and $p = |\mathbf{p}|$.

In Ref. [8], the quark correlator in the Landau gauge is evaluated on the lattice with the quenched approximation, and the quark spectral function is deduced with the two-pole Ansatz

$$\rho_+(\nu, p) = Z_+(p)\delta(\nu - \nu_+(p)) + Z_-(p)\delta(\nu + \nu_-(p)) , \quad (5)$$

where $Z_{\pm}(p)$ and $\nu_{\pm}(p)$ are the residues and dispersions of two quasi-particle states of the normal and plasmino modes, respectively. In Fig. 1, we show the fitting result of each parameter in Eq. (5) for massless quarks as a function of p .

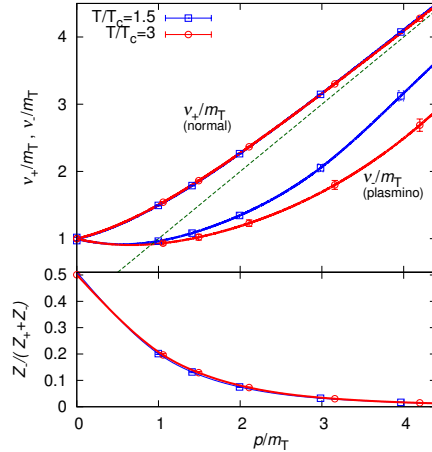


Fig. 1. Open symbols show the momentum dependence of the parameters $\nu_+(p)$, $\nu_-(p)$, and $Z_-(p)/(Z_+(p) + Z_-(p))$ obtained on the lattice in Ref. [8]. The solid lines represent their interpolation obtained by the cubic spline method [6]. The dashed line represents the light cone.

Next, we construct the vertex function Γ_μ . Because of gauge invariance, this function should satisfy the Ward–Takahashi identity

$$Q^\mu \Gamma_\mu(P + Q, P) = S^{-1}(P + Q) - S^{-1}(P) , \quad (6)$$

where $S^{-1}(P)$ is the inverse quark propagator with four momentum P .

In the present work, we use the following form of Γ_μ

$$\begin{aligned} \Gamma_0(i\omega_m + i\nu_n, \mathbf{p} + \mathbf{q}; i\nu_n, \mathbf{q}) &= \frac{1}{2i\omega_m} [S^{-1}(i\omega_m + i\nu_n, \mathbf{p} + \mathbf{q}) \\ &- S^{-1}(i\nu_n, \mathbf{p} + \mathbf{q}) + S^{-1}(i\omega_m + i\nu_n, \mathbf{q}) - S^{-1}(i\nu_n, \mathbf{q})] , \end{aligned} \quad (7)$$

$$\begin{aligned} \Gamma_i(i\omega_m + i\nu_n, \mathbf{p} + \mathbf{q}; i\nu_n, \mathbf{q}) &= \gamma_i - \frac{q_i}{2q^2} [S^{-1}(i\omega_m + i\nu_n, \mathbf{p} + \mathbf{q}) \\ &+ S^{-1}(i\nu_n, \mathbf{p} + \mathbf{q}) - S^{-1}(i\omega_m + i\nu_n, \mathbf{q}) - S^{-1}(i\nu_n, \mathbf{q})] - \frac{q_i(\mathbf{q} \cdot \boldsymbol{\gamma})}{q^2} . \end{aligned} \quad (8)$$

These vertex functions satisfy Eq. (6).

With the lattice quark propagator and the gauge invariant vertex Eqs. (7) and (8), the real photon-production rate is evaluated as

$$\begin{aligned}
\omega \frac{dN_\gamma}{d^3q d^4x} = & -\frac{5\alpha}{6(2\pi)^3} \frac{1}{\omega} \frac{1}{e^{\beta\omega} - 1} \int_0^\infty dp_1 \int_0^\infty dp_2 \sum_{s,t,\eta_1,\eta_2=\pm 1} Z_{\eta_1}(p_1) Z_{\eta_2}(p_2) \\
& \times \left[\left[st \left[(sp_1 + tp_2)^2 - \omega^2 \right] \left[1 - \frac{tp_2 - sp_1}{\omega} \right] \right. \right. \\
& \times \left(2 + \frac{t\eta_2 Z_{\bar{\eta}_2}(p_2) \bar{\nu}(p_2)}{s\eta_1 \nu_{\eta_1}(p_1) + tV(p_2)} + \frac{s\eta_1 Z_{\bar{\eta}_1}(p_1) \bar{\nu}(p_1)}{t\eta_2 \nu_{\eta_2}(p_2) + sV(p_1)} \right) \\
& - 12p_1 p_2 + 2st(sp_1 + tp_2)^2 - 2st(p_1^2 + p_2^2) + \frac{2st}{q^2} \left[(p_1^2 - p_2^2)^2 - q^4 \right] \Big] \\
& \times [f(s\eta_1 \nu_{\eta_1}(p_1)) - f(t\eta_2 \nu_{\eta_2}(p_2))] \delta(\omega + s\eta_1 \nu_{\eta_1}(p_1) - t\eta_2 \nu_{\eta_2}(p_2)) \\
& - st \left[(sp_1 + tp_2)^2 - \omega^2 \right] \left[1 - \frac{tp_2 - sp_1}{\omega} \right] \frac{t\eta_2 Z_{\bar{\eta}_2}(p_2) \bar{\nu}(p_2)}{s\eta_1 \nu_{\eta_1}(p_1) + tV(p_2)} \\
& \times [f(-tV(p_2)) - f(t\eta_2 \nu_{\eta_2}(p_2))] \delta(\omega - tV(p_2) - t\eta_2 \nu_{\eta_2}(p_2)) \\
& - st \left[(sp_1 + tp_2)^2 - \omega^2 \right] \left[1 - \frac{tp_2 - sp_1}{\omega} \right] \frac{s\eta_1 Z_{\bar{\eta}_1}(p_1) \bar{\nu}(p_1)}{t\eta_2 \nu_{\eta_2}(p_2) + sV(p_1)} \\
& \times [f(s\eta_1 \nu_{\eta_1}(p_1)) - f(-sV(p_1))] \delta(\omega + s\eta_1 \nu_{\eta_1}(p_1) + sV(p_1)) \Big], \quad (9)
\end{aligned}$$

where $\bar{\nu}(p_l) \equiv \nu_+(p_l) + \nu_-(p_l)$, $V(p_l) \equiv Z_+(p_l)\nu_-(p_l) - Z_-(p_l)\nu_+(p_l)$ and l takes 1 or 2. We define $Z_{\pm 1}(p_l) = Z_{\pm}(p_l)$, $\nu_{\pm 1}(p_l) = \nu_{\pm}(p_l)$ and $\bar{\eta}_l = -\eta_l$. $f(p)$ is the Fermi distribution function.

The first term in Eq. (9) represents the real photon productions via pair annihilation and the Landau damping of quasi-quarks. We note that the photon production with pair annihilation can manifest itself in our formalism because of the modified dispersion relation of quasi-quarks. On the other hand, the photon productions in second and third terms cannot be interpreted as simple reactions between quasi-quark excitations. These anomalous photon-production mechanism is found in our previous work on the dilepton production rate [6].

3. Numerical results

Next, we show our numerical results of the photon-production rate obtained in the previous section. In Fig. 2 (a), we show the energy dependence of the photon-production rate for $T = 1.5 T_c$. In the figure, we also plot the result calculated with bare vertex but the quark propagator obtained on the lattice. Furthermore, the three thin lines represent the leading order result in Ref. [4] for three values of strong coupling constant α_s .

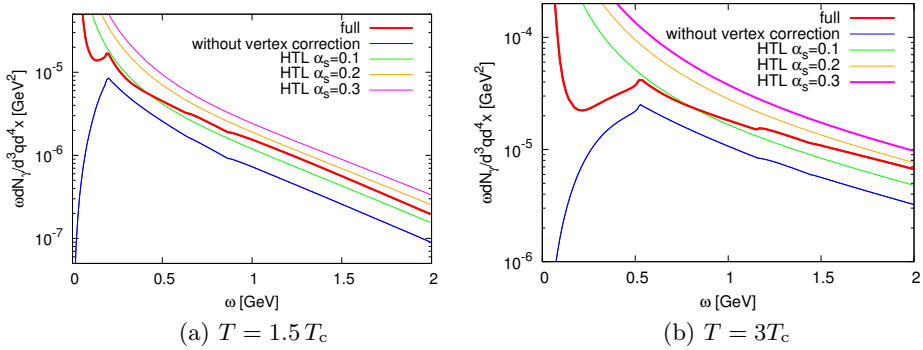


Fig. 2. Photon production rates for $T = 1.5 T_c$ and $3T_c$. The results without vertex correction are also plotted. Thin lines represent the HTL results at the leading order.

In Fig. 2 (a), one sees that our result is comparable to perturbative ones. It is interesting that the obtained result is similar to perturbative ones although the production mechanisms are different. In the perturbative calculation, the production rate is dominated by bremsstrahlung and inelastic pair annihilation processes [4]. On the other hand, these processes are not directly included in our analysis. Instead, the main contribution in our result comes from the Landau damping and the pair annihilation processes of quasi-quark excitations. Figure 2 (a) also shows that the production rate behaves discontinuously at $\omega \simeq 0.63$ GeV. The origin of this discontinuity is that the production rate is given by a superposition of various reactions. The pair annihilation process takes place for $\omega > 0.63$ GeV and the discontinuity corresponds to the threshold energy of this process. Other structures can also be understood similarly. In Fig. 2 (b), the result for $T = 3 T_c$ is shown, which behaves similarly to the result for $T = 1.5 T_c$.

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