S-MATRIX APPROACH TO PION GAS*

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S-matrix formalism is applied to analyze a gas of interacting pions.

1. Introduction

The fact that pions occupy the ground state of the theory of strong interaction follows from a combination of two phenomena: confinement and the spontaneous breaking of chiral symmetry. The former dictates that hadrons, instead of quarks and gluons, fill the physical spectrum of QCD while the latter makes pions exceptionally light due to their role as (pseudo-) Goldstone bosons.

We thus expect at low temperatures the partition function is dominated by the contribution of pions and other low-lying resonances. One well-known approach for describing the system of thermal hadrons is the hadron resonance gas (HRG) model [1–3]. This model assumes that resonance formation dominates the behavior of the confined phase and as first approximation treats the resonances as an ideal gas.

On the other hand, driven by the practical need to interpret the ever-growing precision data from heavy-ion collisions and the theoretical interest in better understanding the QCD phase structure [4], there is a strong incentive to construct a more accurate description of real hadron gas.

A very detailed picture of hadronic interactions has emerged from the impressive volume of experimental data [5], carefully analyzed by theories such as chiral perturbation theory [6, 7], lattice QCD [8], effective hadron models [9] and time-honored potential models [10, 11]. Consequently, we acquire very precise information on particle spectra, production mechanisms

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and decay properties of typical hadrons and even the exotics [12]. These studies, however, have yet to be systematically included in the thermodynamical description of interacting hadrons.

The S-matrix formalism [13] is a promising approach to bridge this gap. It allows the connection of scattering matrix elements to their effects on thermal observables. The matrix elements can be computed within theoretical models or be measured in scattering experiments, making the method ideal for tapping into the rich resources of the field of hadron physics for the study of heavy-ion collisions.

2. S-matrix approach

We shall focus our attention on the system of pions at finite temperature and vanishing chemical potentials. In the S-matrix approach, the interaction contribution to the thermodynamic pressure from $\pi\pi$ scatterings involves an integral over the invariant mass $M = \sqrt{s}$

$$
\begin{aligned}
\Delta P_{\mathrm{int}} &= \frac{T}{V} \ln Z \\
&\approx \sum_I d_{Ij}\, T \int\limits_{m_{\mathrm{th}}}^{\infty} \mathrm{d}M \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{\pi} \frac{\mathrm{d}\delta_j^I}{\mathrm{d}M} \\
&\quad \times \left\{ -\ln \left[1 - e^{-\beta\sqrt{p^2 + M^2}} \right] \right\} .
\end{aligned}
\tag{1}
$$

where δ_j^I is the scattering phase shift for a given isospin channel and d_{Ij} is the corresponding isospin–spin degeneracy factor.

The key input here is the two-body S-matrix elements, *i.e.* the scattering phase shifts. Extensive experimental [14–17] and theoretical [7,18,19] efforts are devoted to the study these quantities. Using a similar method detailed in Ref. [20], we obtain parametrizations of phase shifts that satisfy the stringent constraint on scattering lengths by the chiral perturbation theory. These and their corresponding contributions to pressure are summarized in Fig. 1.

3. Results

Let us now consider several key results in Fig. 1.

3.1. S-wave (I = 0,2) channel

We observe that the $I = 0$ phase shift does not reach $180°$ before $f_0(980)$ emerges. The slow rise of δ_0^0 in the low-mass region, together with the purely repulsive contribution from δ_0^2, severely suppress the overall pressure from

these channels, down to the value of a free gas of $f_0(980)$. This underpins the proposed prescription that σ-meson should not be included in the HRG particle sum [21, 22]. Similar conclusion applies to the κ-meson [20].
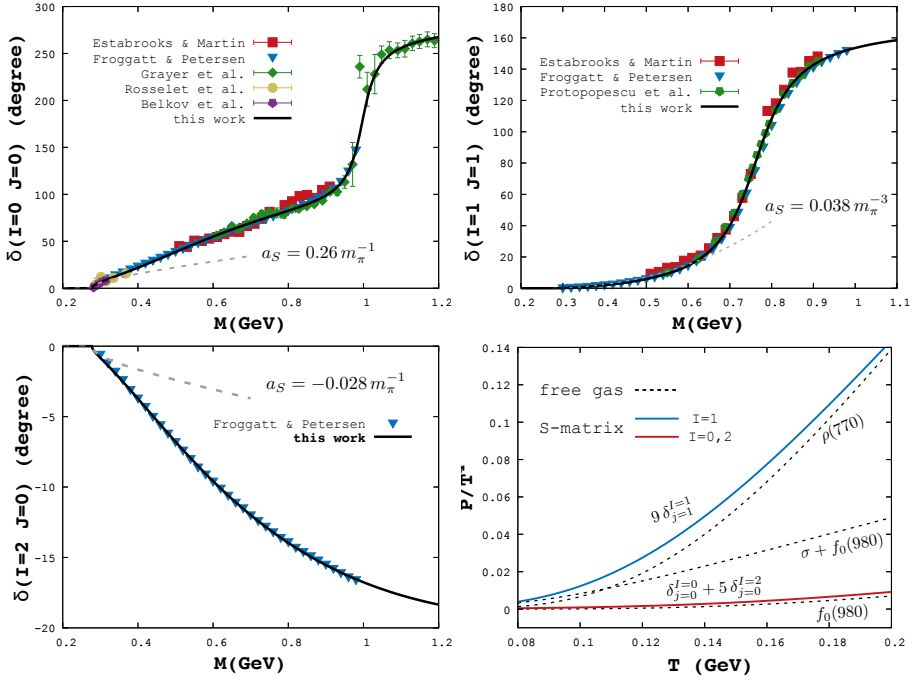

Fig. 1. (Color online) $\pi\pi$ scattering phase shifts ($I = 0, 1, 2$) and their contributions to thermodynamic pressure.

3.2. P-wave (I = 1) channel

In this channel, the S-matrix approach suggests an enhanced effect from the two-body scattering beyond the free gas result. This enhancement originates from a non-resonant threshold effect which is neglected in the HRG model [23, 24]. It may help to account for the excess of low-transverse-momentum pions measured at the LHC [25] over the conventional fluid-dynamical calculations [24].

3.3. Excluded volume effect

In all the channels studied, we observe non-ideal behaviors of the resonance gas, exemplified by the fact that the phase shifts do not act like a theta function at the corresponding pole masses.

The overall pressure computed by the S-matrix approach is less than that of a free gas of σ, $f_0(980)$, and $\rho(770)$. This may be subsumed into an excluded-volume effect [26–28] for a system of classical gas[1]. However, the two approaches are in general incompatible. As we have seen, the strong force between hadrons is highly intricate and heavily interaction-channel-dependent, and is unlikely to be captured by a single phenomenological parameter. The S-matrix approach presented here can nevertheless be used to estimate the magnitude of the effective eigenvolumes.

4. Summary

The S-matrix approach offers a consistent way to incorporate attractive and repulsive forces between hadrons. Using the input of empirical phase shifts from hadron scattering experiments, the important physics of resonance widths and the contribution from purely repulsive channels are naturally included.

Research in extending the method to include three-body scatterings and beyond has begun [29]. We expect interesting results when applying the approach to explore various phenomena of heavy-ion collisions.

REFERENCES

[1] P. Braun-Munzinger, K. Redlich, J. Stachel, in: *Quark–Gluon Plasma 3*, edited by R.C. Hwa *et al.*, World Scientific, Singapore 2004, pp. 491–599.

[2] F. Becattini, `arXiv:0901.3643 [hep-ph]`.

[3] P.M. Lo, M. Marczenko, K. Redlich, C. Sasaki, *Phys. Rev. C* **92**, 055206 (2015).

[4] B. Friman *et al.*, *Lect. Notes Phys.* **814**, 1 (2011).

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[1] Comparable results on pressure can be obtained for a classical gas of π, σ, $f_0(980)$, and $\rho(770)$, provided that one includes repulsion only between pion pairs [28] with an effective eigenvolume of radius $r_0 = 0.3$ fm.

[5] C. Patrignani *et al.* [Particle Data Group], *Chin. Phys. C* **40**, 100001 (2016).

[6] J. Gasser, H. Leutwyler, *Ann. Phys.* **158**, 142 (1984).

[7] J.A. Oller, E. Oset, *Nucl. Phys. A* **620**, 438 (1997) [*Erratum ibid.* **652**, 407 (1999)].

[8] M.R. Shepherd, J.J. Dudek, R.E. Mitchell, *Nature* **534**, 487 (2016).

[9] R. Rapp, J. Wambach, *Adv. Nucl. Phys.* **25**, 1 (2000).

[10] S. Godfrey, N. Isgur, *Phys. Rev. D* **32**, 189 (1985).

[11] T. Barnes, E.S. Swanson, *Phys. Rev. D* **46**, 131 (1992).

[12] R.F. Lebed, R.E. Mitchell, E.S. Swanson, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).

[13] R. Dashen, S.K. Ma, H.J. Bernstein, *Phys. Rev.* **187**, 345 (1969).

[14] P. Estabrooks, A.D. Martin, *Nucl. Phys. B* **79**, 301 (1974).

[15] S.D. Protopopescu *et al.*, *Phys. Rev. D* **7**, 1279 (1973).

[16] C.D. Froggatt, J.L. Petersen, *Nucl. Phys. B* **129**, 89 (1977).

[17] O. Dumbrajs *et al.*, *Nucl. Phys. B* **216**, 277 (1983).

[18] S. Ishida *et al.*, *Prog. Theor. Phys.* **98**, 1005 (1997).

[19] D.J. Wilson *et al.*, *Phys. Rev. D* **92**, 094502 (2015).

[20] B. Friman *et al.*, *Phys. Rev. D* **92**, 074003 (2015).

[21] R. Venugopalan, M. Prakash, *Nucl. Phys. A* **546**, 718 (1992).

[22] W. Broniowski, F. Giacosa, V. Begun, *Phys. Rev. C* **92**, 034905 (2015).

[23] W. Weinhold, B. Friman, W. Nörenberg, *Phys. Lett. B* **433**, 236 (1998).

[24] P. Huovinen *et al.*, *Phys. Lett. B* **769**, 509 (2017).

[25] B. Abelev *et al.* [ALICE Collaboration], *Phys. Rev. C* **88**, 044910 (2013).

[26] A. Andronic, P. Braun-Munzinger, J. Stachel, M. Winn, *Phys. Lett. B* **718**, 80 (2012).

[27] M. Albright, J. Kapusta, C. Young, *Phys. Rev. C* **90**, 024915 (2014).

[28] L.M. Satarov *et al.*, *Phys. Rev. C* **95**, 024902 (2017).

[29] P.M. Lo, in preparation.